

# Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

6-Hyperbolic-functions/6.5-Hyperbolic-secant/178-6.5.2-e-x-<sup>m</sup>-  
a+b-sech-c+d-x<sup>n</sup>-<sup>p</sup>

Nasser M. Abbasi

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# CHAPTER 1

## INTRODUCTION

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This report gives the result of running the computer algebra independent integration test. The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ 84 ]. This is test number [ 178 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 ( 84 )	0.00 ( 0 )
Mathematica	95.24 ( 80 )	4.76 ( 4 )
Fricas	73.81 ( 62 )	26.19 ( 22 )
Maple	59.52 ( 50 )	40.48 ( 34 )
Mupad	55.95 ( 47 )	44.05 ( 37 )
Giac	52.38 ( 44 )	47.62 ( 40 )
Maxima	46.43 ( 39 )	53.57 ( 45 )
Sympy	42.86 ( 36 )	57.14 ( 48 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

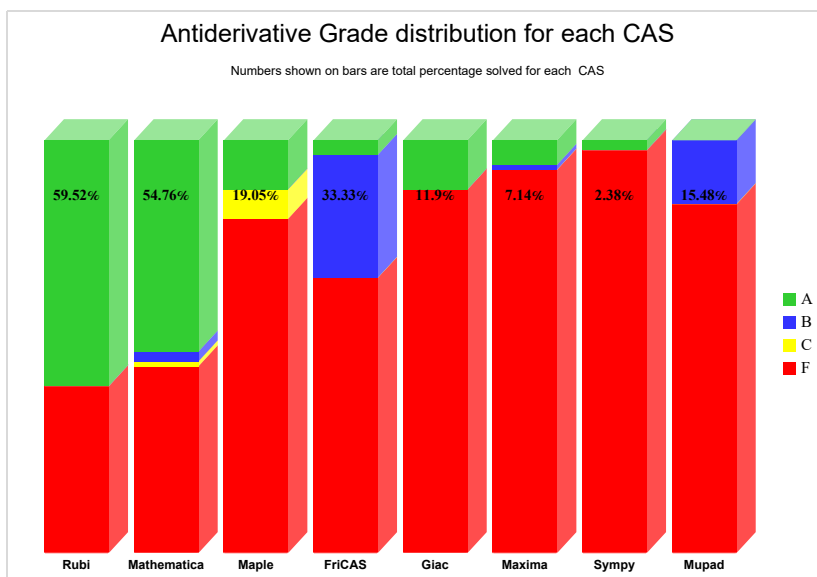
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

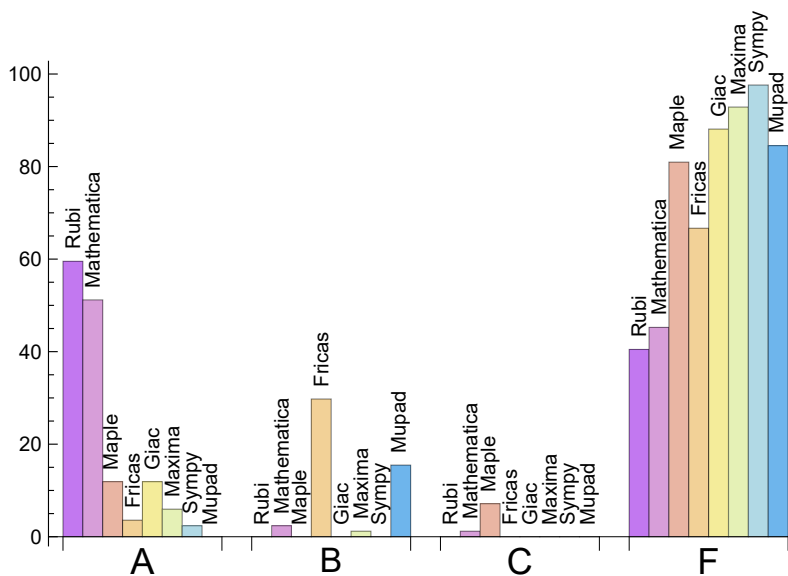
System	% A grade	% B grade	% C grade	% F grade
Rubi	59.524	0.000	0.000	40.476
Mathematica	51.190	2.381	1.190	45.238
Maple	11.905	0.000	7.143	80.952
Giac	11.905	0.000	0.000	88.095
Maxima	5.952	1.190	0.000	92.857
Fricas	3.571	29.762	0.000	66.667
Sympy	2.381	0.000	0.000	97.619
Mupad	0.000	15.476	0.000	84.524

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	22	90.91	9.09	0.00
Maple	34	100.00	0.00	0.00
Mupad	37	0.00	100.00	0.00
Giac	40	100.00	0.00	0.00
Maxima	45	57.78	2.22	40.00
Sympy	48	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.



System	Mean time (sec)
Fricas	0.27
Maxima	0.45
Giac	0.61
Rubi	0.66
Maple	0.74
Mupad	1.96
Sympy	5.19
Mathematica	13.70

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	18.67	0.96	17.00	0.94
Giac	30.77	1.03	20.00	1.11
Mupad	64.66	1.52	22.00	1.22
Maple	70.26	1.22	18.00	1.00
Maxima	114.21	5.74	59.00	3.00
Rubi	273.67	0.98	68.00	1.00
Mathematica	292.99	1.16	52.00	1.10
Fricas	623.32	3.81	44.00	2.15

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the  $y$  axis is the percentage solved which Rubi itself needed the number of rules given the  $x$  axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

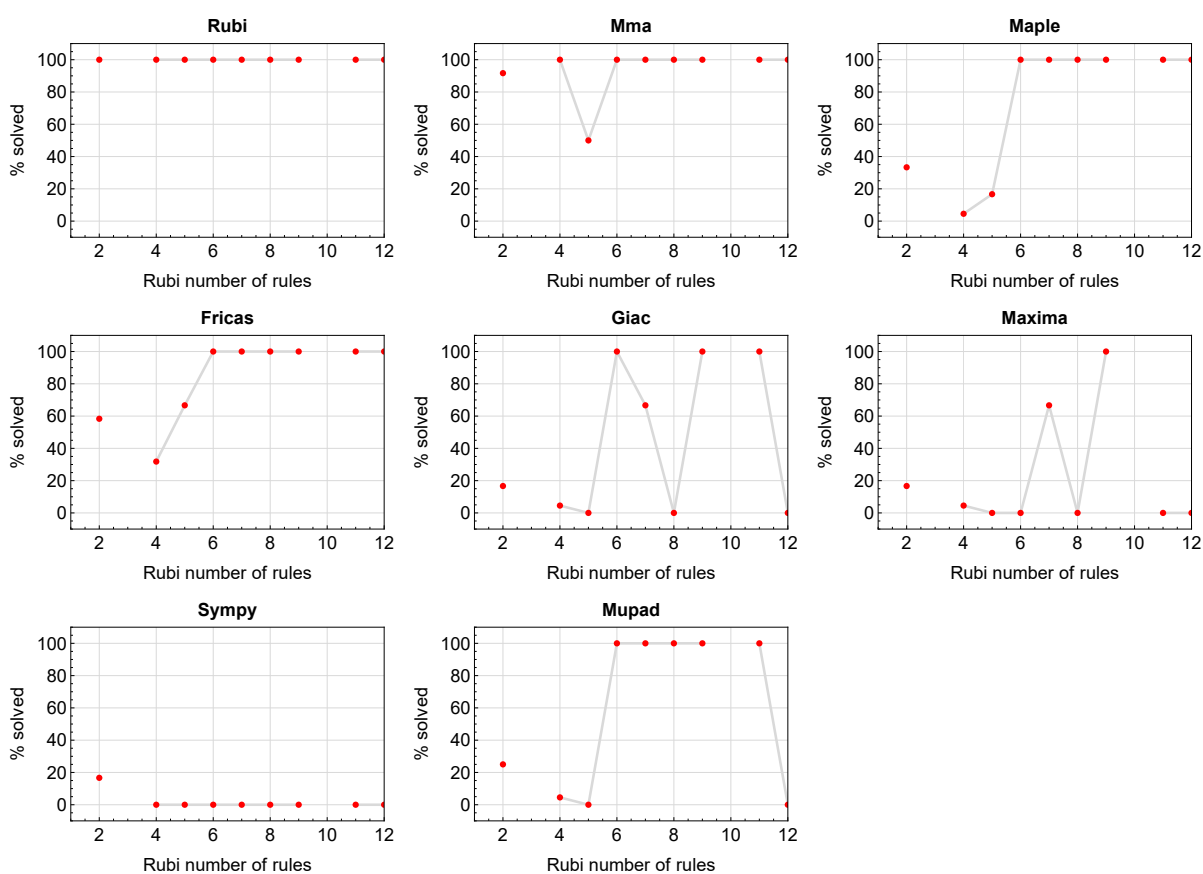


Figure 1.1: Solving statistics per number of Rubi rules used

# 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

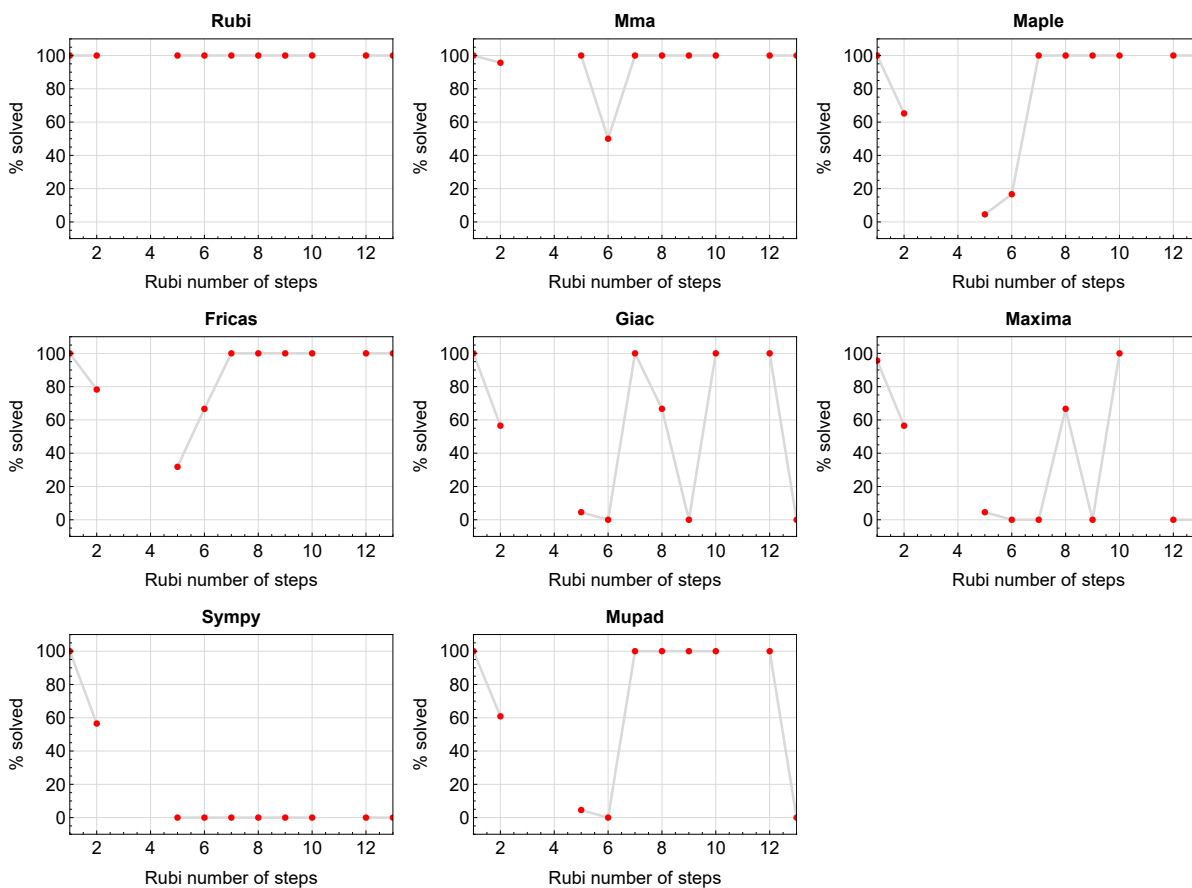


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the percentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

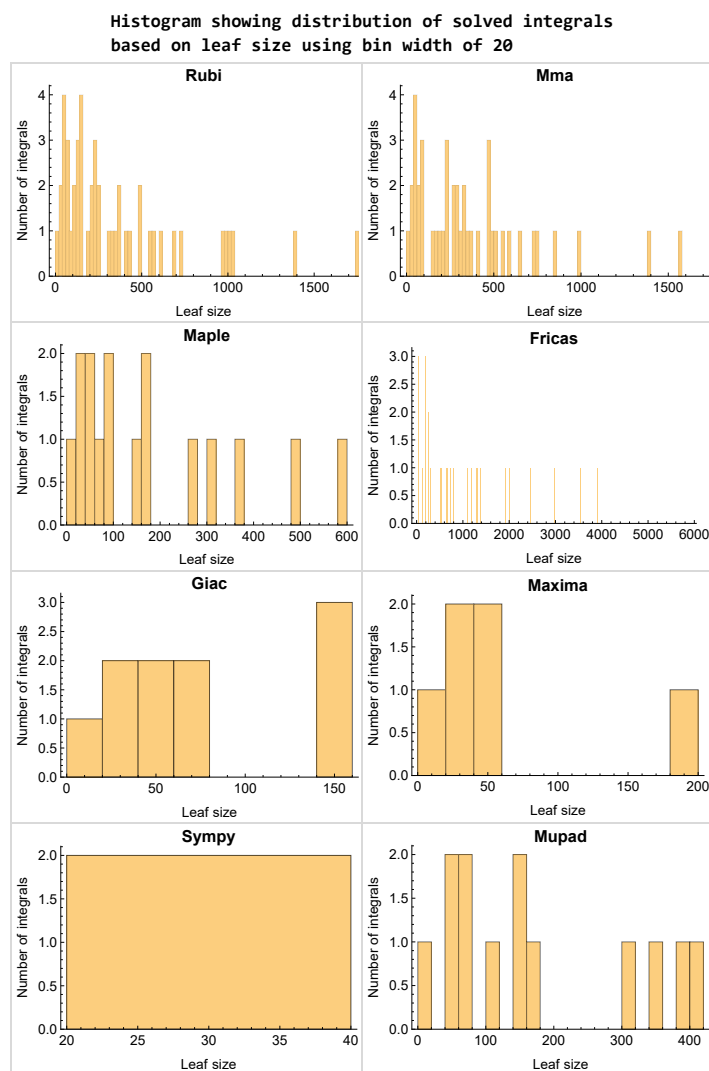


Figure 1.3: Solved integrals based on leaf size distribution

## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

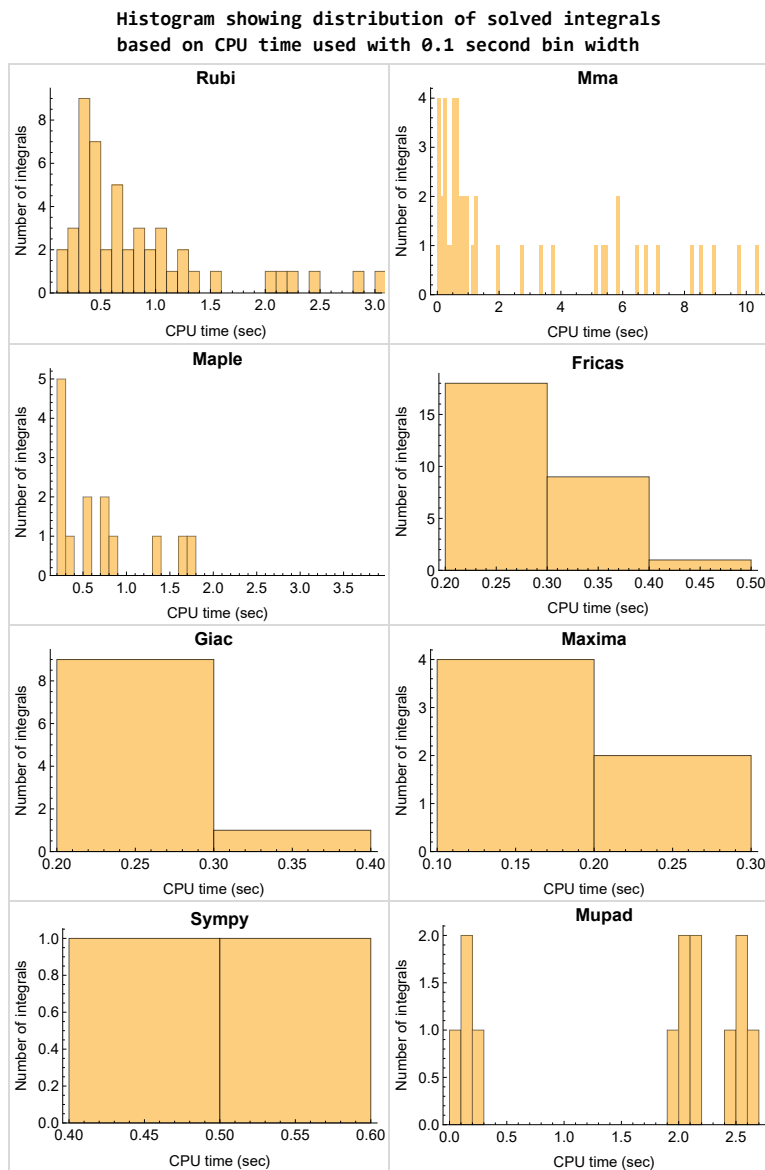


Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

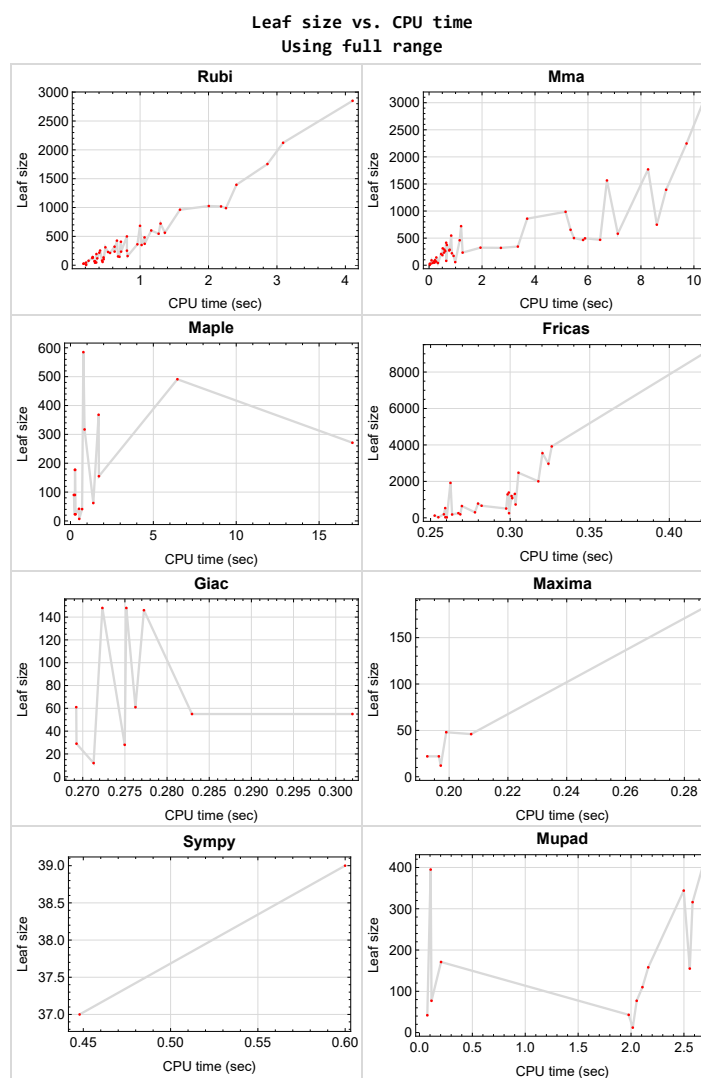


Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 35, 36, 40, 41, 45, 46, 50, 51, 55, 56, 60, 61, 65, 66, 70, 71, 72}

## 1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {39}

Maple {73, 74, 76, 79, 80, 82}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for **Rubi** and **Mathematica**.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.



## 1.14 Important notes about some of the results

### 1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'  
'display2d : false'  
'domain : complex'  
'keepfloat : true'  
'load(to_poly_solve)'  
'load(simplify_sum)'  
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib  
maxima_lib.set('extra_definite_integration_methods', '[]')  
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

### 1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

### 1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

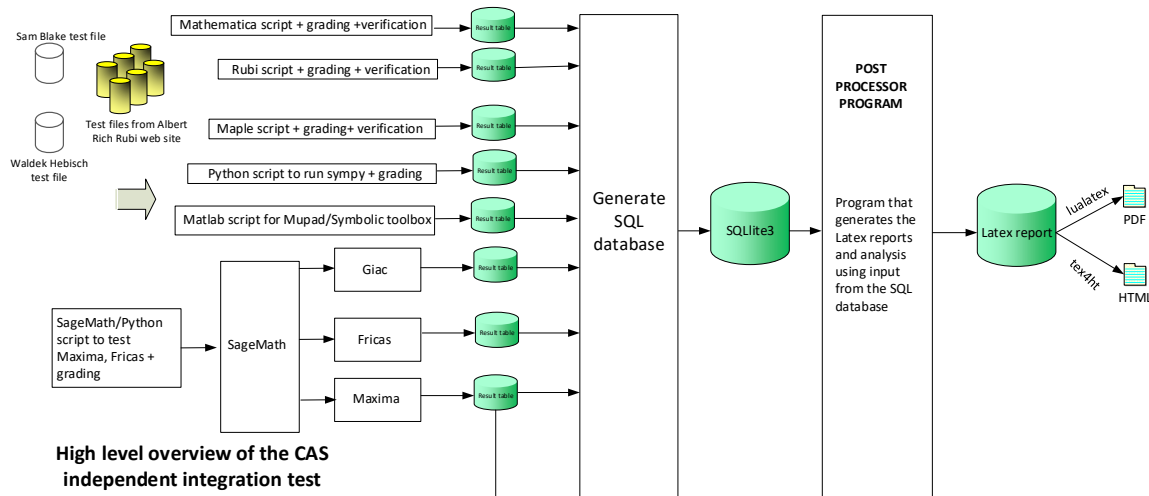
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)
```

Which gives  $\sin(x)^2/2$

# 1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



**High level overview of the CAS independent integration test build system**

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in Rubi Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

Nasser M. Abbasi  
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Design v0.6

# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

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## 2.1 List of integrals sorted by grade for each CAS

2.1.1	Rubi . . . . .	21
2.1.2	Mma . . . . .	21
2.1.3	Maple . . . . .	22
2.1.4	Fricas . . . . .	22
2.1.5	Maxima . . . . .	22
2.1.6	Giac . . . . .	23
2.1.7	Mupad . . . . .	23
2.1.8	Sympy . . . . .	23

### 2.1.1 Rubi

**A grade** { 1, 3, 5, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59, 62, 63, 64, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

**B grade** { }

**C grade** { }

**F normal fail** { }

**F(-1) timeout fail** { }

**F(-2) exception fail** { }

### 2.1.2 Mma

**A grade** { 1, 3, 5, 8, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 54, 57, 58, 59, 62, 63, 64, 67, 68, 69, 73, 74, 76, 79, 82, 83 }

**B grade** { 10, 77 }

**C grade** { 80 }

**F normal fail** { 75, 78, 81, 84 }

**F(-1) timeout fail** { }

**F(-2) exception fail** { }

### 2.1.3 Maple

**A grade** { 5, 12, 15, 20, 27, 31, 54, 59, 64, 69 }

**B grade** { }

**C grade** { 73, 74, 76, 79, 80, 82 }

**F normal fail** { 1, 3, 8, 10, 16, 18, 23, 25, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68, 75, 77, 78, 81, 83, 84 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.4 Fricas

**A grade** { 5, 20, 54 }

**B grade** { 1, 3, 8, 10, 12, 15, 16, 18, 23, 25, 27, 31, 59, 64, 69, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83 }

**C grade** { }

**F normal fail** { 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68 }

**F(-1) timedout fail** { 78, 84 }

**F(-2) exception fail** { }

### 2.1.5 Maxima

**A grade** { 5, 12, 31, 54, 59 }

**B grade** { 15 }

**C grade** { }

**F normal fail** { 1, 3, 8, 10, 32, 33, 34, 37, 38, 39, 52, 53, 57, 58, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

**F(-1) timedout fail** { 61 }

**F(-2) exception fail** { 16, 18, 20, 23, 25, 27, 42, 43, 44, 47, 48, 49, 62, 63, 64, 67, 68, 69 }

### 2.1.6 Giac

A grade { 5, 12, 15, 20, 27, 31, 54, 59, 64, 69 }

B grade { }

C grade { }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

F(-1) timeout fail { }

F(-2) exception fail { }

### 2.1.7 Mupad

A grade { }

B grade { 5, 12, 15, 20, 27, 31, 54, 59, 64, 69, 73, 76, 79 }

C grade { }

F normal fail { }

F(-1) timeout fail { 1, 3, 8, 10, 16, 18, 23, 25, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 62, 63, 67, 68, 74, 75, 77, 78, 80, 81, 82, 83, 84 }

F(-2) exception fail { }

### 2.1.8 Sympy

A grade { 5, 54 }

B grade { }

C grade { }

F normal fail { 1, 3, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 34, 37, 38, 39, 42, 43, 44, 47, 48, 49, 52, 53, 57, 58, 59, 62, 63, 64, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

F(-1) timeout fail { }

F(-2) exception fail { }



## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column N.S. means **normalized size** and is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	143	0	0	256	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.322	0.274	0.000	0.000	0.267	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	38	21	15	18	20
N.S.	1	1.00	1.12	1.00	2.38	1.31	0.94	1.12	1.25
time (sec)	N/A	0.182	5.382	0.047	0.312	0.252	1.179	0.301	1.978

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	92	0	0	184	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	2.39	0.00	0.00	0.00
time (sec)	N/A	0.260	0.223	0.000	0.000	0.264	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	38	21	15	18	20
N.S.	1	1.00	1.12	1.00	2.38	1.31	0.94	1.12	1.25
time (sec)	N/A	0.183	4.694	0.058	0.300	0.247	0.708	0.269	1.987

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	33	39	28	42
N.S.	1	1.00	1.00	0.88	0.85	1.27	1.50	1.08	1.62
time (sec)	N/A	0.189	0.027	0.306	0.197	0.255	0.600	0.275	0.074

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	36	18	14	18	20
N.S.	1	1.00	1.12	1.00	2.25	1.12	0.88	1.12	1.25
time (sec)	N/A	0.182	4.548	0.048	0.306	0.262	1.951	0.288	2.111

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	38	18	15	18	20
N.S.	1	1.00	1.12	1.00	2.38	1.12	0.94	1.12	1.25
time (sec)	N/A	0.184	4.508	0.048	0.303	0.245	0.617	0.329	2.117

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	213	320	0	0	1198	0	0	0
N.S.	1	0.98	1.47	0.00	0.00	5.52	0.00	0.00	0.00
time (sec)	N/A	0.602	2.712	0.000	0.000	0.301	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	79	42	17	20	22
N.S.	1	1.00	1.11	1.00	4.39	2.33	0.94	1.11	1.22
time (sec)	N/A	0.196	11.216	0.077	0.413	0.245	1.556	0.670	2.003

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	<b>F</b>	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	118	324	0	0	782	0	0	0
N.S.	1	0.99	2.72	0.00	0.00	6.57	0.00	0.00	0.00
time (sec)	N/A	0.396	1.942	0.000	0.000	0.280	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	72	42	17	20	22
N.S.	1	1.00	1.11	1.00	4.00	2.33	0.94	1.11	1.22
time (sec)	N/A	0.196	9.295	0.072	0.404	0.256	1.009	0.672	1.992

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	<b>F</b>	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	43	43	41	46	194	0	55	77
N.S.	1	0.98	0.98	0.93	1.05	4.41	0.00	1.25	1.75
time (sec)	N/A	0.362	0.139	0.701	0.207	0.258	0.000	0.302	0.114

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	86	36	15	20	22
N.S.	1	1.00	1.11	1.00	4.78	2.00	0.83	1.11	1.22
time (sec)	N/A	0.195	27.465	0.074	0.399	0.255	6.611	0.310	2.163

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	87	36	17	20	22
N.S.	1	1.00	1.11	1.00	4.83	2.00	0.94	1.11	1.22
time (sec)	N/A	0.194	10.618	0.074	0.434	0.268	1.008	0.818	2.152

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	<b>F</b>	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	104	90	62	182	1918	0	146	395
N.S.	1	1.16	1.00	0.69	2.02	21.31	0.00	1.62	4.39
time (sec)	N/A	0.503	0.094	1.377	0.287	0.263	0.000	0.277	0.106

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F(-2)</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	350	272	0	0	731	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	1.093	0.762	0.000	0.000	0.303	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	59	20	15	20	22
N.S.	1	1.00	1.11	1.00	3.28	1.11	0.83	1.11	1.22
time (sec)	N/A	0.198	3.845	0.053	0.302	0.265	0.580	0.301	1.975

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F(-2)</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	241	235	187	0	0	536	0	0	0
N.S.	1	0.98	0.78	0.00	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.762	0.503	0.000	0.000	0.259	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	59	20	15	20	22
N.S.	1	1.00	1.11	1.00	3.28	1.11	0.83	1.11	1.22
time (sec)	N/A	0.196	3.258	0.051	0.303	0.239	0.486	0.282	1.930

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	<b>F(-2)</b>	A	<b>F</b>	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	67	67	90	0	304	0	61	171
N.S.	1	1.02	1.02	1.36	0.00	4.61	0.00	0.92	2.59
time (sec)	N/A	0.366	0.267	0.204	0.000	0.278	0.000	0.269	0.204

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	58	19	15	20	22
N.S.	1	1.00	1.11	1.00	3.22	1.06	0.83	1.11	1.22
time (sec)	N/A	0.193	2.866	0.052	0.288	0.250	1.133	0.312	2.057

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	38	18	15	18	20
N.S.	1	1.00	1.12	1.00	2.38	1.12	0.94	1.12	1.25
time (sec)	N/A	0.180	0.086	0.010	0.296	0.246	0.703	0.305	0.002

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F(-2)</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	994	990	1565	0	0	3918	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	3.94	0.00	0.00	0.00
time (sec)	N/A	2.390	6.717	0.000	0.000	0.326	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	312	38	17	20	22
N.S.	1	1.00	1.11	1.00	17.33	2.11	0.94	1.11	1.22
time (sec)	N/A	0.190	20.983	0.057	0.418	0.266	1.002	0.402	2.063

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F(-2)</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	555	546	654	0	0	2473	0	0	0
N.S.	1	0.98	1.18	0.00	0.00	4.46	0.00	0.00	0.00
time (sec)	N/A	1.377	5.344	0.000	0.000	0.305	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	300	38	17	20	22
N.S.	1	1.00	1.11	1.00	16.67	2.11	0.94	1.11	1.22
time (sec)	N/A	0.199	19.202	0.054	0.432	0.254	0.808	0.401	2.044

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	<b>F(-2)</b>	B	<b>F</b>	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	146	220	177	0	1314	0	148	316
N.S.	1	1.19	1.79	1.44	0.00	10.68	0.00	1.20	2.57
time (sec)	N/A	0.739	0.862	0.275	0.000	0.303	0.000	0.275	2.582

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	249	38	17	20	22
N.S.	1	1.00	1.11	1.00	13.83	2.11	0.94	1.11	1.22
time (sec)	N/A	0.195	53.753	0.061	0.417	0.250	1.677	0.925	2.547

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	318	44	19	20	22
N.S.	1	1.00	1.11	1.00	17.67	2.44	1.06	1.11	1.22
time (sec)	N/A	0.206	26.111	0.057	0.422	0.266	1.442	0.374	2.439

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	319	44	19	3	22
N.S.	1	1.00	1.11	1.00	17.72	2.44	1.06	0.17	1.22
time (sec)	N/A	0.198	28.548	0.058	0.412	0.254	1.460	3.003	2.637

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	<b>F</b>	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	12	28	0	12	12
N.S.	1	1.00	1.00	1.17	2.00	4.67	0.00	2.00	2.00
time (sec)	N/A	0.226	0.027	0.528	0.197	0.260	0.000	0.271	2.016



Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	426	426	415	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.709	0.646	0.000	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	310	311	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.536	0.516	0.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	207	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.386	0.456	0.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	36	18	15	18	20
N.S.	1	1.00	1.11	0.89	2.00	1.00	0.83	1.00	1.11
time (sec)	N/A	0.181	11.138	0.128	0.428	0.268	2.607	0.283	2.150

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	42	18	17	18	20
N.S.	1	1.00	1.11	0.89	2.33	1.00	0.94	1.00	1.11
time (sec)	N/A	0.181	10.482	0.144	0.429	0.252	1.485	0.301	2.200

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	677	683	748	0	0	0	0	0	0
N.S.	1	1.01	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.058	8.600	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	497	499	582	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.869	7.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	<b>No</b>	N/A	TBD	TBD	TBD	TBD	TBD
size	319	321	466	0	0	0	0	0	0
N.S.	1	1.01	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.689	5.815	0.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	86	36	17	20	22
N.S.	1	1.00	1.10	0.90	4.30	1.80	0.85	1.00	1.10
time (sec)	N/A	0.193	98.183	0.254	0.574	0.260	13.810	0.380	2.418

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	110	36	19	20	22
N.S.	1	1.00	1.10	0.90	5.50	1.80	0.95	1.00	1.10
time (sec)	N/A	0.194	37.837	0.261	0.570	0.282	2.464	0.491	2.375

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F(-2)</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	961	962	721	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.673	1.209	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F(-2)</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	721	722	547	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.393	0.835	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	481	482	373	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.099	0.673	0.000	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	58	19	17	20	22
N.S.	1	1.00	1.10	0.90	2.90	0.95	0.85	1.00	1.10
time (sec)	N/A	0.198	5.432	0.182	0.509	0.251	2.400	0.363	2.041

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	42	18	17	18	20
N.S.	1	1.00	1.11	0.89	2.33	1.00	0.94	1.00	1.11
time (sec)	N/A	0.183	0.105	0.009	0.443	0.261	1.464	0.324	0.002

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2851	2850	3035	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.383	10.368	0.000	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2123	2122	2247	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.306	9.722	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1395	1394	1393	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.586	8.951	0.000	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	253	38	19	20	22
N.S.	1	1.00	1.10	0.90	12.65	1.90	0.95	1.00	1.10
time (sec)	N/A	0.182	179.185	0.178	0.977	0.269	4.715	1.375	2.437

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	324	44	20	3	22
N.S.	1	1.00	1.10	0.90	16.20	2.20	1.00	0.15	1.10
time (sec)	N/A	0.189	91.963	0.181	1.158	0.274	9.896	3.100	2.477

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	254	254	288	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.446	0.576	0.000	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	172	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	0.926	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	33	37	29	43
N.S.	1	1.00	1.00	0.88	0.85	1.27	1.42	1.12	1.65
time (sec)	N/A	0.181	0.058	0.264	0.193	0.259	0.448	0.269	1.978

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	42	25	19	18	20
N.S.	1	1.00	1.10	0.80	2.10	1.25	0.95	0.90	1.00
time (sec)	N/A	0.177	13.992	0.132	0.442	0.264	1.083	0.294	2.195

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	42	25	19	18	20
N.S.	1	1.00	1.10	0.80	2.10	1.25	0.95	0.90	1.00
time (sec)	N/A	0.179	15.408	0.128	0.447	0.247	4.377	0.304	2.279

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	407	407	496	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.778	5.887	0.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	344	0	0	0	0	0	0
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.576	3.352	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	<b>F</b>	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	42	48	194	0	55	77
N.S.	1	1.00	1.00	0.89	1.02	4.13	0.00	1.17	1.64
time (sec)	N/A	0.371	0.205	0.516	0.199	0.269	0.000	0.283	2.054

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	106	46	20	20	22
N.S.	1	1.00	1.09	0.82	4.82	2.09	0.91	0.91	1.00
time (sec)	N/A	0.191	36.169	0.258	0.551	0.259	2.050	0.381	2.375

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	<b>F(-1)</b>	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	46	20	20	22
N.S.	1	1.00	1.09	0.82	0.00	2.09	0.91	0.91	1.00
time (sec)	N/A	0.196	34.808	0.248	0.000	0.274	5.657	0.436	2.410

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F(-2)</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	601	602	460	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.254	1.156	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F(-2)</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	361	362	286	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.007	0.778	0.000	0.000	0.000	0.000	0.000	0.000



Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	<b>F(-2)</b>	B	<b>F</b>	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	69	69	90	0	254	0	61	155
N.S.	1	1.01	1.01	1.32	0.00	3.74	0.00	0.90	2.28
time (sec)	N/A	0.350	0.187	0.268	0.000	0.299	0.000	0.276	2.555

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	66	27	20	20	22
N.S.	1	1.00	1.09	0.82	3.00	1.23	0.91	0.91	1.00
time (sec)	N/A	0.193	10.044	0.169	0.493	0.252	2.189	0.343	2.126

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	66	27	20	20	22
N.S.	1	1.00	1.09	0.82	3.00	1.23	0.91	0.91	1.00
time (sec)	N/A	0.200	10.413	0.166	0.550	0.267	6.603	0.428	2.168

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F(-2)</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1755	1754	1769	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.030	8.270	0.000	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1027	1026	986	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.150	5.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	152	232	177	0	1387	0	148	344
N.S.	1	1.20	1.83	1.39	0.00	10.92	0.00	1.17	2.71
time (sec)	N/A	0.721	0.562	0.273	0.000	0.299	0.000	0.272	2.498

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	317	48	22	20	22
N.S.	1	1.00	1.09	0.82	14.41	2.18	1.00	0.91	1.00
time (sec)	N/A	0.186	81.808	0.176	1.022	0.282	5.812	1.384	2.450

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	324	48	22	3	22
N.S.	1	1.00	1.09	0.82	14.73	2.18	1.00	0.14	1.00
time (sec)	N/A	0.185	81.059	0.146	1.308	0.286	50.987	3.093	2.507

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	24
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.20
time (sec)	N/A	0.264	20.308	0.168	0.329	0.257	44.279	0.782	2.119

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	<b>F</b>	B	<b>F</b>	<b>F</b>	B
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD
size	44	44	41	155	0	122	0	0	110
N.S.	1	1.00	0.93	3.52	0.00	2.77	0.00	0.00	2.50
time (sec)	N/A	0.222	0.335	1.716	0.000	0.253	0.000	0.000	2.108

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD
size	135	135	260	368	0	664	0	0	0
N.S.	1	1.00	1.93	2.73	0.00	4.92	0.00	0.00	0.00
time (sec)	N/A	0.311	0.613	1.698	0.000	0.282	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	<b>F</b>	<b>F</b>	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	1082	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	4.99	0.00	0.00	0.00
time (sec)	N/A	0.412	0.000	0.000	0.000	0.301	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	<b>F</b>	B	<b>F</b>	<b>F</b>	B
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD
size	79	56	57	271	0	646	0	0	158
N.S.	1	0.71	0.72	3.43	0.00	8.18	0.00	0.00	2.00
time (sec)	N/A	0.479	0.988	17.010	0.000	0.270	0.000	0.000	2.163

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	<b>F</b>	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	208	135	501	0	0	2972	0	0	0
N.S.	1	0.65	2.41	0.00	0.00	14.29	0.00	0.00	0.00
time (sec)	N/A	0.488	5.470	0.000	0.000	0.324	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	363	236	0	0	0	0	0	0	0
N.S.	1	0.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.664	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	<b>F</b>	B	<b>F</b>	<b>F</b>	B
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD
size	87	80	80	317	0	511	0	0	409
N.S.	1	0.92	0.92	3.64	0.00	5.87	0.00	0.00	4.70
time (sec)	N/A	0.459	0.648	0.850	0.000	0.298	0.000	0.000	2.685

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD
size	307	252	859	585	0	1286	0	0	0
N.S.	1	0.82	2.80	1.91	0.00	4.19	0.00	0.00	0.00
time (sec)	N/A	0.852	3.704	0.778	0.000	0.298	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	<b>F</b>	<b>F</b>	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	452	371	0	0	0	2005	0	0	0
N.S.	1	0.82	0.00	0.00	0.00	4.44	0.00	0.00	0.00
time (sec)	N/A	1.146	0.000	0.000	0.000	0.318	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD
size	157	159	233	491	0	3547	0	0	0
N.S.	1	1.01	1.48	3.13	0.00	22.59	0.00	0.00	0.00
time (sec)	N/A	0.833	1.267	6.451	0.000	0.320	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	717	563	469	0	0	9020	0	0	0
N.S.	1	0.79	0.65	0.00	0.00	12.58	0.00	0.00	0.00
time (sec)	N/A	1.429	6.458	0.000	0.000	0.422	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1284	1019	0	0	0	0	0	0	0
N.S.	1	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.275	0.000	0.000	0.000	0.000	0.000	0.000	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [15] had the largest ratio of [.7500000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	16	0.125
2	N/A	2	0	1.00	16	0.000
3	A	2	2	1.00	16	0.125
4	N/A	2	0	1.00	16	0.000
5	A	2	2	1.00	14	0.143
6	N/A	2	0	1.00	16	0.000
7	N/A	2	0	1.00	16	0.000
8	A	5	4	0.98	18	0.222
9	N/A	1	0	1.00	18	0.000
10	A	5	4	0.99	18	0.222
11	N/A	1	0	1.00	18	0.000
12	A	8	7	0.98	16	0.438
13	N/A	1	0	1.00	18	0.000
14	N/A	1	0	1.00	18	0.000
15	A	10	9	1.16	12	0.750
16	A	5	4	1.00	18	0.222
17	N/A	1	0	1.00	18	0.000
18	A	5	4	0.98	18	0.222
19	N/A	1	0	1.00	18	0.000
20	A	7	6	1.02	16	0.375
21	N/A	1	0	1.00	18	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	N/A	2	0	1.00	16	0.000
23	A	5	4	1.00	18	0.222
24	N/A	1	0	1.00	18	0.000
25	A	5	4	0.98	18	0.222
26	N/A	1	0	1.00	18	0.000
27	A	12	11	1.19	16	0.688
28	N/A	1	0	1.00	18	0.000
29	N/A	1	0	1.00	18	0.000
30	N/A	1	0	1.00	18	0.000
31	A	5	4	1.00	10	0.400
32	A	2	2	1.00	18	0.111
33	A	2	2	1.00	18	0.111
34	A	2	2	1.00	16	0.125
35	N/A	2	0	1.00	18	0.000
36	N/A	2	0	1.00	18	0.000
37	A	5	4	1.01	20	0.200
38	A	5	4	1.00	20	0.200
39	A	5	4	1.01	18	0.222
40	N/A	1	0	1.00	20	0.000
41	N/A	1	0	1.00	20	0.000
42	A	5	4	1.00	20	0.200
43	A	5	4	1.00	20	0.200
44	A	5	4	1.00	18	0.222
45	N/A	1	0	1.00	20	0.000
46	N/A	2	0	1.00	18	0.000
47	A	5	4	1.00	20	0.200
48	A	5	4	1.00	20	0.200
49	A	5	4	1.00	18	0.222
50	N/A	1	0	1.00	20	0.000
51	N/A	1	0	1.00	20	0.000
52	A	2	2	1.00	20	0.100
53	A	2	2	1.00	20	0.100

Continued on next page



Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	20	0.100
55	N/A	2	0	1.00	20	0.000
56	N/A	2	0	1.00	20	0.000
57	A	5	4	1.00	22	0.182
58	A	5	4	1.00	22	0.182
59	A	8	7	1.00	22	0.318
60	N/A	1	0	1.00	22	0.000
61	N/A	1	0	1.00	22	0.000
62	A	5	4	1.00	22	0.182
63	A	5	4	1.00	22	0.182
64	A	7	6	1.01	22	0.273
65	N/A	1	0	1.00	22	0.000
66	N/A	1	0	1.00	22	0.000
67	A	5	4	1.00	22	0.182
68	A	5	4	1.00	22	0.182
69	A	12	11	1.20	22	0.500
70	N/A	1	0	1.00	22	0.000
71	N/A	1	0	1.00	22	0.000
72	N/A	2	0	1.00	20	0.000
73	A	2	2	1.00	20	0.100
74	A	2	2	1.00	22	0.091
75	A	2	2	1.00	22	0.091
76	A	9	8	0.71	22	0.364
77	A	6	5	0.65	24	0.208
78	A	6	5	0.65	24	0.208
79	A	8	7	0.92	22	0.318
80	A	6	5	0.82	24	0.208
81	A	6	5	0.82	24	0.208
82	A	13	12	1.01	22	0.545
83	A	6	5	0.79	24	0.208
84	A	6	5	0.79	24	0.208

# CHAPTER 3

## LISTING OF INTEGRALS

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3.11	$\int x^2(a + b\operatorname{sech}(c + dx^2))^2 dx$	98
3.12	$\int x(a + b\operatorname{sech}(c + dx^2))^2 dx$	102
3.13	$\int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x} dx$	108
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3.15	$\int x\operatorname{sech}^7(a + bx^2) dx$	116
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3.20	$\int \frac{x}{a+b\operatorname{sech}(c+dx^2)} dx$	142
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3.25	$\int \frac{x^3}{(a+b\operatorname{sech}(c+dx^2))^2} dx$	168
3.26	$\int \frac{x^2}{(a+b\operatorname{sech}(c+dx^2))^2} dx$	175
3.27	$\int \frac{x}{(a+b\operatorname{sech}(c+dx^2))^2} dx$	179
3.28	$\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))^2} dx$	187
3.29	$\int \frac{1}{x^2(a+b\operatorname{sech}(c+dx^2))^2} dx$	191
3.30	$\int \frac{1}{x^3(a+b\operatorname{sech}(c+dx^2))^2} dx$	195
3.31	$\int \frac{\operatorname{sech}^2(\frac{1}{x})}{x^2} dx$	199
3.32	$\int x^3(a+b\operatorname{sech}(c+d\sqrt{x})) dx$	204
3.33	$\int x^2(a+b\operatorname{sech}(c+d\sqrt{x})) dx$	211
3.34	$\int x(a+b\operatorname{sech}(c+d\sqrt{x})) dx$	216
3.35	$\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x} dx$	221
3.36	$\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^2} dx$	225
3.37	$\int x^3(a+b\operatorname{sech}(c+d\sqrt{x}))^2 dx$	229
3.38	$\int x^2(a+b\operatorname{sech}(c+d\sqrt{x}))^2 dx$	237
3.39	$\int x(a+b\operatorname{sech}(c+d\sqrt{x}))^2 dx$	244
3.40	$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x} dx$	250
3.41	$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^2} dx$	254
3.42	$\int \frac{x^3}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$	258
3.43	$\int \frac{x^2}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$	266
3.44	$\int \frac{x}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$	273
3.45	$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$	279
3.46	$\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^2} dx$	283
3.47	$\int \frac{x^3}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$	287
3.48	$\int \frac{x^2}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$	293
3.49	$\int \frac{x}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$	299
3.50	$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$	307
3.51	$\int \frac{1}{x^2(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$	311
3.52	$\int x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x})) dx$	315
3.53	$\int \sqrt{x}(a+b\operatorname{sech}(c+d\sqrt{x})) dx$	320
3.54	$\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{\sqrt{x}} dx$	324

3.55	$\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^{3/2}} dx$	328
3.56	$\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^{5/2}} dx$	332
3.57	$\int x^{3/2} (a + b\operatorname{sech}(c + d\sqrt{x}))^2 dx$	336
3.58	$\int \sqrt{x} (a + b\operatorname{sech}(c + d\sqrt{x}))^2 dx$	342
3.59	$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{\sqrt{x}} dx$	347
3.60	$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{3/2}} dx$	353
3.61	$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}} dx$	357
3.62	$\int \frac{x^{3/2}}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$	361
3.63	$\int \frac{\sqrt{x}}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$	367
3.64	$\int \frac{1}{\sqrt{x}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$	372
3.65	$\int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$	378
3.66	$\int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$	382
3.67	$\int \frac{x^{3/2}}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$	386
3.68	$\int \frac{\sqrt{x}}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$	392
3.69	$\int \frac{1}{\sqrt{x}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$	400
3.70	$\int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$	408
3.71	$\int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$	412
3.72	$\int (ex)^m (a + b\operatorname{sech}(c + dx^n))^p dx$	416
3.73	$\int (ex)^{-1+n} (a + b\operatorname{sech}(c + dx^n)) dx$	420
3.74	$\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n)) dx$	425
3.75	$\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n)) dx$	430
3.76	$\int (ex)^{-1+n} (a + b\operatorname{sech}(c + dx^n))^2 dx$	435
3.77	$\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n))^2 dx$	441
3.78	$\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx$	447
3.79	$\int \frac{(ex)^{-1+n}}{a+b\operatorname{sech}(c+dx^n)} dx$	452
3.80	$\int \frac{(ex)^{-1+2n}}{a+b\operatorname{sech}(c+dx^n)} dx$	458
3.81	$\int \frac{(ex)^{-1+3n}}{a+b\operatorname{sech}(c+dx^n)} dx$	465
3.82	$\int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$	471
3.83	$\int \frac{(ex)^{-1+2n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$	479
3.84	$\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$	486

### 3.1 $\int x^5(a + b\operatorname{sech}(c + dx^2)) dx$

3.1.1	Optimal result . . . . .	52
3.1.2	Mathematica [A] (verified) . . . . .	52
3.1.3	Rubi [A] (verified) . . . . .	53
3.1.4	Maple [F] . . . . .	54
3.1.5	Fricas [B] (verification not implemented) . . . . .	54
3.1.6	Sympy [F] . . . . .	55
3.1.7	Maxima [F] . . . . .	55
3.1.8	Giac [F] . . . . .	55
3.1.9	Mupad [F(-1)] . . . . .	56

#### 3.1.1 Optimal result

Integrand size = 16, antiderivative size = 125

$$\int x^5(a + b\operatorname{sech}(c + dx^2)) dx = \frac{ax^6}{6} + \frac{bx^4 \arctan(e^{c+dx^2})}{d} - \frac{ibx^2 \operatorname{PolyLog}(2, -ie^{c+dx^2})}{d^2} + \frac{ibx^2 \operatorname{PolyLog}(2, ie^{c+dx^2})}{d^2} + \frac{ib \operatorname{PolyLog}(3, -ie^{c+dx^2})}{d^3} - \frac{ib \operatorname{PolyLog}(3, ie^{c+dx^2})}{d^3}$$

```
output 1/6*a*x^6+b*x^4*arctan(exp(d*x^2+c))/d-I*b*x^2*polylog(2,-I*exp(d*x^2+c))/d^2+I*b*x^2*polylog(2,I*exp(d*x^2+c))/d^2+I*b*polylog(3,-I*exp(d*x^2+c))/d^3-I*b*polylog(3,I*exp(d*x^2+c))/d^3
```

#### 3.1.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.14

$$\int x^5(a + b\operatorname{sech}(c + dx^2)) dx = \frac{ax^6}{6} + \frac{ib(d^2x^4 \log(1 - ie^{c+dx^2}) - d^2x^4 \log(1 + ie^{c+dx^2}) - 2dx^2 \operatorname{PolyLog}(2, -ie^{c+dx^2}) + 2dx^2 \operatorname{PolyLog}(2, ie^{c+dx^2}))}{2d^3}$$

input `Integrate[x^5*(a + b*Sech[c + d*x^2]),x]`

output `(a*x^6)/6 + ((I/2)*b*(d^2*x^4*Log[1 - I*E^(c + d*x^2)] - d^2*x^4*Log[1 + I*E^(c + d*x^2)] - 2*d*x^2*PolyLog[2, (-I)*E^(c + d*x^2)] + 2*d*x^2*PolyLog[2, I*E^(c + d*x^2)] + 2*PolyLog[3, (-I)*E^(c + d*x^2)] - 2*PolyLog[3, I*E^(c + d*x^2)]))/d^3`

### 3.1.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a + b\operatorname{sech}(c + dx^2)) dx$$

↓ 2010

$$\int (ax^5 + bx^5\operatorname{sech}(c + dx^2)) dx$$

↓ 2009

$$\frac{ax^6}{6} + \frac{bx^4 \arctan(e^{c+dx^2})}{d} + \frac{ib \operatorname{PolyLog}(3, -ie^{dx^2+c})}{d^3} - \frac{ib \operatorname{PolyLog}(3, ie^{dx^2+c})}{d^3} - \frac{ibx^2 \operatorname{PolyLog}(2, -ie^{dx^2+c})}{d^2} + \frac{ibx^2 \operatorname{PolyLog}(2, ie^{dx^2+c})}{d^2}$$

input `Int[x^5*(a + b*Sech[c + d*x^2]),x]`

output `(a*x^6)/6 + (b*x^4*ArcTan[E^(c + d*x^2)])/d - (I*b*x^2*PolyLog[2, (-I)*E^(c + d*x^2)])/d^2 + (I*b*x^2*PolyLog[2, I*E^(c + d*x^2)])/d^2 + (I*b*PolyLog[3, (-I)*E^(c + d*x^2)])/d^3 - (I*b*PolyLog[3, I*E^(c + d*x^2)])/d^3`

### 3.1.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

### 3.1.4 Maple [F]

$$\int x^5 (a + b \operatorname{sech}(dx^2 + c)) dx$$

input `int(x^5*(a+b*sech(d*x^2+c)),x)`

output `int(x^5*(a+b*sech(d*x^2+c)),x)`

### 3.1.5 Fracas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs.  $2(100) = 200$ .

Time = 0.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.05

$$\int x^5 (a + b \operatorname{sech}(c + dx^2)) dx$$

$$= \frac{ad^3x^6 + 6i bdx^2 \operatorname{Li}_2(i \cosh(dx^2 + c) + i \sinh(dx^2 + c)) - 6i bdx^2 \operatorname{Li}_2(-i \cosh(dx^2 + c) - i \sinh(dx^2 + c))}{1}$$

input `integrate(x^5*(a+b*sech(d*x^2+c)),x, algorithm="fricas")`

output `1/6*(a*d^3*x^6 + 6*I*b*d*x^2*dilog(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c)) - 6*I*b*d*x^2*dilog(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)) + 3*I*b*c^2*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + I) - 3*I*b*c^2*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) - I) - 3*(I*b*d^2*x^4 - I*b*c^2)*log(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c) + 1) - 3*(-I*b*d^2*x^4 + I*b*c^2)*log(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c) + 1) - 6*I*b*polylog(3, I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c)) + 6*I*b*polylog(3, -I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)))/d^3`

---

3.1.  $\int x^5 (a + b \operatorname{sech}(c + dx^2)) dx$

**3.1.6 Sympy [F]**

$$\int x^5(a + b\operatorname{sech}(c + dx^2)) dx = \int x^5(a + b\operatorname{sech}(c + dx^2)) dx$$

input `integrate(x**5*(a+b*sech(d*x**2+c)),x)`

output `Integral(x**5*(a + b*sech(c + d*x**2)), x)`

**3.1.7 Maxima [F]**

$$\int x^5(a + b\operatorname{sech}(c + dx^2)) dx = \int (b\operatorname{sech}(dx^2 + c) + a)x^5 dx$$

input `integrate(x^5*(a+b*sech(d*x^2+c)),x, algorithm="maxima")`

output `1/6*a*x^6 + 2*b*integrate(x^5/(e^(d*x^2 + c) + e^(-d*x^2 - c)), x)`

**3.1.8 Giac [F]**

$$\int x^5(a + b\operatorname{sech}(c + dx^2)) dx = \int (b\operatorname{sech}(dx^2 + c) + a)x^5 dx$$

input `integrate(x^5*(a+b*sech(d*x^2+c)),x, algorithm="giac")`

output `integrate((b*sech(d*x^2 + c) + a)*x^5, x)`



**3.1.9 Mupad [F(-1)]**

Timed out.

$$\int x^5 (a + b \operatorname{sech}(c + dx^2)) dx = \int x^5 \left( a + \frac{b}{\cosh(dx^2 + c)} \right) dx$$

input `int(x^5*(a + b/cosh(c + d*x^2)),x)`output `int(x^5*(a + b/cosh(c + d*x^2)), x)`

## 3.2 $\int x^4(a + b\operatorname{sech}(c + dx^2)) dx$

3.2.1	Optimal result	57
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3.2.3	Rubi [N/A]	58
3.2.4	Maple [N/A] (verified)	59
3.2.5	Fricas [N/A]	59
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3.2.7	Maxima [N/A]	60
3.2.8	Giac [N/A]	60
3.2.9	Mupad [N/A]	60

### 3.2.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^4(a + b\operatorname{sech}(c + dx^2)) dx = \frac{ax^5}{5} + b\operatorname{Int}(x^4\operatorname{sech}(c + dx^2), x)$$

output `1/5*a*x^5+b*Unintegrable(x^4*sech(d*x^2+c),x)`

### 3.2.2 Mathematica [N/A]

Not integrable

Time = 5.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4(a + b\operatorname{sech}(c + dx^2)) dx = \int x^4(a + b\operatorname{sech}(c + dx^2)) dx$$

input `Integrate[x^4*(a + b*Sech[c + d*x^2]),x]`

output `Integrate[x^4*(a + b*Sech[c + d*x^2]), x]`

### 3.2.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a + b\operatorname{sech}(c + dx^2)) dx$$

$$\downarrow \text{2010}$$

$$\int (ax^4 + bx^4\operatorname{sech}(c + dx^2)) dx$$

$$\downarrow \text{2009}$$

$$b \int x^4\operatorname{sech}(dx^2 + c) dx + \frac{ax^5}{5}$$

input `Int[x^4*(a + b*Sech[c + d*x^2]),x]`

output `$Aborted`

#### 3.2.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

**3.2.4 Maple [N/A] (verified)**

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^4(a + b \operatorname{sech}(dx^2 + c)) dx$$

input `int(x^4*(a+b*sech(d*x^2+c)),x)`output `int(x^4*(a+b*sech(d*x^2+c)),x)`**3.2.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^4(a + b \operatorname{sech}(c + dx^2)) dx = \int (b \operatorname{sech}(dx^2 + c) + a)x^4 dx$$

input `integrate(x^4*(a+b*sech(d*x^2+c)),x, algorithm="fricas")`output `integral(b*x^4*sech(d*x^2 + c) + a*x^4, x)`**3.2.6 Sympy [N/A]**

Not integrable

Time = 1.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^4(a + b \operatorname{sech}(c + dx^2)) dx = \int x^4(a + b \operatorname{sech}(c + dx^2)) dx$$

input `integrate(x**4*(a+b*sech(d*x**2+c)),x)`output `Integral(x**4*(a + b*sech(c + d*x**2)), x)`

**3.2.7 Maxima [N/A]**

Not integrable

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int x^4(a + b\operatorname{sech}(c + dx^2)) dx = \int (b\operatorname{sech}(dx^2 + c) + a)x^4 dx$$

input `integrate(x^4*(a+b*sech(d*x^2+c)),x, algorithm="maxima")`output `1/5*a*x^5 + 2*b*integrate(x^4/(e^(d*x^2 + c) + e^(-d*x^2 - c)), x)`**3.2.8 Giac [N/A]**

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4(a + b\operatorname{sech}(c + dx^2)) dx = \int (b\operatorname{sech}(dx^2 + c) + a)x^4 dx$$

input `integrate(x^4*(a+b*sech(d*x^2+c)),x, algorithm="giac")`output `integrate((b*sech(d*x^2 + c) + a)*x^4, x)`**3.2.9 Mupad [N/A]**

Not integrable

Time = 1.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^4(a + b\operatorname{sech}(c + dx^2)) dx = \int x^4 \left( a + \frac{b}{\cosh(dx^2 + c)} \right) dx$$

input `int(x^4*(a + b/cosh(c + d*x^2)),x)`output `int(x^4*(a + b/cosh(c + d*x^2)), x)`

### 3.3 $\int x^3(a + b\operatorname{sech}(c + dx^2)) dx$

3.3.1	Optimal result . . . . .	61
3.3.2	Mathematica [A] (verified) . . . . .	61
3.3.3	Rubi [A] (verified) . . . . .	62
3.3.4	Maple [F] . . . . .	63
3.3.5	Fricas [B] (verification not implemented) . . . . .	63
3.3.6	Sympy [F] . . . . .	63
3.3.7	Maxima [F] . . . . .	64
3.3.8	Giac [F] . . . . .	64
3.3.9	Mupad [F(-1)] . . . . .	64

#### 3.3.1 Optimal result

Integrand size = 16, antiderivative size = 77

$$\int x^3(a + b\operatorname{sech}(c + dx^2)) dx = \frac{ax^4}{4} + \frac{bx^2 \arctan(e^{c+dx^2})}{d} - \frac{ib \operatorname{PolyLog}\left(2, -ie^{c+dx^2}\right)}{2d^2} + \frac{ib \operatorname{PolyLog}\left(2, ie^{c+dx^2}\right)}{2d^2}$$

output `1/4*a*x^4+b*x^2*arctan(exp(d*x^2+c))/d-1/2*I*b*polylog(2,-I*exp(d*x^2+c))/d^2+1/2*I*b*polylog(2,I*exp(d*x^2+c))/d^2`

#### 3.3.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int x^3(a + b\operatorname{sech}(c + dx^2)) dx = \frac{ax^4}{4} + \frac{ib(dx^2(\log(1 - ie^{c+dx^2}) - \log(1 + ie^{c+dx^2}))) - \operatorname{PolyLog}(2, -ie^{c+dx^2}) + \operatorname{PolyLog}(2, ie^{c+dx^2})}{2d^2}$$

input `Integrate[x^3*(a + b*Sech[c + d*x^2]),x]`

output `(a*x^4)/4 + ((I/2)*b*(d*x^2*(Log[1 - I*E^(c + d*x^2)] - Log[1 + I*E^(c + d*x^2)]) - PolyLog[2, (-I)*E^(c + d*x^2)] + PolyLog[2, I*E^(c + d*x^2)]))/d^2`

### 3.3.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a + b\operatorname{sech}(c + dx^2)) dx$$

$$\downarrow \text{2010}$$

$$\int (ax^3 + bx^3\operatorname{sech}(c + dx^2)) dx$$

$$\downarrow \text{2009}$$

$$\frac{ax^4}{4} + \frac{bx^2 \arctan(e^{c+dx^2})}{d} - \frac{ib \operatorname{PolyLog}(2, -ie^{dx^2+c})}{2d^2} + \frac{ib \operatorname{PolyLog}(2, ie^{dx^2+c})}{2d^2}$$

input `Int[x^3*(a + b*Sech[c + d*x^2]),x]`

output `(a*x^4)/4 + (b*x^2*ArcTan[E^(c + d*x^2)])/d - ((I/2)*b*PolyLog[2, (-I)*E^(c + d*x^2)])/d^2 + ((I/2)*b*PolyLog[2, I*E^(c + d*x^2)])/d^2`

#### 3.3.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

### 3.3.4 Maple [F]

$$\int x^3(a + b \operatorname{sech}(dx^2 + c)) dx$$

input `int(x^3*(a+b*sech(d*x^2+c)),x)`

output `int(x^3*(a+b*sech(d*x^2+c)),x)`

### 3.3.5 Fracas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs.  $2(58) = 116$ .

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.39

$$\int x^3(a + b \operatorname{sech}(c + dx^2)) dx$$

$$= \frac{ad^2x^4 - 2i bc \log(\cosh(dx^2 + c) + \sinh(dx^2 + c) + i) + 2i bc \log(\cosh(dx^2 + c) + \sinh(dx^2 + c) - i) + 2}{d^2}$$

input `integrate(x^3*(a+b*sech(d*x^2+c)),x, algorithm="fracas")`

output `1/4*(a*d^2*x^4 - 2*I*b*c*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + I) + 2*I*b*c*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) - I) + 2*I*b*dilog(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c)) - 2*I*b*dilog(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)) - 2*(I*b*d*x^2 + I*b*c)*log(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c) + 1) - 2*(-I*b*d*x^2 - I*b*c)*log(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c) + 1))/d^2`

### 3.3.6 Sympy [F]

$$\int x^3(a + b \operatorname{sech}(c + dx^2)) dx = \int x^3(a + b \operatorname{sech}(c + dx^2)) dx$$

input `integrate(x**3*(a+b*sech(d*x**2+c)),x)`

output `Integral(x**3*(a + b*sech(c + d*x**2)), x)`



**3.3.7 Maxima [F]**

$$\int x^3(a + b\operatorname{sech}(c + dx^2)) dx = \int (b\operatorname{sech}(dx^2 + c) + a)x^3 dx$$

input `integrate(x^3*(a+b*sech(d*x^2+c)),x, algorithm="maxima")`

output `1/4*a*x^4 + 2*b*integrate(x^3/(e^(d*x^2 + c) + e^(-d*x^2 - c)), x)`

**3.3.8 Giac [F]**

$$\int x^3(a + b\operatorname{sech}(c + dx^2)) dx = \int (b\operatorname{sech}(dx^2 + c) + a)x^3 dx$$

input `integrate(x^3*(a+b*sech(d*x^2+c)),x, algorithm="giac")`

output `integrate((b*sech(d*x^2 + c) + a)*x^3, x)`

**3.3.9 Mupad [F(-1)]**

Timed out.

$$\int x^3(a + b\operatorname{sech}(c + dx^2)) dx = \int x^3 \left( a + \frac{b}{\cosh(dx^2 + c)} \right) dx$$

input `int(x^3*(a + b/cosh(c + d*x^2)),x)`

output `int(x^3*(a + b/cosh(c + d*x^2)), x)`

### 3.4 $\int x^2(a + b\operatorname{sech}(c + dx^2)) dx$

3.4.1	Optimal result . . . . .	65
3.4.2	Mathematica [N/A] . . . . .	65
3.4.3	Rubi [N/A] . . . . .	66
3.4.4	Maple [N/A] (verified) . . . . .	67
3.4.5	Fricas [N/A] . . . . .	67
3.4.6	Sympy [N/A] . . . . .	67
3.4.7	Maxima [N/A] . . . . .	68
3.4.8	Giac [N/A] . . . . .	68
3.4.9	Mupad [N/A] . . . . .	68

#### 3.4.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^2(a + b\operatorname{sech}(c + dx^2)) dx = \frac{ax^3}{3} + b\operatorname{Int}(x^2\operatorname{sech}(c + dx^2), x)$$

output `1/3*a*x^3+b*Unintegrable(x^2*sech(d*x^2+c),x)`

#### 3.4.2 Mathematica [N/A]

Not integrable

Time = 4.69 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2(a + b\operatorname{sech}(c + dx^2)) dx = \int x^2(a + b\operatorname{sech}(c + dx^2)) dx$$

input `Integrate[x^2*(a + b*Sech[c + d*x^2]),x]`

output `Integrate[x^2*(a + b*Sech[c + d*x^2]), x]`

### 3.4.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a + b\operatorname{sech}(c + dx^2)) dx$$

$$\downarrow \text{2010}$$

$$\int (ax^2 + bx^2\operatorname{sech}(c + dx^2)) dx$$

$$\downarrow \text{2009}$$

$$b \int x^2\operatorname{sech}(dx^2 + c) dx + \frac{ax^3}{3}$$

input `Int[x^2*(a + b*Sech[c + d*x^2]),x]`

output `$Aborted`

#### 3.4.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]]`

**3.4.4 Maple [N/A] (verified)**

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^2(a + b \operatorname{sech}(dx^2 + c)) dx$$

input `int(x^2*(a+b*sech(d*x^2+c)),x)`output `int(x^2*(a+b*sech(d*x^2+c)),x)`**3.4.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^2(a + b \operatorname{sech}(c + dx^2)) dx = \int (b \operatorname{sech}(dx^2 + c) + a)x^2 dx$$

input `integrate(x^2*(a+b*sech(d*x^2+c)),x, algorithm="fricas")`output `integral(b*x^2*sech(d*x^2 + c) + a*x^2, x)`**3.4.6 Sympy [N/A]**

Not integrable

Time = 0.71 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^2(a + b \operatorname{sech}(c + dx^2)) dx = \int x^2(a + b \operatorname{sech}(c + dx^2)) dx$$

input `integrate(x**2*(a+b*sech(d*x**2+c)),x)`output `Integral(x**2*(a + b*sech(c + d*x**2)), x)`

**3.4.7 Maxima [N/A]**

Not integrable

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int x^2(a + b \operatorname{sech}(c + dx^2)) dx = \int (b \operatorname{sech}(dx^2 + c) + a)x^2 dx$$

input `integrate(x^2*(a+b*sech(d*x^2+c)),x, algorithm="maxima")`output `1/3*a*x^3 + 2*b*integrate(x^2/(e^(d*x^2 + c) + e^(-d*x^2 - c)), x)`**3.4.8 Giac [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2(a + b \operatorname{sech}(c + dx^2)) dx = \int (b \operatorname{sech}(dx^2 + c) + a)x^2 dx$$

input `integrate(x^2*(a+b*sech(d*x^2+c)),x, algorithm="giac")`output `integrate((b*sech(d*x^2 + c) + a)*x^2, x)`**3.4.9 Mupad [N/A]**

Not integrable

Time = 1.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^2(a + b \operatorname{sech}(c + dx^2)) dx = \int x^2 \left( a + \frac{b}{\cosh(dx^2 + c)} \right) dx$$

input `int(x^2*(a + b/cosh(c + d*x^2)),x)`output `int(x^2*(a + b/cosh(c + d*x^2)), x)`

## 3.5 $\int x(a + b\operatorname{sech}(c + dx^2)) dx$

3.5.1	Optimal result . . . . .	69
3.5.2	Mathematica [A] (verified) . . . . .	69
3.5.3	Rubi [A] (verified) . . . . .	70
3.5.4	Maple [A] (verified) . . . . .	71
3.5.5	Fricas [A] (verification not implemented) . . . . .	71
3.5.6	Sympy [A] (verification not implemented) . . . . .	71
3.5.7	Maxima [A] (verification not implemented) . . . . .	72
3.5.8	Giac [A] (verification not implemented) . . . . .	72
3.5.9	Mupad [B] (verification not implemented) . . . . .	72

### 3.5.1 Optimal result

Integrand size = 14, antiderivative size = 26

$$\int x(a + b\operatorname{sech}(c + dx^2)) dx = \frac{ax^2}{2} + \frac{b \arctan(\sinh(c + dx^2))}{2d}$$

output `1/2*a*x^2+1/2*b*arctan(sinh(d*x^2+c))/d`

### 3.5.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x(a + b\operatorname{sech}(c + dx^2)) dx = \frac{ax^2}{2} + \frac{b \arctan(\sinh(c + dx^2))}{2d}$$

input `Integrate[x*(a + b*Sech[c + d*x^2]),x]`

output `(a*x^2)/2 + (b*ArcTan[Sinh[c + d*x^2]])/(2*d)`

### 3.5.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a + b \operatorname{sech}(c + dx^2)) dx$$

$$\downarrow \text{2010}$$

$$\int (ax + bx \operatorname{sech}(c + dx^2)) dx$$

$$\downarrow \text{2009}$$

$$\frac{ax^2}{2} + \frac{b \arctan(\sinh(c + dx^2))}{2d}$$

input `Int[x*(a + b*Sech[c + d*x^2]),x]`

output `(a*x^2)/2 + (b*ArcTan[Sinh[c + d*x^2]])/(2*d)`

#### 3.5.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

### 3.5.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
parts	$\frac{ax^2}{2} + \frac{b \arctan(\sinh(dx^2+c))}{2d}$	23
derivativedivides	$\frac{(dx^2+c)a+b \arctan(\sinh(dx^2+c))}{2d}$	27
default	$\frac{(dx^2+c)a+b \arctan(\sinh(dx^2+c))}{2d}$	27
risch	$\frac{ax^2}{2} + \frac{ib \ln(e^{dx^2+c+i})}{2d} - \frac{ib \ln(e^{dx^2+c-i})}{2d}$	46
parallelrisc	$\frac{adx^2 - ib \ln(\tanh(\frac{dx^2}{2} + \frac{c}{2}) - i) + ib \ln(\tanh(\frac{dx^2}{2} + \frac{c}{2}) + i)}{2d}$	51

input `int(x*(a+b*sech(d*x^2+c)),x,method=_RETURNVERBOSE)`

output `1/2*a*x^2+1/2*b*arctan(sinh(d*x^2+c))/d`

### 3.5.5 Fracas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int x(a + b \operatorname{sech}(c + dx^2)) dx = \frac{adx^2 + 2b \arctan(\cosh(dx^2 + c) + \sinh(dx^2 + c))}{2d}$$

input `integrate(x*(a+b*sech(d*x^2+c)),x, algorithm="fricas")`

output `1/2*(a*d*x^2 + 2*b*arctan(cosh(d*x^2 + c) + sinh(d*x^2 + c)))/d`

### 3.5.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int x(a + b \operatorname{sech}(c + dx^2)) dx = \begin{cases} \frac{a(c+dx^2)+2b \operatorname{atan}\left(\tanh\left(\frac{c}{2} + \frac{dx^2}{2}\right)\right)}{2d} & \text{for } d \neq 0 \\ \frac{x^2(a+b \operatorname{sech}(c))}{2} & \text{otherwise} \end{cases}$$



input `integrate(x*(a+b*sech(d*x**2+c)),x)`

output `Piecewise(((a*(c + d*x**2) + 2*b*atan(tanh(c/2 + d*x**2/2)))/(2*d), Ne(d, 0)), (x**2*(a + b*sech(c))/2, True))`

### 3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x(a + b\operatorname{sech}(c + dx^2)) dx = \frac{1}{2}ax^2 + \frac{b \arctan(\sinh(dx^2 + c))}{2d}$$

input `integrate(x*(a+b*sech(d*x^2+c)),x, algorithm="maxima")`

output `1/2*a*x^2 + 1/2*b*arctan(sinh(d*x^2 + c))/d`

### 3.5.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x(a + b\operatorname{sech}(c + dx^2)) dx = \frac{(dx^2 + c)a}{2d} + \frac{b \arctan\left(e^{(dx^2+c)}\right)}{d}$$

input `integrate(x*(a+b*sech(d*x^2+c)),x, algorithm="giac")`

output `1/2*(d*x^2 + c)*a/d + b*arctan(e^(d*x^2 + c))/d`

### 3.5.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int x(a + b\operatorname{sech}(c + dx^2)) dx = \frac{ax^2}{2} + \frac{\operatorname{atan}\left(\frac{be^{dx^2}e^c\sqrt{d^2}}{d\sqrt{b^2}}\right)\sqrt{b^2}}{\sqrt{d^2}}$$

input `int(x*(a + b/cosh(c + d*x^2)),x)`

output  $(a*x^2)/2 + (\operatorname{atan}((b*\exp(d*x^2)*\exp(c)*(d^2)^{(1/2)})/(d*(b^2)^{(1/2)}))* (b^2)^{(1/2)})/(d^2)^{(1/2)}$

### 3.6 $\int \frac{a+b\operatorname{sech}(c+dx^2)}{x} dx$

3.6.1	Optimal result	74
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3.6.9	Mupad [N/A]	77

#### 3.6.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x} dx = a \log(x) + b\operatorname{Int}\left(\frac{\operatorname{sech}(c + dx^2)}{x}, x\right)$$

output `a*ln(x)+b*Unintegrable(sech(d*x^2+c)/x,x)`

#### 3.6.2 Mathematica [N/A]

Not integrable

Time = 4.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x} dx = \int \frac{a + b\operatorname{sech}(c + dx^2)}{x} dx$$

input `Integrate[(a + b*Sech[c + d*x^2])/x,x]`

output `Integrate[(a + b*Sech[c + d*x^2])/x, x]`

### 3.6.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x} dx$$

↓ 2010

$$\int \left( \frac{a}{x} + \frac{b \operatorname{sech}(c + dx^2)}{x} \right) dx$$

↓ 2009

$$b \int \frac{\operatorname{sech}(dx^2 + c)}{x} dx + a \log(x)$$

input `Int[(a + b*Sech[c + d*x^2])/x,x]`

output `$Aborted`

#### 3.6.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

**3.6.4 Maple [N/A] (verified)**

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a + b \operatorname{sech}(dx^2 + c)}{x} dx$$

input `int((a+b*sech(d*x^2+c))/x,x)`output `int((a+b*sech(d*x^2+c))/x,x)`**3.6.5 Fricas [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x} dx = \int \frac{b \operatorname{sech}(dx^2 + c) + a}{x} dx$$

input `integrate((a+b*sech(d*x^2+c))/x,x, algorithm="fricas")`output `integral((b*sech(d*x^2 + c) + a)/x, x)`**3.6.6 Sympy [N/A]**

Not integrable

Time = 1.95 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x} dx = \int \frac{a + b \operatorname{sech}(c + dx^2)}{x} dx$$

input `integrate((a+b*sech(d*x**2+c))/x,x)`output `Integral((a + b*sech(c + d*x**2))/x, x)`

**3.6.7 Maxima [N/A]**

Not integrable

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x} dx = \int \frac{b \operatorname{sech}(dx^2 + c) + a}{x} dx$$

input `integrate((a+b*sech(d*x^2+c))/x,x, algorithm="maxima")`output `2*b*integrate(1/(x*(e^(d*x^2 + c) + e^(-d*x^2 - c))), x) + a*log(x)`**3.6.8 Giac [N/A]**

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x} dx = \int \frac{b \operatorname{sech}(dx^2 + c) + a}{x} dx$$

input `integrate((a+b*sech(d*x^2+c))/x,x, algorithm="giac")`output `integrate((b*sech(d*x^2 + c) + a)/x, x)`**3.6.9 Mupad [N/A]**

Not integrable

Time = 2.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x} dx = \int \frac{a + \frac{b}{\cosh(dx^2+c)}}{x} dx$$

input `int((a + b/cosh(c + d*x^2))/x,x)`output `int((a + b/cosh(c + d*x^2))/x, x)`

---

3.6.  $\int \frac{a+b \operatorname{sech}(c+dx^2)}{x} dx$

### 3.7 $\int \frac{a+b\operatorname{sech}(c+dx^2)}{x^2} dx$

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3.7.9	Mupad [N/A]	81

#### 3.7.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx = -\frac{a}{x} + b\operatorname{Int}\left(\frac{\operatorname{sech}(c + dx^2)}{x^2}, x\right)$$

output `-a/x+b*Unintegrable(sech(d*x^2+c)/x^2,x)`

#### 3.7.2 Mathematica [N/A]

Not integrable

Time = 4.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx$$

input `Integrate[(a + b*Sech[c + d*x^2])/x^2,x]`

output `Integrate[(a + b*Sech[c + d*x^2])/x^2, x]`

### 3.7.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx$$

↓ 2010

$$\int \left( \frac{a}{x^2} + \frac{b \operatorname{sech}(c + dx^2)}{x^2} \right) dx$$

↓ 2009

$$b \int \frac{\operatorname{sech}(dx^2 + c)}{x^2} dx - \frac{a}{x}$$

input `Int[(a + b*Sech[c + d*x^2])/x^2,x]`

output `$Aborted`

#### 3.7.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`



**3.7.4 Maple [N/A] (verified)**

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a + b \operatorname{sech}(dx^2 + c)}{x^2} dx$$

input `int((a+b*sech(d*x^2+c))/x^2,x)`output `int((a+b*sech(d*x^2+c))/x^2,x)`**3.7.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{b \operatorname{sech}(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="fricas")`output `integral((b*sech(d*x^2 + c) + a)/x^2, x)`**3.7.6 Sympy [N/A]**

Not integrable

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx$$

input `integrate((a+b*sech(d*x**2+c))/x**2,x)`output `Integral((a + b*sech(c + d*x**2))/x**2, x)`

**3.7.7 Maxima [N/A]**

Not integrable

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{b \operatorname{sech}(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="maxima")`output `2*b*integrate(1/(x^2*(e^(d*x^2 + c) + e^(-d*x^2 - c))), x) - a/x`**3.7.8 Giac [N/A]**

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{b \operatorname{sech}(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="giac")`output `integrate((b*sech(d*x^2 + c) + a)/x^2, x)`**3.7.9 Mupad [N/A]**

Not integrable

Time = 2.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{a + \frac{b}{\cosh(dx^2+c)}}{x^2} dx$$

input `int((a + b/cosh(c + d*x^2))/x^2,x)`output `int((a + b/cosh(c + d*x^2))/x^2, x)`

---

3.7.  $\int \frac{a+b \operatorname{sech}(c+dx^2)}{x^2} dx$

### 3.8 $\int x^5(a + b\operatorname{sech}(c + dx^2))^2 dx$

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3.8.4	Maple [F] . . . . .	85
3.8.5	Fricas [B] (verification not implemented) . . . . .	85
3.8.6	Sympy [F] . . . . .	86
3.8.7	Maxima [F] . . . . .	86
3.8.8	Giac [F] . . . . .	86
3.8.9	Mupad [F(-1)] . . . . .	87

#### 3.8.1 Optimal result

Integrand size = 18, antiderivative size = 217

$$\int x^5(a + b\operatorname{sech}(c + dx^2))^2 dx = \frac{b^2x^4}{2d} + \frac{a^2x^6}{6} + \frac{2abx^4 \arctan(e^{c+dx^2})}{d} - \frac{b^2x^2 \log(1 + e^{2(c+dx^2)})}{d^2} - \frac{2iabx^2 \operatorname{PolyLog}(2, -ie^{c+dx^2})}{d^2} + \frac{2iabx^2 \operatorname{PolyLog}(2, ie^{c+dx^2})}{d^2} - \frac{b^2 \operatorname{PolyLog}(2, -e^{2(c+dx^2)})}{2d^3} + \frac{2iab \operatorname{PolyLog}(3, -ie^{c+dx^2})}{d^3} - \frac{2iab \operatorname{PolyLog}(3, ie^{c+dx^2})}{d^3} + \frac{b^2x^4 \tanh(c + dx^2)}{2d}$$

output `1/2*b^2*x^4/d+1/6*a^2*x^6+2*a*b*x^4*arctan(exp(d*x^2+c))/d-b^2*x^2*ln(1+exp(2*d*x^2+2*c))/d^2-2*I*a*b*x^2*polylog(2,-I*exp(d*x^2+c))/d^2+2*I*a*b*x^2*polylog(2,I*exp(d*x^2+c))/d^2-1/2*b^2*polylog(2,-exp(2*d*x^2+2*c))/d^3+2*I*a*b*polylog(3,-I*exp(d*x^2+c))/d^3-2*I*a*b*polylog(3,I*exp(d*x^2+c))/d^3+1/2*b^2*x^4*tanh(d*x^2+c)/d`

### 3.8.2 Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.47

$$\int x^5 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

$$= \frac{\cosh(c + dx^2) (a + b \operatorname{sech}(c + dx^2))^2 \left( a^2 x^6 \cosh(c + dx^2) + \frac{3b \cosh(c + dx^2) (2bd^2 e^{2c} x^4 - 2bd^2 (1 + e^{2c}) x^4 + b(1 + e^{2c}) (2d^2 x^4 - \log[1 + E^{2(c + dx^2)]]) - \operatorname{PolyLog}[2, -E^{2(c + dx^2)]]) + (2I) a (1 + E^{2c}) (d^2 x^4 \log[1 - I E^{c + dx^2}] - d^2 x^4 \log[1 + I E^{c + dx^2}] - 2 d x^2 \operatorname{PolyLog}[2, (-I) E^{c + dx^2}] + 2 d x^2 \operatorname{PolyLog}[2, I E^{c + dx^2}] + 2 \operatorname{PolyLog}[3, (-I) E^{c + dx^2}] - 2 \operatorname{PolyLog}[3, I E^{c + dx^2}])]}{d^3 (1 + E^{2c})} + (3 b^2 x^4 \operatorname{Sech}[c + dx^2] \operatorname{Sinh}[dx^2]) / d \right)}{6 (b + a \cosh(c + dx^2))^2}$$

input `Integrate[x^5*(a + b*Sech[c + d*x^2])^2,x]`

output `(Cosh[c + d*x^2]*(a + b*Sech[c + d*x^2])^2*(a^2*x^6*Cosh[c + d*x^2] + (3*b*Cosh[c + d*x^2]*(2*b*d^2*E^(2*c)*x^4 - 2*b*d^2*(1 + E^(2*c))*x^4 + b*(1 + E^(2*c))*(2*d*x^2*(d*x^2 - Log[1 + E^(2*(c + d*x^2)])) - PolyLog[2, -E^(2*(c + d*x^2)]]) + (2*I)*a*(1 + E^(2*c))*(d^2*x^4*Log[1 - I*E^(c + d*x^2)] - d^2*x^4*Log[1 + I*E^(c + d*x^2)] - 2*d*x^2*PolyLog[2, (-I)*E^(c + d*x^2)] + 2*d*x^2*PolyLog[2, I*E^(c + d*x^2)] + 2*PolyLog[3, (-I)*E^(c + d*x^2)] - 2*PolyLog[3, I*E^(c + d*x^2)])))/d^3*(1 + E^(2*c))) + (3*b^2*x^4*Sech[c]*Sinh[d*x^2])/d)/(6*(b + a*Cosh[c + d*x^2])^2)`

### 3.8.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {5959, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

$$\downarrow \text{5959}$$

$$\frac{1}{2} \int x^4 (a + b \operatorname{sech}(dx^2 + c))^2 dx^2$$

$$\downarrow \text{3042}$$

$$\frac{1}{2} \int x^4 \left( a + b \operatorname{csc} \left( i dx^2 + ic + \frac{\pi}{2} \right) \right)^2 dx^2$$

$$\begin{array}{c} \downarrow 4678 \\ \frac{1}{2} \int (a^2 x^4 + b^2 \operatorname{sech}^2(dx^2 + c) x^4 + 2ab \operatorname{sech}(dx^2 + c) x^4) dx^2 \\ \downarrow 2009 \\ \frac{1}{2} \left( \frac{a^2 x^6}{3} + \frac{4abx^4 \arctan(e^{c+dx^2})}{d} + \frac{4iab \operatorname{PolyLog}(3, -ie^{dx^2+c})}{d^3} - \frac{4iab \operatorname{PolyLog}(3, ie^{dx^2+c})}{d^3} - \frac{4iabx^2 \operatorname{PolyLog}}{d^2} \right) \end{array}$$

input `Int[x^5*(a + b*Sech[c + d*x^2])^2,x]`

output `((b^2*x^4)/d + (a^2*x^6)/3 + (4*a*b*x^4*ArcTan[E^(c + d*x^2)])/d - (2*b^2*x^2*Log[1 + E^(2*(c + d*x^2))])/d^2 - ((4*I)*a*b*x^2*PolyLog[2, (-I)*E^(c + d*x^2)])/d^2 + ((4*I)*a*b*x^2*PolyLog[2, I*E^(c + d*x^2)])/d^2 - (b^2*PolyLog[2, -E^(2*(c + d*x^2))])/d^3 + ((4*I)*a*b*PolyLog[3, (-I)*E^(c + d*x^2)])/d^3 - ((4*I)*a*b*PolyLog[3, I*E^(c + d*x^2)])/d^3 + (b^2*x^4*Tanh[c + d*x^2])/d)/2`

### 3.8.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

### 3.8.4 Maple [F]

$$\int x^5 (a + b \operatorname{sech}(dx^2 + c))^2 dx$$

input `int(x^5*(a+b*sech(d*x^2+c))^2,x)`

output `int(x^5*(a+b*sech(d*x^2+c))^2,x)`

### 3.8.5 Fracas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1198 vs.  $2(185) = 370$ .

Time = 0.30 (sec) , antiderivative size = 1198, normalized size of antiderivative = 5.52

$$\int x^5 (a + b \operatorname{sech}(c + dx^2))^2 dx = \text{Too large to display}$$

input `integrate(x^5*(a+b*sech(d*x^2+c))^2,x, algorithm="fracas")`

output

```

1/6*(a^2*d^3*x^6 - 6*b^2*c^2 + (a^2*d^3*x^6 + 6*b^2*d^2*x^4 - 6*b^2*c^2)*c
osh(d*x^2 + c)^2 + 2*(a^2*d^3*x^6 + 6*b^2*d^2*x^4 - 6*b^2*c^2)*cosh(d*x^2
+ c)*sinh(d*x^2 + c) + (a^2*d^3*x^6 + 6*b^2*d^2*x^4 - 6*b^2*c^2)*sinh(d*x^
2 + c)^2 - 6*(-2*I*a*b*d*x^2 + (-2*I*a*b*d*x^2 + b^2)*cosh(d*x^2 + c)^2 +
2*(-2*I*a*b*d*x^2 + b^2)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (-2*I*a*b*d*x^2
+ b^2)*sinh(d*x^2 + c)^2 + b^2)*dilog(I*cosh(d*x^2 + c) + I*sinh(d*x^2 +
c)) - 6*(2*I*a*b*d*x^2 + (2*I*a*b*d*x^2 + b^2)*cosh(d*x^2 + c)^2 + 2*(2*I
a*b*d*x^2 + b^2)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (2*I*a*b*d*x^2 + b^2)*s
inh(d*x^2 + c)^2 + b^2)*dilog(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)) - 6*
(-I*a*b*c^2 - b^2*c + (-I*a*b*c^2 - b^2*c)*cosh(d*x^2 + c)^2 + 2*(-I*a*b*c
^2 - b^2*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (-I*a*b*c^2 - b^2*c)*sinh(d*
x^2 + c)^2)*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + I) - 6*(I*a*b*c^2 - b^
2*c + (I*a*b*c^2 - b^2*c)*cosh(d*x^2 + c)^2 + 2*(I*a*b*c^2 - b^2*c)*cosh(d
*x^2 + c)*sinh(d*x^2 + c) + (I*a*b*c^2 - b^2*c)*sinh(d*x^2 + c)^2)*log(cos
h(d*x^2 + c) + sinh(d*x^2 + c) - I) - 6*(I*a*b*d^2*x^4 + b^2*d*x^2 - I*a*b
*c^2 + b^2*c + (I*a*b*d^2*x^4 + b^2*d*x^2 - I*a*b*c^2 + b^2*c)*cosh(d*x^2
+ c)^2 + 2*(I*a*b*d^2*x^4 + b^2*d*x^2 - I*a*b*c^2 + b^2*c)*cosh(d*x^2 + c)
*sinh(d*x^2 + c) + (I*a*b*d^2*x^4 + b^2*d*x^2 - I*a*b*c^2 + b^2*c)*sinh(d*
x^2 + c)^2)*log(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c) + 1) - 6*(-I*a*b*d^2
*x^4 + b^2*d*x^2 + I*a*b*c^2 + b^2*c + (-I*a*b*d^2*x^4 + b^2*d*x^2 + I*...

```

**3.8.6 Sympy [F]**

$$\int x^5 (a + b \operatorname{sech}(c + dx^2))^2 dx = \int x^5 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

input `integrate(x**5*(a+b*sech(d*x**2+c))**2,x)`

output `Integral(x**5*(a + b*sech(c + d*x**2))**2, x)`

**3.8.7 Maxima [F]**

$$\int x^5 (a + b \operatorname{sech}(c + dx^2))^2 dx = \int (b \operatorname{sech}(dx^2 + c) + a)^2 x^5 dx$$

input `integrate(x^5*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

output `1/6*a^2*x^6 - b^2*x^4/(d*e^(2*d*x^2 + 2*c) + d) + integrate(4*(a*b*d*x^5*e^(d*x^2 + c) + b^2*x^3)/(d*e^(2*d*x^2 + 2*c) + d), x)`

**3.8.8 Giac [F]**

$$\int x^5 (a + b \operatorname{sech}(c + dx^2))^2 dx = \int (b \operatorname{sech}(dx^2 + c) + a)^2 x^5 dx$$

input `integrate(x^5*(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`

output `integrate((b*sech(d*x^2 + c) + a)^2*x^5, x)`

**3.8.9 Mupad [F(-1)]**

Timed out.

$$\int x^5 (a + b \operatorname{sech}(c + dx^2))^2 dx = \int x^5 \left( a + \frac{b}{\cosh(dx^2 + c)} \right)^2 dx$$

input `int(x^5*(a + b/cosh(c + d*x^2))^2,x)`output `int(x^5*(a + b/cosh(c + d*x^2))^2, x)`



### 3.9 $\int x^4(a + b\operatorname{sech}(c + dx^2))^2 dx$

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#### 3.9.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^4(a + b\operatorname{sech}(c + dx^2))^2 dx = \operatorname{Int}\left(x^4(a + b\operatorname{sech}(c + dx^2))^2, x\right)$$

output `Unintegrable(x^4*(a+b*sech(d*x^2+c))^2,x)`

#### 3.9.2 Mathematica [N/A]

Not integrable

Time = 11.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4(a + b\operatorname{sech}(c + dx^2))^2 dx = \int x^4(a + b\operatorname{sech}(c + dx^2))^2 dx$$

input `Integrate[x^4*(a + b*Sech[c + d*x^2])^2,x]`

output `Integrate[x^4*(a + b*Sech[c + d*x^2])^2, x]`

### 3.9.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

↓ 5961

$$\int x^4 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

input `Int[x^4*(a + b*Sech[c + d*x^2])^2,x]`

output `$Aborted`

#### 3.9.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.9.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^4 (a + b \operatorname{sech}(dx^2 + c))^2 dx$$

input `int(x^4*(a+b*sech(d*x^2+c))^2,x)`

output `int(x^4*(a+b*sech(d*x^2+c))^2,x)`

**3.9.5 Fracas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^4(a + b\operatorname{sech}(c + dx^2))^2 dx = \int (b\operatorname{sech}(dx^2 + c) + a)^2 x^4 dx$$

input `integrate(x^4*(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")`output `integral(b^2*x^4*sech(d*x^2 + c)^2 + 2*a*b*x^4*sech(d*x^2 + c) + a^2*x^4, x)`**3.9.6 Sympy [N/A]**

Not integrable

Time = 1.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^4(a + b\operatorname{sech}(c + dx^2))^2 dx = \int x^4(a + b\operatorname{sech}(c + dx^2))^2 dx$$

input `integrate(x**4*(a+b*sech(d*x**2+c))**2,x)`output `Integral(x**4*(a + b*sech(c + d*x**2))**2, x)`**3.9.7 Maxima [N/A]**

Not integrable

Time = 0.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.39

$$\int x^4(a + b\operatorname{sech}(c + dx^2))^2 dx = \int (b\operatorname{sech}(dx^2 + c) + a)^2 x^4 dx$$

input `integrate(x^4*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`output `1/5*a^2*x^5 - b^2*x^3/(d*e^(2*d*x^2 + 2*c) + d) + integrate((4*a*b*d*x^4*e^(d*x^2 + c) + 3*b^2*x^2)/(d*e^(2*d*x^2 + 2*c) + d), x)`

**3.9.8 Giac [N/A]**

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4 (a + b \operatorname{sech}(c + dx^2))^2 dx = \int (b \operatorname{sech}(dx^2 + c) + a)^2 x^4 dx$$

input `integrate(x^4*(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`output `integrate((b*sech(d*x^2 + c) + a)^2*x^4, x)`**3.9.9 Mupad [N/A]**

Not integrable

Time = 2.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^4 (a + b \operatorname{sech}(c + dx^2))^2 dx = \int x^4 \left( a + \frac{b}{\cosh(dx^2 + c)} \right)^2 dx$$

input `int(x^4*(a + b/cosh(c + d*x^2))^2,x)`output `int(x^4*(a + b/cosh(c + d*x^2))^2, x)`

### 3.10 $\int x^3(a + b\operatorname{sech}(c + dx^2))^2 dx$

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#### 3.10.1 Optimal result

Integrand size = 18, antiderivative size = 119

$$\int x^3(a + b\operatorname{sech}(c + dx^2))^2 dx = \frac{a^2x^4}{4} + \frac{2abx^2 \arctan(e^{c+dx^2})}{d} - \frac{b^2 \log(\cosh(c + dx^2))}{2d^2} - \frac{iab \operatorname{PolyLog}(2, -ie^{c+dx^2})}{d^2} + \frac{iab \operatorname{PolyLog}(2, ie^{c+dx^2})}{d^2} + \frac{b^2x^2 \tanh(c + dx^2)}{2d}$$

```
output 1/4*a^2*x^4+2*a*b*x^2*arctan(exp(d*x^2+c))/d-1/2*b^2*ln(cosh(d*x^2+c))/d^2
-I*a*b*polylog(2,-I*exp(d*x^2+c))/d^2+I*a*b*polylog(2,I*exp(d*x^2+c))/d^2+
1/2*b^2*x^2*tanh(d*x^2+c)/d
```

#### 3.10.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 324 vs. 2(119) = 238.

Time = 1.94 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.72

$$\int x^3(a + b\operatorname{sech}(c + dx^2))^2 dx = \frac{4b^2de^{2c}x^2 + a^2d^2x^4 + a^2d^2e^{2c}x^4 + 4iabdx^2 \log(1 - ie^{c+dx^2}) + 4iabde^{2c}x^2 \log(1 - ie^{c+dx^2}) - 4iabdx^2 \log(1 + ie^{c+dx^2}) - 4iabde^{2c}x^2 \log(1 + ie^{c+dx^2})}{d^2}$$

input `Integrate[x^3*(a + b*Sech[c + d*x^2])^2,x]`

output  $(4*b^2*d*E^{(2*c)}*x^2 + a^2*d^2*x^4 + a^2*d^2*E^{(2*c)}*x^4 + (4*I)*a*b*d*x^2 * \text{Log}[1 - I*E^{(c + d*x^2)}] + (4*I)*a*b*d*E^{(2*c)}*x^2*\text{Log}[1 - I*E^{(c + d*x^2)}]) - (4*I)*a*b*d*x^2*\text{Log}[1 + I*E^{(c + d*x^2)}] - (4*I)*a*b*d*E^{(2*c)}*x^2*\text{Log}[1 + I*E^{(c + d*x^2)}] - 2*b^2*\text{Log}[1 + E^{(2*(c + d*x^2))}] - 2*b^2*E^{(2*c)}*\text{Log}[1 + E^{(2*(c + d*x^2))}] - (4*I)*a*b*(1 + E^{(2*c)})*\text{PolyLog}[2, (-I)*E^{(c + d*x^2)}] + (4*I)*a*b*(1 + E^{(2*c)})*\text{PolyLog}[2, I*E^{(c + d*x^2)}] + 2*b^2*d*x^2*\text{Sech}[c]*\text{Sech}[c + d*x^2]*\text{Sinh}[d*x^2] + 2*b^2*d*E^{(2*c)}*x^2*\text{Sech}[c]*\text{Sech}[c + d*x^2]*\text{Sinh}[d*x^2])/(4*d^2*(1 + E^{(2*c)}))$

### 3.10.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {5959, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x^3 (a + b \operatorname{sech}(c + dx^2))^2 dx \\ & \quad \downarrow \text{5959} \\ & \frac{1}{2} \int x^2 (a + b \operatorname{sech}(dx^2 + c))^2 dx^2 \\ & \quad \downarrow \text{3042} \\ & \frac{1}{2} \int x^2 \left( a + b \operatorname{csc} \left( i dx^2 + ic + \frac{\pi}{2} \right) \right)^2 dx^2 \\ & \quad \downarrow \text{4678} \\ & \frac{1}{2} \int (a^2 x^2 + b^2 \operatorname{sech}^2(dx^2 + c) x^2 + 2ab \operatorname{sech}(dx^2 + c) x^2) dx^2 \\ & \quad \downarrow \text{2009} \\ & \frac{1}{2} \left( \frac{a^2 x^4}{2} + \frac{4abx^2 \arctan(e^{c+dx^2})}{d} - \frac{2iab \operatorname{PolyLog}(2, -ie^{dx^2+c})}{d^2} + \frac{2iab \operatorname{PolyLog}(2, ie^{dx^2+c})}{d^2} - \frac{b^2 \log(\cosh(c + dx^2))}{d^2} \right) \end{aligned}$$

input `Int[x^3*(a + b*Sech[c + d*x^2])^2,x]`

output `((a^2*x^4)/2 + (4*a*b*x^2*ArcTan[E^(c + d*x^2)])/d - (b^2*Log[Cosh[c + d*x^2]])/d^2 - ((2*I)*a*b*PolyLog[2, (-I)*E^(c + d*x^2)])/d^2 + ((2*I)*a*b*PolyLog[2, I*E^(c + d*x^2)])/d^2 + (b^2*x^2*Tanh[c + d*x^2])/d)/2`

### 3.10.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

### 3.10.4 Maple [F]

$$\int x^3(a + b \operatorname{sech}(dx^2 + c))^2 dx$$

input `int(x^3*(a+b*sech(d*x^2+c))^2,x)`

output `int(x^3*(a+b*sech(d*x^2+c))^2,x)`

### 3.10.5 Fracas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 782 vs.  $2(100) = 200$ .

Time = 0.28 (sec) , antiderivative size = 782, normalized size of antiderivative = 6.57

$$\int x^3 (a + b \operatorname{sech}(c + dx^2))^2 dx$$

$$= \frac{a^2 d^2 x^4 + 4 b^2 c + (a^2 d^2 x^4 + 4 b^2 dx^2 + 4 b^2 c) \cosh(dx^2 + c)^2 + 2(a^2 d^2 x^4 + 4 b^2 dx^2 + 4 b^2 c) \cosh(dx^2 + c) \sinh(dx^2 + c) + \dots}{d^2 \cosh(dx^2 + c)^2 + 2 d^2 \cosh(dx^2 + c) \sinh(dx^2 + c) + d^2 \sinh(dx^2 + c)^2 + d^2}$$

input `integrate(x^3*(a+b*sech(d*x^2+c))^2,x, algorithm="fracas")`

output

```
1/4*(a^2*d^2*x^4 + 4*b^2*c + (a^2*d^2*x^4 + 4*b^2*d*x^2 + 4*b^2*c)*cosh(d*x^2 + c)^2 + 2*(a^2*d^2*x^4 + 4*b^2*d*x^2 + 4*b^2*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (a^2*d^2*x^4 + 4*b^2*d*x^2 + 4*b^2*c)*sinh(d*x^2 + c)^2 - 4*(-I*a*b*cosh(d*x^2 + c)^2 - 2*I*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) - I*a*b*sinh(d*x^2 + c)^2 - I*a*b)*dilog(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c)) - 4*(I*a*b*cosh(d*x^2 + c)^2 + 2*I*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) + I*a*b*sinh(d*x^2 + c)^2 + I*a*b)*dilog(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c)) - 2*(2*I*a*b*c + (2*I*a*b*c + b^2)*cosh(d*x^2 + c)^2 + 2*(2*I*a*b*c + b^2)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (2*I*a*b*c + b^2)*sinh(d*x^2 + c)^2 + b^2)*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) + I) - 2*(-2*I*a*b*c + (-2*I*a*b*c + b^2)*cosh(d*x^2 + c)^2 + 2*(-2*I*a*b*c + b^2)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (-2*I*a*b*c + b^2)*sinh(d*x^2 + c)^2 + b^2)*log(cosh(d*x^2 + c) + sinh(d*x^2 + c) - I) - 4*(I*a*b*d*x^2 + I*a*b*c + (I*a*b*d*x^2 + I*a*b*c)*cosh(d*x^2 + c)^2 + 2*(I*a*b*d*x^2 + I*a*b*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (I*a*b*d*x^2 + I*a*b*c)*sinh(d*x^2 + c)^2)*log(I*cosh(d*x^2 + c) + I*sinh(d*x^2 + c) + 1) - 4*(-I*a*b*d*x^2 - I*a*b*c + (-I*a*b*d*x^2 - I*a*b*c)*cosh(d*x^2 + c)^2 + 2*(-I*a*b*d*x^2 - I*a*b*c)*cosh(d*x^2 + c)*sinh(d*x^2 + c) + (-I*a*b*d*x^2 - I*a*b*c)*sinh(d*x^2 + c)^2)*log(-I*cosh(d*x^2 + c) - I*sinh(d*x^2 + c) + 1))/(d^2*cosh(d*x^2 + c)^2 + 2*d^2*cosh(d*x^2 + c)*sinh(d*x^2 + c) + d^2*sinh(d*x^2 + c)^2 + d^2)
```



**3.10.6 Sympy [F]**

$$\int x^3(a + b\operatorname{sech}(c + dx^2))^2 dx = \int x^3(a + b\operatorname{sech}(c + dx^2))^2 dx$$

input `integrate(x**3*(a+b*sech(d*x**2+c))**2,x)`

output `Integral(x**3*(a + b*sech(c + d*x**2))**2, x)`

**3.10.7 Maxima [F]**

$$\int x^3(a + b\operatorname{sech}(c + dx^2))^2 dx = \int (b\operatorname{sech}(dx^2 + c) + a)^2 x^3 dx$$

input `integrate(x^3*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

output `1/4*a^2*x^4 + 1/2*(2*x^2*e^(2*d*x^2 + 2*c)/(d*e^(2*d*x^2 + 2*c) + d) - log((e^(2*d*x^2 + 2*c) + 1)*e^(-2*c))/d^2)*b^2 + 4*a*b*integrate(x^3*e^(d*x^2 + c)/(e^(2*d*x^2 + 2*c) + 1), x)`

**3.10.8 Giac [F]**

$$\int x^3(a + b\operatorname{sech}(c + dx^2))^2 dx = \int (b\operatorname{sech}(dx^2 + c) + a)^2 x^3 dx$$

input `integrate(x^3*(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`

output `integrate((b*sech(d*x^2 + c) + a)^2*x^3, x)`

**3.10.9 Mupad [F(-1)]**

Timed out.

$$\int x^3 (a + b \operatorname{sech}(c + dx^2))^2 dx = \int x^3 \left( a + \frac{b}{\cosh(dx^2 + c)} \right)^2 dx$$

input `int(x^3*(a + b/cosh(c + d*x^2))^2,x)`output `int(x^3*(a + b/cosh(c + d*x^2))^2, x)`

### 3.11 $\int x^2(a + b\operatorname{sech}(c + dx^2))^2 dx$

3.11.1	Optimal result . . . . .	98
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3.11.3	Rubi [N/A] . . . . .	99
3.11.4	Maple [N/A] (verified) . . . . .	99
3.11.5	Fricas [N/A] . . . . .	100
3.11.6	Sympy [N/A] . . . . .	100
3.11.7	Maxima [N/A] . . . . .	100
3.11.8	Giac [N/A] . . . . .	101
3.11.9	Mupad [N/A] . . . . .	101

#### 3.11.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^2(a + b\operatorname{sech}(c + dx^2))^2 dx = \operatorname{Int}\left(x^2(a + b\operatorname{sech}(c + dx^2))^2, x\right)$$

output `Unintegrable(x^2*(a+b*sech(d*x^2+c))^2,x)`

#### 3.11.2 Mathematica [N/A]

Not integrable

Time = 9.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2(a + b\operatorname{sech}(c + dx^2))^2 dx = \int x^2(a + b\operatorname{sech}(c + dx^2))^2 dx$$

input `Integrate[x^2*(a + b*Sech[c + d*x^2])^2,x]`

output `Integrate[x^2*(a + b*Sech[c + d*x^2])^2, x]`

### 3.11.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a + b\operatorname{sech}(c + dx^2))^2 dx$$

↓ 5961

$$\int x^2(a + b\operatorname{sech}(c + dx^2))^2 dx$$

input `Int[x^2*(a + b*Sech[c + d*x^2])^2,x]`

output `$Aborted`

#### 3.11.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.11.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2(a + b \operatorname{sech}(dx^2 + c))^2 dx$$

input `int(x^2*(a+b*sech(d*x^2+c))^2,x)`

output `int(x^2*(a+b*sech(d*x^2+c))^2,x)`

**3.11.5 Fracas [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^2(a + b \operatorname{sech}(c + dx^2))^2 dx = \int (b \operatorname{sech}(dx^2 + c) + a)^2 x^2 dx$$

input `integrate(x^2*(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")`output `integral(b^2*x^2*sech(d*x^2 + c)^2 + 2*a*b*x^2*sech(d*x^2 + c) + a^2*x^2, x)`**3.11.6 Sympy [N/A]**

Not integrable

Time = 1.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^2(a + b \operatorname{sech}(c + dx^2))^2 dx = \int x^2(a + b \operatorname{sech}(c + dx^2))^2 dx$$

input `integrate(x**2*(a+b*sech(d*x**2+c))**2,x)`output `Integral(x**2*(a + b*sech(c + d*x**2))**2, x)`**3.11.7 Maxima [N/A]**

Not integrable

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.00

$$\int x^2(a + b \operatorname{sech}(c + dx^2))^2 dx = \int (b \operatorname{sech}(dx^2 + c) + a)^2 x^2 dx$$

input `integrate(x^2*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`output `1/3*a^2*x^3 - b^2*x/(d*e^(2*d*x^2 + 2*c) + d) + integrate((4*a*b*d*x^2*e^(d*x^2 + c) + b^2)/(d*e^(2*d*x^2 + 2*c) + d), x)`

---

3.11.  $\int x^2(a + b \operatorname{sech}(c + dx^2))^2 dx$

**3.11.8 Giac [N/A]**

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2 (a + b \operatorname{sech}(c + dx^2))^2 dx = \int (b \operatorname{sech}(dx^2 + c) + a)^2 x^2 dx$$

input `integrate(x^2*(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`output `integrate((b*sech(d*x^2 + c) + a)^2*x^2, x)`**3.11.9 Mupad [N/A]**

Not integrable

Time = 1.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^2 (a + b \operatorname{sech}(c + dx^2))^2 dx = \int x^2 \left( a + \frac{b}{\cosh(dx^2 + c)} \right)^2 dx$$

input `int(x^2*(a + b/cosh(c + d*x^2))^2,x)`output `int(x^2*(a + b/cosh(c + d*x^2))^2, x)`

### 3.12 $\int x(a + b\operatorname{sech}(c + dx^2))^2 dx$

3.12.1	Optimal result . . . . .	102
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3.12.7	Maxima [A] (verification not implemented) . . . . .	106
3.12.8	Giac [A] (verification not implemented) . . . . .	106
3.12.9	Mupad [B] (verification not implemented) . . . . .	107

#### 3.12.1 Optimal result

Integrand size = 16, antiderivative size = 44

$$\int x(a + b\operatorname{sech}(c + dx^2))^2 dx = \frac{a^2x^2}{2} + \frac{ab \arctan(\sinh(c + dx^2))}{d} + \frac{b^2 \tanh(c + dx^2)}{2d}$$

output `1/2*a^2*x^2+a*b*arctan(sinh(d*x^2+c))/d+1/2*b^2*tanh(d*x^2+c)/d`

#### 3.12.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int x(a + b\operatorname{sech}(c + dx^2))^2 dx = \frac{1}{2} \left( a^2x^2 + \frac{2ab \arctan(\sinh(c + dx^2))}{d} + \frac{b^2 \tanh(c + dx^2)}{d} \right)$$

input `Integrate[x*(a + b*Sech[c + d*x^2])^2,x]`

output `(a^2*x^2 + (2*a*b*ArcTan[Sinh[c + d*x^2]]))/d + (b^2*Tanh[c + d*x^2])/d)/2`

### 3.12.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {5959, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x(a + b \operatorname{sech}(c + dx^2))^2 dx \\
 & \quad \downarrow \text{5959} \\
 & \frac{1}{2} \int (a + b \operatorname{sech}(dx^2 + c))^2 dx^2 \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \int \left( a + b \operatorname{csc} \left( idx^2 + ic + \frac{\pi}{2} \right) \right)^2 dx^2 \\
 & \quad \downarrow \text{4260} \\
 & \frac{1}{2} \left( 2ab \int \operatorname{sech}(dx^2 + c) dx^2 + b^2 \int \operatorname{sech}^2(dx^2 + c) dx^2 + a^2 x^2 \right) \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \left( 2ab \int \operatorname{csc} \left( idx^2 + ic + \frac{\pi}{2} \right) dx^2 + b^2 \int \operatorname{csc} \left( idx^2 + ic + \frac{\pi}{2} \right)^2 dx^2 + a^2 x^2 \right) \\
 & \quad \downarrow \text{4254} \\
 & \frac{1}{2} \left( 2ab \int \operatorname{csc} \left( idx^2 + ic + \frac{\pi}{2} \right) dx^2 + \frac{ib^2 \int 1d(-i \tanh(dx^2 + c))}{d} + a^2 x^2 \right) \\
 & \quad \downarrow \text{24} \\
 & \frac{1}{2} \left( 2ab \int \operatorname{csc} \left( idx^2 + ic + \frac{\pi}{2} \right) dx^2 + a^2 x^2 + \frac{b^2 \tanh(c + dx^2)}{d} \right) \\
 & \quad \downarrow \text{4257} \\
 & \frac{1}{2} \left( a^2 x^2 + \frac{2ab \arctan(\sinh(c + dx^2))}{d} + \frac{b^2 \tanh(c + dx^2)}{d} \right)
 \end{aligned}$$

input `Int[x*(a + b*Sech[c + d*x^2])^2,x]`



output  $(a^2x^2 + (2ab\text{ArcTan}[\text{Sinh}[c + dx^2]])/d + (b^2\text{Tanh}[c + dx^2])/d)/2$

### 3.12.3.1 Defintions of rubi rules used

rule 24  $\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

rule 3042  $\text{Int}[u_, x\_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4254  $\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[-d^{(-1)} \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x], x] /; \text{FreeQ}[\{c, d\}, x] \ \&\& \ \text{IGtQ}[n/2, 0]$

rule 4257  $\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

rule 4260  $\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^{2}, x\_Symbol] \rightarrow \text{Simp}[a^2*x, x] + (\text{Simp}[2*a*b \text{Int}[\text{Csc}[c + d*x], x], x] + \text{Simp}[b^2 \text{Int}[\text{Csc}[c + d*x]^2, x], x]) /; \text{FreeQ}[\{a, b, c, d\}, x]$

rule 5959  $\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*\text{Sech}[(c_.) + (d_.)*(x_.)^{(n_.)])}^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[1/n \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*\text{Sech}[c + d*x])^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{IGtQ}[\text{Simplify}[(m + 1)/n], 0] \ \&\& \ \text{IntegerQ}[p]$

### 3.12.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result
parts	$\frac{a^2 x^2}{2} + \frac{ab \arctan(\sinh(dx^2+c))}{d} + \frac{b^2 \tanh(dx^2+c)}{2d}$
derivativedivides	$\frac{a^2(dx^2+c)+4ab \arctan(e^{dx^2+c})+b^2 \tanh(dx^2+c)}{2d}$
default	$\frac{a^2(dx^2+c)+4ab \arctan(e^{dx^2+c})+b^2 \tanh(dx^2+c)}{2d}$
risch	$\frac{a^2 x^2}{2} - \frac{b^2}{d(1+e^{2dx^2+2c})} + \frac{iba \ln(e^{dx^2+c+i})}{d} - \frac{iba \ln(e^{dx^2+c-i})}{d}$
parallelrisch	$\frac{a^2 dx^2 \cosh(dx^2+c) - 2i \cosh(dx^2+c) \ln(\tanh(\frac{dx^2}{2} + \frac{c}{2}) - i) ab + 2i \cosh(dx^2+c) \ln(\tanh(\frac{dx^2}{2} + \frac{c}{2}) + i) ab + b^2 \sinh(dx^2+c)}{2d \cosh(dx^2+c)}$

input `int(x*(a+b*sech(d*x^2+c))^2,x,method=_RETURNVERBOSE)`

output `1/2*a^2*x^2+a*b*arctan(sinh(d*x^2+c))/d+1/2*b^2*tanh(d*x^2+c)/d`

### 3.12.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(40) = 80.

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.41

$$\int x(a + b \operatorname{sech}(c + dx^2))^2 dx$$

$$= \frac{a^2 dx^2 \cosh(dx^2 + c)^2 + 2a^2 dx^2 \cosh(dx^2 + c) \sinh(dx^2 + c) + a^2 dx^2 \sinh(dx^2 + c)^2 + a^2 dx^2 - 2b^2 + 4}{2(d \cosh(dx^2 + c))^2 + 2d \cosh(dx^2 + c)}$$

input `integrate(x*(a+b*sech(d*x^2+c))^2,x, algorithm="fracas")`

output `1/2*(a^2*d*x^2*cosh(d*x^2 + c)^2 + 2*a^2*d*x^2*cosh(d*x^2 + c)*sinh(d*x^2 + c) + a^2*d*x^2*sinh(d*x^2 + c)^2 + a^2*d*x^2 - 2*b^2 + 4*(a*b*cosh(d*x^2 + c)^2 + 2*a*b*cosh(d*x^2 + c)*sinh(d*x^2 + c) + a*b*sinh(d*x^2 + c)^2 + a*b)*arctan(cosh(d*x^2 + c) + sinh(d*x^2 + c)))/(d*cosh(d*x^2 + c)^2 + 2*d*cosh(d*x^2 + c)*sinh(d*x^2 + c) + d*sinh(d*x^2 + c)^2 + d)`

---

3.12.  $\int x(a + b \operatorname{sech}(c + dx^2))^2 dx$

### 3.12.6 Sympy [F]

$$\int x(a + b \operatorname{sech}(c + dx^2))^2 dx = \int x(a + b \operatorname{sech}(c + dx^2))^2 dx$$

input `integrate(x*(a+b*sech(d*x**2+c))**2,x)`

output `Integral(x*(a + b*sech(c + d*x**2))**2, x)`

### 3.12.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int x(a + b \operatorname{sech}(c + dx^2))^2 dx = \frac{1}{2} a^2 x^2 + \frac{ab \arctan(\sinh(dx^2 + c))}{d} + \frac{b^2}{d(e^{(-2dx^2-2c)} + 1)}$$

input `integrate(x*(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

output `1/2*a^2*x^2 + a*b*arctan(sinh(d*x^2 + c))/d + b^2/(d*(e^(-2*d*x^2 - 2*c) + 1))`

### 3.12.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int x(a + b \operatorname{sech}(c + dx^2))^2 dx = \frac{(dx^2 + c)a^2}{2d} + \frac{2ab \arctan(e^{(dx^2+c)})}{d} - \frac{b^2}{d(e^{(2dx^2+2c)} + 1)}$$

input `integrate(x*(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`

output `1/2*(d*x^2 + c)*a^2/d + 2*a*b*arctan(e^(d*x^2 + c))/d - b^2/(d*(e^(2*d*x^2 + 2*c) + 1))`

**3.12.9 Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.75

$$\int x(a + b \operatorname{sech}(c + dx^2))^2 dx = \frac{a^2 x^2}{2} + \frac{2 \operatorname{atan}\left(\frac{a b e^{dx^2} e^c \sqrt{d^2}}{d \sqrt{a^2 b^2}}\right) \sqrt{a^2 b^2}}{\sqrt{d^2}} - \frac{b^2}{d (e^{2dx^2+2c} + 1)}$$

input `int(x*(a + b/cosh(c + d*x^2))^2,x)`

output `(a^2*x^2)/2 + (2*atan((a*b*exp(d*x^2)*exp(c)*(d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d^2)^(1/2) - b^2/(d*(exp(2*c + 2*d*x^2) + 1))`

**3.13**  $\int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x} dx$

3.13.1	Optimal result	108
3.13.2	Mathematica [N/A]	108
3.13.3	Rubi [N/A]	109
3.13.4	Maple [N/A] (verified)	109
3.13.5	Fricas [N/A]	110
3.13.6	Sympy [N/A]	110
3.13.7	Maxima [N/A]	110
3.13.8	Giac [N/A]	111
3.13.9	Mupad [N/A]	111

**3.13.1 Optimal result**

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a + b\operatorname{sech}(c + dx^2))^2}{x} dx = \operatorname{Int}\left(\frac{(a + b\operatorname{sech}(c + dx^2))^2}{x}, x\right)$$

output `Unintegrable((a+b*sech(d*x^2+c))^2/x,x)`

**3.13.2 Mathematica [N/A]**

Not integrable

Time = 27.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a + b\operatorname{sech}(c + dx^2))^2}{x} dx = \int \frac{(a + b\operatorname{sech}(c + dx^2))^2}{x} dx$$

input `Integrate[(a + b*Sech[c + d*x^2])^2/x,x]`

output `Integrate[(a + b*Sech[c + d*x^2])^2/x, x]`

---

3.13.  $\int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x} dx$

### 3.13.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x} dx$$

↓ 5961

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x} dx$$

input `Int[(a + b*Sech[c + d*x^2])^2/x,x]`

output `$Aborted`

#### 3.13.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.13.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \operatorname{sech}(dx^2 + c))^2}{x} dx$$

input `int((a+b*sech(d*x^2+c))^2/x,x)`

output `int((a+b*sech(d*x^2+c))^2/x,x)`

---

3.13.  $\int \frac{(a+b \operatorname{sech}(c+dx^2))^2}{x} dx$

**3.13.5 Fracas [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x} dx = \int \frac{(b \operatorname{sech}(dx^2 + c) + a)^2}{x} dx$$

input `integrate((a+b*sech(d*x^2+c))^2/x,x, algorithm="fricas")`output `integral((b^2*sech(d*x^2 + c)^2 + 2*a*b*sech(d*x^2 + c) + a^2)/x, x)`**3.13.6 Sympy [N/A]**

Not integrable

Time = 6.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x} dx = \int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x} dx$$

input `integrate((a+b*sech(d*x**2+c))**2/x,x)`output `Integral((a + b*sech(c + d*x**2))**2/x, x)`**3.13.7 Maxima [N/A]**

Not integrable

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.78

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x} dx = \int \frac{(b \operatorname{sech}(dx^2 + c) + a)^2}{x} dx$$

input `integrate((a+b*sech(d*x^2+c))^2/x,x, algorithm="maxima")`output `a^2*log(x) - b^2/(d*x^2*e^(2*d*x^2 + 2*c) + d*x^2) + integrate(2*(2*a*b*d*x^2*e^(d*x^2 + c) - b^2)/(d*x^3*e^(2*d*x^2 + 2*c) + d*x^3), x)`

---

3.13.  $\int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x} dx$

**3.13.8 Giac [N/A]**

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x} dx = \int \frac{(b \operatorname{sech}(dx^2 + c) + a)^2}{x} dx$$

input `integrate((a+b*sech(d*x^2+c))^2/x,x, algorithm="giac")`output `integrate((b*sech(d*x^2 + c) + a)^2/x, x)`**3.13.9 Mupad [N/A]**

Not integrable

Time = 2.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x} dx = \int \frac{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2}{x} dx$$

input `int((a + b/cosh(c + d*x^2))^2/x,x)`output `int((a + b/cosh(c + d*x^2))^2/x, x)`



$$3.14 \quad \int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x^2} dx$$

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### 3.14.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a + b\operatorname{sech}(c + dx^2))^2}{x^2} dx = \operatorname{Int}\left(\frac{(a + b\operatorname{sech}(c + dx^2))^2}{x^2}, x\right)$$

output `Unintegrable((a+b*sech(d*x^2+c))^2/x^2,x)`

### 3.14.2 Mathematica [N/A]

Not integrable

Time = 10.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a + b\operatorname{sech}(c + dx^2))^2}{x^2} dx = \int \frac{(a + b\operatorname{sech}(c + dx^2))^2}{x^2} dx$$

input `Integrate[(a + b*Sech[c + d*x^2])^2/x^2,x]`

output `Integrate[(a + b*Sech[c + d*x^2])^2/x^2, x]`

---


$$3.14. \quad \int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x^2} dx$$

### 3.14.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x^2} dx$$

↓ 5961

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x^2} dx$$

input `Int[(a + b*Sech[c + d*x^2])^2/x^2,x]`

output `$Aborted`

#### 3.14.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.14.4 Maple [N/A] (verified)

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \operatorname{sech}(dx^2 + c))^2}{x^2} dx$$

input `int((a+b*sech(d*x^2+c))^2/x^2,x)`

output `int((a+b*sech(d*x^2+c))^2/x^2,x)`

---

3.14.  $\int \frac{(a+b \operatorname{sech}(c+dx^2))^2}{x^2} dx$

**3.14.5 Fricas [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x^2} dx = \int \frac{(b \operatorname{sech}(dx^2 + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sech(d*x^2+c))^2/x^2,x, algorithm="fricas")`output `integral((b^2*sech(d*x^2 + c)^2 + 2*a*b*sech(d*x^2 + c) + a^2)/x^2, x)`**3.14.6 Sympy [N/A]**

Not integrable

Time = 1.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x^2} dx = \int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x^2} dx$$

input `integrate((a+b*sech(d*x**2+c))**2/x**2,x)`output `Integral((a + b*sech(c + d*x**2))**2/x**2, x)`**3.14.7 Maxima [N/A]**

Not integrable

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.83

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x^2} dx = \int \frac{(b \operatorname{sech}(dx^2 + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sech(d*x^2+c))^2/x^2,x, algorithm="maxima")`output `-b^2/(d*x^3*e^(2*d*x^2 + 2*c) + d*x^3) - a^2/x + integrate((4*a*b*d*x^2*e^(d*x^2 + c) - 3*b^2)/(d*x^4*e^(2*d*x^2 + 2*c) + d*x^4), x)`

---

3.14.  $\int \frac{(a+b\operatorname{sech}(c+dx^2))^2}{x^2} dx$

**3.14.8 Giac [N/A]**

Not integrable

Time = 0.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x^2} dx = \int \frac{(b \operatorname{sech}(dx^2 + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sech(d*x^2+c))^2/x^2,x, algorithm="giac")`output `integrate((b*sech(d*x^2 + c) + a)^2/x^2, x)`**3.14.9 Mupad [N/A]**

Not integrable

Time = 2.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a + b \operatorname{sech}(c + dx^2))^2}{x^2} dx = \int \frac{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2}{x^2} dx$$

input `int((a + b/cosh(c + d*x^2))^2/x^2,x)`output `int((a + b/cosh(c + d*x^2))^2/x^2, x)`

## 3.15 $\int x \operatorname{sech}^7(a + bx^2) dx$

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### 3.15.1 Optimal result

Integrand size = 12, antiderivative size = 90

$$\int x \operatorname{sech}^7(a + bx^2) dx = \frac{5 \arctan(\sinh(a + bx^2))}{32b} + \frac{5 \operatorname{sech}(a + bx^2) \tanh(a + bx^2)}{32b} \\ + \frac{5 \operatorname{sech}^3(a + bx^2) \tanh(a + bx^2)}{48b} + \frac{\operatorname{sech}^5(a + bx^2) \tanh(a + bx^2)}{12b}$$

output `5/32*arctan(sinh(b*x^2+a))/b+5/32*sech(b*x^2+a)*tanh(b*x^2+a)/b+5/48*sech(b*x^2+a)^3*tanh(b*x^2+a)/b+1/12*sech(b*x^2+a)^5*tanh(b*x^2+a)/b`

### 3.15.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int x \operatorname{sech}^7(a + bx^2) dx = \frac{5 \arctan(\sinh(a + bx^2))}{32b} + \frac{5 \operatorname{sech}(a + bx^2) \tanh(a + bx^2)}{32b} \\ + \frac{5 \operatorname{sech}^3(a + bx^2) \tanh(a + bx^2)}{48b} + \frac{\operatorname{sech}^5(a + bx^2) \tanh(a + bx^2)}{12b}$$

input `Integrate[x*Sech[a + b*x^2]^7,x]`

output `(5*ArcTan[Sinh[a + b*x^2]])/(32*b) + (5*Sech[a + b*x^2]*Tanh[a + b*x^2])/(32*b) + (5*Sech[a + b*x^2]^3*Tanh[a + b*x^2])/(48*b) + (Sech[a + b*x^2]^5*Tanh[a + b*x^2])/(12*b)`

**3.15.3 Rubi [A] (verified)**

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {5959, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x \operatorname{sech}^7(a + bx^2) dx \\
 & \quad \downarrow \text{5959} \\
 & \frac{1}{2} \int \operatorname{sech}^7(bx^2 + a) dx^2 \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \int \csc\left(ibx^2 + ia + \frac{\pi}{2}\right)^7 dx^2 \\
 & \quad \downarrow \text{4255} \\
 & \frac{1}{2} \left( \frac{5}{6} \int \operatorname{sech}^5(bx^2 + a) dx^2 + \frac{\tanh(a + bx^2) \operatorname{sech}^5(a + bx^2)}{6b} \right) \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \left( \frac{\tanh(a + bx^2) \operatorname{sech}^5(a + bx^2)}{6b} + \frac{5}{6} \int \csc\left(ibx^2 + ia + \frac{\pi}{2}\right)^5 dx^2 \right) \\
 & \quad \downarrow \text{4255} \\
 & \frac{1}{2} \left( \frac{5}{6} \left( \frac{3}{4} \int \operatorname{sech}^3(bx^2 + a) dx^2 + \frac{\tanh(a + bx^2) \operatorname{sech}^3(a + bx^2)}{4b} \right) + \frac{\tanh(a + bx^2) \operatorname{sech}^5(a + bx^2)}{6b} \right) \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \left( \frac{\tanh(a + bx^2) \operatorname{sech}^5(a + bx^2)}{6b} + \frac{5}{6} \left( \frac{\tanh(a + bx^2) \operatorname{sech}^3(a + bx^2)}{4b} + \frac{3}{4} \int \csc\left(ibx^2 + ia + \frac{\pi}{2}\right)^3 dx^2 \right) \right) \\
 & \quad \downarrow \text{4255} \\
 & \frac{1}{2} \left( \frac{5}{6} \left( \frac{3}{4} \left( \frac{1}{2} \int \operatorname{sech}(bx^2 + a) dx^2 + \frac{\tanh(a + bx^2) \operatorname{sech}(a + bx^2)}{2b} \right) + \frac{\tanh(a + bx^2) \operatorname{sech}^3(a + bx^2)}{4b} \right) + \frac{\tanh(a + bx^2) \operatorname{sech}^5(a + bx^2)}{6b} \right)
 \end{aligned}$$

$$\begin{aligned} & \downarrow 3042 \\ & \frac{1}{2} \left( \frac{\tanh(a+bx^2) \operatorname{sech}^5(a+bx^2)}{6b} + \frac{5}{6} \left( \frac{\tanh(a+bx^2) \operatorname{sech}^3(a+bx^2)}{4b} + \frac{3}{4} \left( \frac{\tanh(a+bx^2) \operatorname{sech}(a+bx^2)}{2b} + \frac{1}{2} \right) \right) \right) \\ & \downarrow 4257 \\ & \frac{1}{2} \left( \frac{5}{6} \left( \frac{3}{4} \left( \frac{\arctan(\sinh(a+bx^2))}{2b} + \frac{\tanh(a+bx^2) \operatorname{sech}(a+bx^2)}{2b} \right) + \frac{\tanh(a+bx^2) \operatorname{sech}^3(a+bx^2)}{4b} \right) + \frac{\tanh(a+bx^2) \operatorname{sech}^5(a+bx^2)}{6b} \right) \end{aligned}$$

input `Int[x*Sech[a + b*x^2]^7,x]`

output `((Sech[a + b*x^2]^5*Tanh[a + b*x^2])/(6*b) + (5*((Sech[a + b*x^2]^3*Tanh[a + b*x^2])/(4*b) + (3*(ArcTan[Sinh[a + b*x^2]]/(2*b) + (Sech[a + b*x^2]*Tanh[a + b*x^2])/(2*b))))/4))/6)/2`

### 3.15.3.1 Defintions of rubi rules used

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 4255 `Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1)) Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

rule 4257 `Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

### 3.15.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{\left(\frac{\operatorname{sech}(bx^2+a)^5}{6} + \frac{5 \operatorname{sech}(bx^2+a)^3}{24} + \frac{5 \operatorname{sech}(bx^2+a)}{16}\right) \tanh(bx^2+a) + \frac{5 \arctan\left(\frac{e^{bx^2+a}}{8}\right)}{8}}{2b}$
default	$\frac{\left(\frac{\operatorname{sech}(bx^2+a)^5}{6} + \frac{5 \operatorname{sech}(bx^2+a)^3}{24} + \frac{5 \operatorname{sech}(bx^2+a)}{16}\right) \tanh(bx^2+a) + \frac{5 \arctan\left(\frac{e^{bx^2+a}}{8}\right)}{8}}{2b}$
risch	$\frac{e^{bx^2+a} \left(15 e^{10bx^2+10a} + 85 e^{8bx^2+8a} + 198 e^{6bx^2+6a} - 198 e^{4bx^2+4a} - 85 e^{2bx^2+2a} - 15\right)}{48b \left(e^{2bx^2+2a} + 1\right)^6} + \frac{5i \ln\left(e^{bx^2+a} + i\right)}{32b} - \frac{5i \ln\left(e^{bx^2+a} - i\right)}{32b}$
parallelrisch	$\frac{15i(-10 - \cosh(6bx^2+6a)) - 6 \cosh(4bx^2+4a) - 15 \cosh(2bx^2+2a)}{96b(10 + \cosh(6bx^2+6a) + 6 \cosh(4bx^2+4a) + 6 \cosh(2bx^2+2a))} \ln\left(\tanh\left(\frac{bx^2+a}{2}\right) - i\right) + 15i(10 + \cosh(6bx^2+6a) + 6 \cosh(4bx^2+4a) + 6 \cosh(2bx^2+2a))$

input `int(x*sech(b*x^2+a)^7,x,method=_RETURNVERBOSE)`

output `1/2/b*((1/6*sech(b*x^2+a)^5+5/24*sech(b*x^2+a)^3+5/16*sech(b*x^2+a))*tanh(b*x^2+a)+5/8*arctan(exp(b*x^2+a)))`

### 3.15.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1918 vs. 2(82) = 164.

Time = 0.26 (sec) , antiderivative size = 1918, normalized size of antiderivative = 21.31

$$\int x \operatorname{sech}^7(a + bx^2) dx = \text{Too large to display}$$

input `integrate(x*sech(b*x^2+a)^7,x, algorithm="fricas")`



```

output 1/48*(15*cosh(b*x^2 + a)^11 + 165*cosh(b*x^2 + a)*sinh(b*x^2 + a)^10 + 15*
sinh(b*x^2 + a)^11 + 5*(165*cosh(b*x^2 + a)^2 + 17)*sinh(b*x^2 + a)^9 + 85
*cosh(b*x^2 + a)^9 + 45*(55*cosh(b*x^2 + a)^3 + 17*cosh(b*x^2 + a))*sinh(b
*x^2 + a)^8 + 18*(275*cosh(b*x^2 + a)^4 + 170*cosh(b*x^2 + a)^2 + 11)*sinh
(b*x^2 + a)^7 + 198*cosh(b*x^2 + a)^7 + 42*(165*cosh(b*x^2 + a)^5 + 170*co
sh(b*x^2 + a)^3 + 33*cosh(b*x^2 + a))*sinh(b*x^2 + a)^6 + 18*(385*cosh(b*x
^2 + a)^6 + 595*cosh(b*x^2 + a)^4 + 231*cosh(b*x^2 + a)^2 - 11)*sinh(b*x^2
+ a)^5 - 198*cosh(b*x^2 + a)^5 + 90*(55*cosh(b*x^2 + a)^7 + 119*cosh(b*x^
2 + a)^5 + 77*cosh(b*x^2 + a)^3 - 11*cosh(b*x^2 + a))*sinh(b*x^2 + a)^4 +
5*(495*cosh(b*x^2 + a)^8 + 1428*cosh(b*x^2 + a)^6 + 1386*cosh(b*x^2 + a)^4
- 396*cosh(b*x^2 + a)^2 - 17)*sinh(b*x^2 + a)^3 - 85*cosh(b*x^2 + a)^3 +
3*(275*cosh(b*x^2 + a)^9 + 1020*cosh(b*x^2 + a)^7 + 1386*cosh(b*x^2 + a)^5
- 660*cosh(b*x^2 + a)^3 - 85*cosh(b*x^2 + a))*sinh(b*x^2 + a)^2 + 15*(cos
h(b*x^2 + a)^12 + 12*cosh(b*x^2 + a)*sinh(b*x^2 + a)^11 + sinh(b*x^2 + a)^
12 + 6*(11*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^10 + 6*cosh(b*x^2 + a)^1
0 + 20*(11*cosh(b*x^2 + a)^3 + 3*cosh(b*x^2 + a))*sinh(b*x^2 + a)^9 + 15*(
33*cosh(b*x^2 + a)^4 + 18*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^8 + 15*co
sh(b*x^2 + a)^8 + 24*(33*cosh(b*x^2 + a)^5 + 30*cosh(b*x^2 + a)^3 + 5*cosh
(b*x^2 + a))*sinh(b*x^2 + a)^7 + 4*(231*cosh(b*x^2 + a)^6 + 315*cosh(b*x^2
+ a)^4 + 105*cosh(b*x^2 + a)^2 + 5)*sinh(b*x^2 + a)^6 + 20*cosh(b*x^2 ...

```

### 3.15.6 Sympy [F]

$$\int x \operatorname{sech}^7(a + bx^2) dx = \int x \operatorname{sech}^7(a + bx^2) dx$$

```
input integrate(x*sech(b*x**2+a)**7,x)
```

```
output Integral(x*sech(a + b*x**2)**7, x)
```

### 3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(82) = 164.

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.02

$$\int x \operatorname{sech}^7(a + bx^2) dx = -\frac{5 \arctan\left(e^{(-bx^2-a)}\right)}{16b} + \frac{15e^{(-bx^2-a)} + 85e^{(-3bx^2-3a)} + 198e^{(-5bx^2-5a)} - 198e^{(-7bx^2-7a)} - 85e^{(-9bx^2-9a)} - 15e^{(-11bx^2-11a)}}{48b(6e^{(-2bx^2-2a)} + 15e^{(-4bx^2-4a)} + 20e^{(-6bx^2-6a)} + 15e^{(-8bx^2-8a)} + 6e^{(-10bx^2-10a)} + e^{(-12bx^2-12a)} + 1)}$$

input `integrate(x*sech(b*x^2+a)^7,x, algorithm="maxima")`

output `-5/16*arctan(e^(-b*x^2 - a))/b + 1/48*(15*e^(-b*x^2 - a) + 85*e^(-3*b*x^2 - 3*a) + 198*e^(-5*b*x^2 - 5*a) - 198*e^(-7*b*x^2 - 7*a) - 85*e^(-9*b*x^2 - 9*a) - 15*e^(-11*b*x^2 - 11*a))/(b*(6*e^(-2*b*x^2 - 2*a) + 15*e^(-4*b*x^2 - 4*a) + 20*e^(-6*b*x^2 - 6*a) + 15*e^(-8*b*x^2 - 8*a) + 6*e^(-10*b*x^2 - 10*a) + e^(-12*b*x^2 - 12*a) + 1))`

### 3.15.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int x \operatorname{sech}^7(a + bx^2) dx = \frac{5 \left( \pi + 2 \arctan\left(\frac{1}{2} \left( e^{(2bx^2+2a)} - 1 \right) e^{(-bx^2-a)} \right) \right)}{64b} + \frac{15 \left( e^{(bx^2+a)} - e^{(-bx^2-a)} \right)^5 + 160 \left( e^{(bx^2+a)} - e^{(-bx^2-a)} \right)^3 + 528 e^{(bx^2+a)} - 528 e^{(-bx^2-a)}}{48 \left( \left( e^{(bx^2+a)} - e^{(-bx^2-a)} \right)^2 + 4 \right)^3 b}$$

input `integrate(x*sech(b*x^2+a)^7,x, algorithm="giac")`

output `5/64*(pi + 2*arctan(1/2*(e^(2*b*x^2 + 2*a) - 1)*e^(-b*x^2 - a)))/b + 1/48*(15*(e^(b*x^2 + a) - e^(-b*x^2 - a))^5 + 160*(e^(b*x^2 + a) - e^(-b*x^2 - a))^3 + 528*e^(b*x^2 + a) - 528*e^(-b*x^2 - a))/(((e^(b*x^2 + a) - e^(-b*x^2 - a))^2 + 4)^3*b)`

**3.15.9 Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 395, normalized size of antiderivative = 4.39

$$\int x \operatorname{sech}^7(a + bx^2) dx = \frac{5 \operatorname{atan}\left(\frac{e^a e^{bx^2} \sqrt{b^2}}{b}\right)}{16 \sqrt{b^2}} - \frac{8 e^{3bx^2+3a}}{3b(5e^{2bx^2+2a} + 10e^{4bx^2+4a} + 10e^{6bx^2+6a} + 5e^{8bx^2+8a} + e^{10bx^2+10a} + 1)}{e^{bx^2+a}} - \frac{b(4e^{2bx^2+2a} + 6e^{4bx^2+4a} + 4e^{6bx^2+6a} + e^{8bx^2+8a} + 1)}{5e^{bx^2+a}} + \frac{5e^{bx^2+a}}{24b(2e^{2bx^2+2a} + e^{4bx^2+4a} + 1)} - \frac{16e^{5bx^2+5a}}{3b(6e^{2bx^2+2a} + 15e^{4bx^2+4a} + 20e^{6bx^2+6a} + 15e^{8bx^2+8a} + 6e^{10bx^2+10a} + e^{12bx^2+12a} + 1)} + \frac{e^{bx^2+a}}{6b(3e^{2bx^2+2a} + 3e^{4bx^2+4a} + e^{6bx^2+6a} + 1)} + \frac{5e^{bx^2+a}}{16b(e^{2bx^2+2a} + 1)}$$

input `int(x/cosh(a + b*x^2)^7,x)`

output

```
(5*atan((exp(a)*exp(b*x^2)*(b^2)^(1/2))/b))/(16*(b^2)^(1/2)) - (8*exp(3*a + 3*b*x^2))/(3*b*(5*exp(2*a + 2*b*x^2) + 10*exp(4*a + 4*b*x^2) + 10*exp(6*a + 6*b*x^2) + 5*exp(8*a + 8*b*x^2) + exp(10*a + 10*b*x^2) + 1)) - exp(a + b*x^2)/(b*(4*exp(2*a + 2*b*x^2) + 6*exp(4*a + 4*b*x^2) + 4*exp(6*a + 6*b*x^2) + exp(8*a + 8*b*x^2) + 1)) + (5*exp(a + b*x^2))/(24*b*(2*exp(2*a + 2*b*x^2) + exp(4*a + 4*b*x^2) + 1)) - (16*exp(5*a + 5*b*x^2))/(3*b*(6*exp(2*a + 2*b*x^2) + 15*exp(4*a + 4*b*x^2) + 20*exp(6*a + 6*b*x^2) + 15*exp(8*a + 8*b*x^2) + 6*exp(10*a + 10*b*x^2) + exp(12*a + 12*b*x^2) + 1)) + exp(a + b*x^2)/(6*b*(3*exp(2*a + 2*b*x^2) + 3*exp(4*a + 4*b*x^2) + exp(6*a + 6*b*x^2) + 1)) + (5*exp(a + b*x^2))/(16*b*(exp(2*a + 2*b*x^2) + 1))
```

### 3.16 $\int \frac{x^5}{a+b\operatorname{sech}(c+dx^2)} dx$

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#### 3.16.1 Optimal result

Integrand size = 18, antiderivative size = 349

$$\int \frac{x^5}{a+b\operatorname{sech}(c+dx^2)} dx = \frac{x^6}{6a} - \frac{bx^4 \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^4 \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{b \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{b \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

output  $\frac{1}{6}x^6/a - 1/2*b*x^4*\ln(1+a*\exp(d*x^2+c)/(b-(-a^2+b^2)^{(1/2)}))/a/d/(-a^2+b^2)^{(1/2)} + 1/2*b*x^4*\ln(1+a*\exp(d*x^2+c)/(b+(-a^2+b^2)^{(1/2)}))/a/d/(-a^2+b^2)^{(1/2)} - b*x^2*polylog(2,-a*\exp(d*x^2+c)/(b-(-a^2+b^2)^{(1/2)}))/a/d^2/(-a^2+b^2)^{(1/2)} + b*x^2*polylog(2,-a*\exp(d*x^2+c)/(b+(-a^2+b^2)^{(1/2)}))/a/d^2/(-a^2+b^2)^{(1/2)} + b*polylog(3,-a*\exp(d*x^2+c)/(b-(-a^2+b^2)^{(1/2)}))/a/d^3/(-a^2+b^2)^{(1/2)} - b*polylog(3,-a*\exp(d*x^2+c)/(b+(-a^2+b^2)^{(1/2)}))/a/d^3/(-a^2+b^2)^{(1/2)}$

### 3.16.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{a + b \operatorname{sech}(c + dx^2)} dx$$

$$= \frac{\sqrt{-a^2 + b^2} d^3 x^6 - 3bd^2 x^4 \log\left(1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}}\right) + 3bd^2 x^4 \log\left(1 + \frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}}\right) - 6bdx^2 \operatorname{PolyLog}\left(2, \frac{ae^{c+dx^2}}{-b + \sqrt{-a^2 + b^2}}\right) + 6bdx^2 \operatorname{PolyLog}\left(2, \frac{ae^{c+dx^2}}{-b - \sqrt{-a^2 + b^2}}\right) + 6b \operatorname{PolyLog}\left(3, \frac{ae^{c+dx^2}}{-b + \sqrt{-a^2 + b^2}}\right) - 6b \operatorname{PolyLog}\left(3, \frac{ae^{c+dx^2}}{-b - \sqrt{-a^2 + b^2}}\right)}{6a\sqrt{-a^2}}$$

input `Integrate[x^5/(a + b*Sech[c + d*x^2]),x]`

output `(Sqrt[-a^2 + b^2]*d^3*x^6 - 3*b*d^2*x^4*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2])] + 3*b*d^2*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2])] - 6*b*d*x^2*PolyLog[2, (a*E^(c + d*x^2))/(-b + Sqrt[-a^2 + b^2])] + 6*b*d*x^2*PolyLog[2, -(a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2])] + 6*b*PolyLog[3, (a*E^(c + d*x^2))/(-b + Sqrt[-a^2 + b^2])] - 6*b*PolyLog[3, -(a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2])])/(6*a*Sqrt[-a^2 + b^2]*d^3)`

### 3.16.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a + b \operatorname{sech}(c + dx^2)} dx$$

$$\downarrow \text{5959}$$

$$\frac{1}{2} \int \frac{x^4}{a + b \operatorname{sech}(dx^2 + c)} dx^2$$

$$\downarrow \text{3042}$$

$$\frac{1}{2} \int \frac{x^4}{a + b \operatorname{csc}(idx^2 + ic + \frac{\pi}{2})} dx^2$$

$$\downarrow \text{4679}$$

$$\frac{1}{2} \int \left( \frac{x^4}{a} - \frac{bx^4}{a(b + a \cosh(dx^2 + c))} \right) dx^2$$

↓ 2009

$$\frac{1}{2} \left( \frac{2b \operatorname{PolyLog}\left(3, -\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{2b \operatorname{PolyLog}\left(3, -\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{2bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{2bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} \right)$$

input `Int[x^5/(a + b*Sech[c + d*x^2]),x]`

output `(x^6/(3*a) - (b*x^4*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]])/(a*Sqrt[-a^2 + b^2]*d) + (b*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]])/(a*Sqrt[-a^2 + b^2]*d) - (2*b*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]]))/(a*Sqrt[-a^2 + b^2]*d^2) + (2*b*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]]))/(a*Sqrt[-a^2 + b^2]*d^2) + (2*b*PolyLog[3, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]]))/(a*Sqrt[-a^2 + b^2]*d^3) - (2*b*PolyLog[3, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]]))/(a*Sqrt[-a^2 + b^2]*d^3))/2`

### 3.16.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

### 3.16.4 Maple [F]

$$\int \frac{x^5}{a + b \operatorname{sech}(dx^2 + c)} dx$$

input `int(x^5/(a+b*sech(d*x^2+c)),x)`

output `int(x^5/(a+b*sech(d*x^2+c)),x)`

### 3.16.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs.  $2(311) = 622$ .

Time = 0.30 (sec) , antiderivative size = 731, normalized size of antiderivative = 2.09

$$\int \frac{x^5}{a + b \operatorname{sech}(c + dx^2)} dx$$

$$= \frac{(a^2 - b^2)d^3 x^6 + 6 abdx^2 \sqrt{-\frac{a^2 - b^2}{a^2}} \operatorname{Li}_2\left(-\frac{b \cosh(dx^2 + c) + b \sinh(dx^2 + c) + (a \cosh(dx^2 + c) + a \sinh(dx^2 + c)) \sqrt{-\frac{a^2 - b^2}{a^2} + a}}{a} + 1\right)}{1}$$

input `integrate(x^5/(a+b*sech(d*x^2+c)),x, algorithm="fricas")`

output

```
1/6*((a^2 - b^2)*d^3*x^6 + 6*a*b*d*x^2*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 6*a*b*d*x^2*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 3*a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cosh(d*x^2 + c) + 2*a*sinh(d*x^2 + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) + 3*a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*log(2*a*cosh(d*x^2 + c) + 2*a*sinh(d*x^2 + c) - 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) - 6*a*b*sqrt(-(a^2 - b^2)/a^2)*polylog(3, -(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) + 6*a*b*sqrt(-(a^2 - b^2)/a^2)*polylog(3, -(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) + 3*(a*b*d^2*x^4 - a*b*c^2)*sqrt(-(a^2 - b^2)/a^2)*log((b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a) - 3*(a*b*d^2*x^4 - a*b*c^2)*sqrt(-(a^2 - b^2)/a^2)*log((b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a))/((a^3 - a*b^2)*d^3)
```

---

3.16.  $\int \frac{x^5}{a + b \operatorname{sech}(c + dx^2)} dx$

**3.16.6 Sympy [F]**

$$\int \frac{x^5}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^5}{a + b \operatorname{sech}(c + dx^2)} dx$$

input `integrate(x**5/(a+b*sech(d*x**2+c)),x)`

output `Integral(x**5/(a + b*sech(c + d*x**2)), x)`

**3.16.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{x^5}{a + b \operatorname{sech}(c + dx^2)} dx = \text{Exception raised: ValueError}$$

input `integrate(x^5/(a+b*sech(d*x^2+c)),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

**3.16.8 Giac [F]**

$$\int \frac{x^5}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^5}{b \operatorname{sech}(dx^2 + c) + a} dx$$

input `integrate(x^5/(a+b*sech(d*x^2+c)),x, algorithm="giac")`

output `integrate(x^5/(b*sech(d*x^2 + c) + a), x)`



**3.16.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x^5}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^5}{a + \frac{b}{\cosh(dx^2+c)}} dx$$

input `int(x^5/(a + b/cosh(c + d*x^2)),x)`output `int(x^5/(a + b/cosh(c + d*x^2)), x)`

### 3.17 $\int \frac{x^4}{a+b\operatorname{sech}(c+dx^2)} dx$

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#### 3.17.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{a + b\operatorname{sech}(c + dx^2)} dx = \operatorname{Int}\left(\frac{x^4}{a + b\operatorname{sech}(c + dx^2)}, x\right)$$

output `Unintegrable(x^4/(a+b*sech(d*x^2+c)),x)`

#### 3.17.2 Mathematica [N/A]

Not integrable

Time = 3.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a + b\operatorname{sech}(c + dx^2)} dx = \int \frac{x^4}{a + b\operatorname{sech}(c + dx^2)} dx$$

input `Integrate[x^4/(a + b*Sech[c + d*x^2]),x]`

output `Integrate[x^4/(a + b*Sech[c + d*x^2]), x]`

### 3.17.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a + b \operatorname{sech}(c + dx^2)} dx$$

↓ 5961

$$\int \frac{x^4}{a + b \operatorname{sech}(c + dx^2)} dx$$

input `Int[x^4/(a + b*Sech[c + d*x^2]),x]`

output `$Aborted`

#### 3.17.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.17.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a + b \operatorname{sech}(dx^2 + c)} dx$$

input `int(x^4/(a+b*sech(d*x^2+c)),x)`

output `int(x^4/(a+b*sech(d*x^2+c)),x)`

**3.17.5 Fricas [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^4}{b \operatorname{sech}(dx^2 + c) + a} dx$$

input `integrate(x^4/(a+b*sech(d*x^2+c)),x, algorithm="fricas")`output `integral(x^4/(b*sech(d*x^2 + c) + a), x)`**3.17.6 Sympy [N/A]**

Not integrable

Time = 0.58 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^4}{a + b \operatorname{sech}(c + dx^2)} dx$$

input `integrate(x**4/(a+b*sech(d*x**2+c)),x)`output `Integral(x**4/(a + b*sech(c + d*x**2)), x)`**3.17.7 Maxima [N/A]**

Not integrable

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.28

$$\int \frac{x^4}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^4}{b \operatorname{sech}(dx^2 + c) + a} dx$$

input `integrate(x^4/(a+b*sech(d*x^2+c)),x, algorithm="maxima")`output `1/5*x^5/a - 2*b*integrate(x^4*e^(d*x^2 + c)/(a^2*e^(2*d*x^2 + 2*c) + 2*a*b*e^(d*x^2 + c) + a^2), x)`

---

3.17.  $\int \frac{x^4}{a+b\operatorname{sech}(c+dx^2)} dx$

**3.17.8 Giac [N/A]**

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^4}{b \operatorname{sech}(dx^2 + c) + a} dx$$

input `integrate(x^4/(a+b*sech(d*x^2+c)),x, algorithm="giac")`output `integrate(x^4/(b*sech(d*x^2 + c) + a), x)`**3.17.9 Mupad [N/A]**

Not integrable

Time = 1.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^4}{a + \frac{b}{\cosh(dx^2+c)}} dx$$

input `int(x^4/(a + b/cosh(c + d*x^2)),x)`output `int(x^4/(a + b/cosh(c + d*x^2)), x)`

### 3.18 $\int \frac{x^3}{a+b\operatorname{sech}(c+dx^2)} dx$

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3.18.8	Giac [F]	137
3.18.9	Mupad [F(-1)]	137

#### 3.18.1 Optimal result

Integrand size = 18, antiderivative size = 241

$$\int \frac{x^3}{a + b\operatorname{sech}(c + dx^2)} dx = \frac{x^4}{4a} - \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2} + \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2}$$

output

```
1/4*x^4/a-1/2*b*x^2*ln(1+a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2)+1/2*b*x^2*ln(1+a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2)-1/2*b*polylog(2,-a*exp(d*x^2+c)/(b-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+1/2*b*polylog(2,-a*exp(d*x^2+c)/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)
```

#### 3.18.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{a + b\operatorname{sech}(c + dx^2)} dx = \frac{dx^2\left(\sqrt{-a^2+b^2}dx^2 - 2b \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right) + 2b \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)\right) - 2b \operatorname{PolyLog}\left(2, \frac{ae^{c+dx^2}}{-b+\sqrt{-a^2+b^2}}\right)}{4a\sqrt{-a^2+b^2}d^2}$$

input `Integrate[x^3/(a + b*Sech[c + d*x^2]),x]`

output `(d*x^2*(Sqrt[-a^2 + b^2]*d*x^2 - 2*b*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]]) + 2*b*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]]) - 2*b*PolyLog[2, (a*E^(c + d*x^2))/(-b + Sqrt[-a^2 + b^2])] + 2*b*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]))])/(4*a*Sqrt[-a^2 + b^2]*d^2)`

### 3.18.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3}{a + b \operatorname{sech}(c + dx^2)} dx \\
 & \quad \downarrow \text{5959} \\
 & \frac{1}{2} \int \frac{x^2}{a + b \operatorname{sech}(dx^2 + c)} dx^2 \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \int \frac{x^2}{a + b \csc\left(idx^2 + ic + \frac{\pi}{2}\right)} dx^2 \\
 & \quad \downarrow \text{4679} \\
 & \frac{1}{2} \int \left( \frac{x^2}{a} - \frac{bx^2}{a(b + a \cosh(dx^2 + c))} \right) dx^2 \\
 & \quad \downarrow \text{2009} \\
 & \frac{1}{2} \left( -\frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{bx^2 \log\left(\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}} + 1\right)}{ad\sqrt{b^2-a^2}} + \frac{bx^2 \log\left(\frac{ae^{c+dx^2}}{\sqrt{b^2-a^2}+b} + 1\right)}{ad\sqrt{b^2-a^2}} + 1 \right)
 \end{aligned}$$

input `Int[x^3/(a + b*Sech[c + d*x^2]),x]`

```
output (x^4/(2*a) - (b*x^2*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]])/(a*
Sqrt[-a^2 + b^2]*d) + (b*x^2*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^
2]]))/(a*Sqrt[-a^2 + b^2]*d) - (b*PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt
[-a^2 + b^2]))))/(a*Sqrt[-a^2 + b^2]*d^2) + (b*PolyLog[2, -((a*E^(c + d*x^
2))/(b + Sqrt[-a^2 + b^2]))))/(a*Sqrt[-a^2 + b^2]*d^2))/2
```

### 3.18.3.1 Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

```
rule 4679 Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

### 3.18.4 Maple [F]

$$\int \frac{x^3}{a + b \operatorname{sech}(dx^2 + c)} dx$$

```
input int(x^3/(a+b*sech(d*x^2+c)),x)
```

```
output int(x^3/(a+b*sech(d*x^2+c)),x)
```



### 3.18.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs.  $2(209) = 418$ .

Time = 0.26 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.22

$$\int \frac{x^3}{a + b \operatorname{sech}(c + dx^2)} dx$$

$$= \frac{(a^2 - b^2)d^2 x^4 + 2abc \sqrt{-\frac{a^2 - b^2}{a^2}} \log \left( 2a \cosh(dx^2 + c) + 2a \sinh(dx^2 + c) + 2a \sqrt{-\frac{a^2 - b^2}{a^2}} + 2b \right) - 2abc \sqrt{-\frac{a^2 - b^2}{a^2}}}{(a^2 - b^2)d^2 x^4 + 2abc \sqrt{-\frac{a^2 - b^2}{a^2}} \log \left( 2a \cosh(dx^2 + c) + 2a \sinh(dx^2 + c) + 2a \sqrt{-\frac{a^2 - b^2}{a^2}} + 2b \right) - 2abc \sqrt{-\frac{a^2 - b^2}{a^2}}}$$

input `integrate(x^3/(a+b*sech(d*x^2+c)),x, algorithm="fracas")`

output  $\frac{1}{4} * ((a^2 - b^2) * d^2 * x^4 + 2 * a * b * c * \sqrt{-(a^2 - b^2) / a^2} * \log(2 * a * \cosh(d * x^2 + c) + 2 * a * \sinh(d * x^2 + c) + 2 * a * \sqrt{-(a^2 - b^2) / a^2} + 2 * b) - 2 * a * b * c * \sqrt{-(a^2 - b^2) / a^2} * \log(2 * a * \cosh(d * x^2 + c) + 2 * a * \sinh(d * x^2 + c) - 2 * a * \sqrt{-(a^2 - b^2) / a^2} + 2 * b) + 2 * a * b * \sqrt{-(a^2 - b^2) / a^2} * \operatorname{dilog}(-(b * \cosh(d * x^2 + c) + b * \sinh(d * x^2 + c) + (a * \cosh(d * x^2 + c) + a * \sinh(d * x^2 + c))) * \sqrt{-(a^2 - b^2) / a^2} + a) / a + 1) - 2 * a * b * \sqrt{-(a^2 - b^2) / a^2} * \operatorname{dilog}(-(b * \cosh(d * x^2 + c) + b * \sinh(d * x^2 + c) - (a * \cosh(d * x^2 + c) + a * \sinh(d * x^2 + c))) * \sqrt{-(a^2 - b^2) / a^2} + a) / a + 1) + 2 * (a * b * d * x^2 + a * b * c) * \sqrt{-(a^2 - b^2) / a^2} * \log((b * \cosh(d * x^2 + c) + b * \sinh(d * x^2 + c) + (a * \cosh(d * x^2 + c) + a * \sinh(d * x^2 + c)) * \sqrt{-(a^2 - b^2) / a^2} + a) / a) - 2 * (a * b * d * x^2 + a * b * c) * \sqrt{-(a^2 - b^2) / a^2} * \log((b * \cosh(d * x^2 + c) + b * \sinh(d * x^2 + c) - (a * \cosh(d * x^2 + c) + a * \sinh(d * x^2 + c)) * \sqrt{-(a^2 - b^2) / a^2} + a) / a) / ((a^3 - a * b^2) * d^2)$

### 3.18.6 Sympy [F]

$$\int \frac{x^3}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^3}{a + b \operatorname{sech}(c + dx^2)} dx$$

input `integrate(x**3/(a+b*sech(d*x**2+c)),x)`

output `Integral(x**3/(a + b*sech(c + d*x**2)), x)`

---

3.18.  $\int \frac{x^3}{a + b \operatorname{sech}(c + dx^2)} dx$

**3.18.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{x^3}{a + b \operatorname{sech}(c + dx^2)} dx = \text{Exception raised: ValueError}$$

input `integrate(x^3/(a+b*sech(d*x^2+c)),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

**3.18.8 Giac [F]**

$$\int \frac{x^3}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^3}{b \operatorname{sech}(dx^2 + c) + a} dx$$

input `integrate(x^3/(a+b*sech(d*x^2+c)),x, algorithm="giac")`

output `integrate(x^3/(b*sech(d*x^2 + c) + a), x)`

**3.18.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x^3}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^3}{a + \frac{b}{\cosh(dx^2+c)}} dx$$

input `int(x^3/(a + b/cosh(c + d*x^2)),x)`

output `int(x^3/(a + b/cosh(c + d*x^2)), x)`

### 3.19 $\int \frac{x^2}{a+b\operatorname{sech}(c+dx^2)} dx$

3.19.1	Optimal result	138
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3.19.9	Mupad [N/A]	141

#### 3.19.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{a + b\operatorname{sech}(c + dx^2)} dx = \operatorname{Int}\left(\frac{x^2}{a + b\operatorname{sech}(c + dx^2)}, x\right)$$

output `Unintegrable(x^2/(a+b*sech(d*x^2+c)),x)`

#### 3.19.2 Mathematica [N/A]

Not integrable

Time = 3.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a + b\operatorname{sech}(c + dx^2)} dx = \int \frac{x^2}{a + b\operatorname{sech}(c + dx^2)} dx$$

input `Integrate[x^2/(a + b*Sech[c + d*x^2]),x]`

output `Integrate[x^2/(a + b*Sech[c + d*x^2]), x]`

### 3.19.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a + b \operatorname{sech}(c + dx^2)} dx$$

↓ 5961

$$\int \frac{x^2}{a + b \operatorname{sech}(c + dx^2)} dx$$

input `Int[x^2/(a + b*Sech[c + d*x^2]),x]`

output `$Aborted`

#### 3.19.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.19.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a + b \operatorname{sech}(dx^2 + c)} dx$$

input `int(x^2/(a+b*sech(d*x^2+c)),x)`

output `int(x^2/(a+b*sech(d*x^2+c)),x)`

**3.19.5 Fricas [N/A]**

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^2}{b \operatorname{sech}(dx^2 + c) + a} dx$$

input `integrate(x^2/(a+b*sech(d*x^2+c)),x, algorithm="fricas")`output `integral(x^2/(b*sech(d*x^2 + c) + a), x)`**3.19.6 Sympy [N/A]**

Not integrable

Time = 0.49 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^2}{a + b \operatorname{sech}(c + dx^2)} dx$$

input `integrate(x**2/(a+b*sech(d*x**2+c)),x)`output `Integral(x**2/(a + b*sech(c + d*x**2)), x)`**3.19.7 Maxima [N/A]**

Not integrable

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.28

$$\int \frac{x^2}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^2}{b \operatorname{sech}(dx^2 + c) + a} dx$$

input `integrate(x^2/(a+b*sech(d*x^2+c)),x, algorithm="maxima")`output `1/3*x^3/a - 2*b*integrate(x^2*e^(d*x^2 + c)/(a^2*e^(2*d*x^2 + 2*c) + 2*a*b*e^(d*x^2 + c) + a^2), x)`

---

3.19.  $\int \frac{x^2}{a+b \operatorname{sech}(c+dx^2)} dx$

**3.19.8 Giac [N/A]**

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^2}{b \operatorname{sech}(dx^2 + c) + a} dx$$

input `integrate(x^2/(a+b*sech(d*x^2+c)),x, algorithm="giac")`output `integrate(x^2/(b*sech(d*x^2 + c) + a), x)`**3.19.9 Mupad [N/A]**

Not integrable

Time = 1.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x^2}{a + \frac{b}{\cosh(dx^2+c)}} dx$$

input `int(x^2/(a + b/cosh(c + d*x^2)),x)`output `int(x^2/(a + b/cosh(c + d*x^2)), x)`

### 3.20 $\int \frac{x}{a+b\operatorname{sech}(c+dx^2)} dx$

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3.20.8	Giac [A] (verification not implemented)	146
3.20.9	Mupad [B] (verification not implemented)	147

#### 3.20.1 Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{x}{a + b\operatorname{sech}(c + dx^2)} dx = \frac{x^2}{2a} - \frac{b \arctan\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+bd}}$$

output  $\frac{1}{2}x^2/a-b*\arctan((a-b)^{(1/2)}*\tanh(1/2*d*x^2+1/2*c)/(a+b)^{(1/2)})/a/d/(a-b)^{(1/2)}/(a+b)^{(1/2)}$

#### 3.20.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x}{a + b\operatorname{sech}(c + dx^2)} dx = \frac{c}{d} + x^2 + \frac{2b \arctan\left(\frac{(-a+b) \tanh\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{2a}$$

input `Integrate[x/(a + b*Sech[c + d*x^2]),x]`

output  $(c/d + x^2 + (2*b*\operatorname{ArcTan}[((-a + b)*\operatorname{Tanh}[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*d)/(2*a)$

### 3.20.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5959, 3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x}{a + b \operatorname{sech}(c + dx^2)} dx \\
 & \quad \downarrow \text{5959} \\
 & \frac{1}{2} \int \frac{1}{a + b \operatorname{sech}(dx^2 + c)} dx^2 \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \int \frac{1}{a + b \csc\left(idx^2 + ic + \frac{\pi}{2}\right)} dx^2 \\
 & \quad \downarrow \text{4270} \\
 & \frac{1}{2} \left( \frac{x^2}{a} - \frac{\int \frac{1}{\frac{a \cosh(dx^2 + c)}{b} + 1} dx^2}{a} \right) \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \left( \frac{x^2}{a} - \frac{\int \frac{1}{\frac{a \sin\left(idx^2 + ic + \frac{\pi}{2}\right)}{b} + 1} dx^2}{a} \right) \\
 & \quad \downarrow \text{3138} \\
 & \frac{1}{2} \left( \frac{x^2}{a} + \frac{2i \int \frac{1}{\left(1 - \frac{a}{b}\right)x^4 + \frac{a+b}{b}} d\left(i \tanh\left(\frac{1}{2}(dx^2 + c)\right)\right)}{ad} \right) \\
 & \quad \downarrow \text{221} \\
 & \frac{1}{2} \left( \frac{x^2}{a} - \frac{2b \arctan\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c + dx^2)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}} \right)
 \end{aligned}$$

input `Int[x/(a + b*Sech[c + d*x^2]),x]`



output  $(x^2/a - (2*b*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x^2)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/2$

### 3.20.3.1 Defintions of rubi rules used

rule 221  $Int[((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /;$  FreeQ[{a, b}, x] && NegQ[a/b]

rule 3042  $Int[u_, x\_Symbol] := Int[DeactivateTrig[u, x], x] /;$  FunctionOfTrigOfLinearQ[u, x]

rule 3138  $Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^{-1}, x\_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

rule 4270  $Int[(csc[(c_) + (d_)*(x_)])*(b_) + (a_)]^{-1}, x\_Symbol] := Simp[x/a, x] - Simp[1/a Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

rule 5959  $Int[(x_)^{(m_)}*((a_) + (b_)*Sech[(c_) + (d_)*(x_)^{(n_)}])^{(p_)}, x\_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /;$  FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]

### 3.20.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

method	result	size
derivativedivides	$-\frac{2b \arctan\left(\frac{(a-b) \tanh\left(\frac{dx^2}{2} + \frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}} + \frac{\ln\left(1 + \tanh\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right)}{a} - \frac{\ln\left(\tanh\left(\frac{dx^2}{2} + \frac{c}{2}\right) - 1\right)}{a}$	90
default	$-\frac{2b \arctan\left(\frac{(a-b) \tanh\left(\frac{dx^2}{2} + \frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}} + \frac{\ln\left(1 + \tanh\left(\frac{dx^2}{2} + \frac{c}{2}\right)\right)}{a} - \frac{\ln\left(\tanh\left(\frac{dx^2}{2} + \frac{c}{2}\right) - 1\right)}{a}$	90
risch	$\frac{x^2}{2a} - \frac{b \ln\left(e^{dx^2+c} + \frac{b\sqrt{-a^2+b^2} + a^2 - b^2}{\sqrt{-a^2+b^2}a}\right)}{2\sqrt{-a^2+b^2}da} + \frac{b \ln\left(e^{dx^2+c} + \frac{b\sqrt{-a^2+b^2} - a^2 + b^2}{\sqrt{-a^2+b^2}a}\right)}{2\sqrt{-a^2+b^2}da}$	144

input `int(x/(a+b*sech(d*x^2+c)),x,method=_RETURNVERBOSE)`

output `1/2/d*(-2*b/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x^2+1/2*c)/((a+b)*(a-b))^(1/2))+1/a*ln(1+tanh(1/2*d*x^2+1/2*c))-1/a*ln(tanh(1/2*d*x^2+1/2*c)-1))`

### 3.20.5 Fracas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 304, normalized size of antiderivative = 4.61

$$\int \frac{x}{a + b \operatorname{sech}(c + dx^2)} dx$$

$$= \left[ \frac{(a^2 - b^2)dx^2 - \sqrt{-a^2 + b^2}b \log\left(\frac{a^2 \cosh(dx^2+c)^2 + a^2 \sinh(dx^2+c)^2 + 2ab \cosh(dx^2+c) - a^2 + 2b^2 + 2(a^2 \cosh(dx^2+c) + ab) \sinh(dx^2+c)}{a \cosh(dx^2+c)^2 + a \sinh(dx^2+c)^2 + 2b \cosh(dx^2+c) + 2(a \cosh(dx^2+c) + b \sinh(dx^2+c))}\right)}{2(a^3 - ab^2)d} \right]$$

input `integrate(x/(a+b*sech(d*x^2+c)),x, algorithm="fricas")`

output `[1/2*((a^2 - b^2)*d*x^2 - sqrt(-a^2 + b^2)*b*log((a^2*cosh(d*x^2 + c)^2 + a^2*sinh(d*x^2 + c)^2 + 2*a*b*cosh(d*x^2 + c) - a^2 + 2*b^2 + 2*(a^2*cosh(d*x^2 + c) + a*b)*sinh(d*x^2 + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c) + b))/(a*cosh(d*x^2 + c)^2 + a*sinh(d*x^2 + c)^2 + 2*b*cosh(d*x^2 + c) + 2*(a*cosh(d*x^2 + c) + b)*sinh(d*x^2 + c) + a)))/((a^3 - a*b^2)*d), 1/2*((a^2 - b^2)*d*x^2 + 2*sqrt(a^2 - b^2)*b*arctan(-(a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c) + b)/sqrt(a^2 - b^2)))/((a^3 - a*b^2)*d)]`

---

3.20.  $\int \frac{x}{a+b\operatorname{sech}(c+dx^2)} dx$

### 3.20.6 Sympy [F]

$$\int \frac{x}{a + b \operatorname{sech}(c + dx^2)} dx = \int \frac{x}{a + b \operatorname{sech}(c + dx^2)} dx$$

input `integrate(x/(a+b*sech(d*x**2+c)),x)`

output `Integral(x/(a + b*sech(c + d*x**2)), x)`

### 3.20.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a + b \operatorname{sech}(c + dx^2)} dx = \text{Exception raised: ValueError}$$

input `integrate(x/(a+b*sech(d*x^2+c)),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f or more de`

### 3.20.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x}{a + b \operatorname{sech}(c + dx^2)} dx = -\frac{b \arctan\left(\frac{ae^{(dx^2+c)}+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}ad} + \frac{dx^2+c}{2ad}$$

input `integrate(x/(a+b*sech(d*x^2+c)),x, algorithm="giac")`

output `-b*arctan((a*e^(d*x^2 + c) + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a*d) + 1/2*(d*x^2 + c)/(a*d)`

---

3.20.  $\int \frac{x}{a+b\operatorname{sech}(c+dx^2)} dx$

**3.20.9 Mupad [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.59

$$\int \frac{x}{a + b \operatorname{sech}(c + dx^2)} dx$$

$$= \frac{x^2}{2a} - \frac{\operatorname{atan}\left(\frac{ad\sqrt{b^2}}{\sqrt{a^4d^2 - a^2b^2d^2}} + \frac{be^{dx^2}e^c\sqrt{a^4d^2 - a^2b^2d^2}}{a^2d\sqrt{b^2}} + \frac{a^2bde^{dx^2}e^c\sqrt{b^2}\sqrt{a^4d^2 - a^2b^2d^2}}{a^6d^2 - a^4b^2d^2}\right)\sqrt{b^2}}{\sqrt{a^4d^2 - a^2b^2d^2}}$$

input `int(x/(a + b/cosh(c + d*x^2)),x)`output `x^2/(2*a) - (atan((a*d*(b^2)^(1/2))/(a^4*d^2 - a^2*b^2*d^2)^(1/2) + (b*exp(d*x^2)*exp(c)*(a^4*d^2 - a^2*b^2*d^2)^(1/2))/(a^2*d*(b^2)^(1/2)) + (a^2*b*d*exp(d*x^2)*exp(c)*(b^2)^(1/2)*(a^4*d^2 - a^2*b^2*d^2)^(1/2))/(a^6*d^2 - a^4*b^2*d^2))*(b^2)^(1/2))/(a^4*d^2 - a^2*b^2*d^2)^(1/2)`

**3.21** 
$$\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))} dx$$

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**3.21.1 Optimal result**

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))} dx = \operatorname{Int}\left(\frac{1}{x(a+b\operatorname{sech}(c+dx^2))}, x\right)$$

output `Unintegrable(1/x/(a+b*sech(d*x^2+c)),x)`

**3.21.2 Mathematica [N/A]**

Not integrable

Time = 2.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))} dx = \int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))} dx$$

input `Integrate[1/(x*(a + b*Sech[c + d*x^2])),x]`

output `Integrate[1/(x*(a + b*Sech[c + d*x^2])), x]`

### 3.21.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))} dx$$

↓ 5961

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))} dx$$

input `Int[1/(x*(a + b*Sech[c + d*x^2])),x]`

output `$Aborted`

#### 3.21.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.21.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a + b \operatorname{sech}(dx^2 + c))} dx$$

input `int(1/x/(a+b*sech(d*x^2+c)),x)`

output `int(1/x/(a+b*sech(d*x^2+c)),x)`

**3.21.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)x} dx$$

input `integrate(1/x/(a+b*sech(d*x^2+c)),x, algorithm="fricas")`output `integral(1/(b*x*sech(d*x^2 + c) + a*x), x)`**3.21.6 Sympy [N/A]**

Not integrable

Time = 1.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))} dx = \int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))} dx$$

input `integrate(1/x/(a+b*sech(d*x**2+c)),x)`output `Integral(1/(x*(a + b*sech(c + d*x**2))), x)`**3.21.7 Maxima [N/A]**

Not integrable

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.22

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)x} dx$$

input `integrate(1/x/(a+b*sech(d*x^2+c)),x, algorithm="maxima")`output `-2*b*integrate(e^(d*x^2 + c)/(a^2*x*e^(2*d*x^2 + 2*c) + 2*a*b*x*e^(d*x^2 + c) + a^2*x), x) + log(x)/a`

---

3.21.  $\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))} dx$

**3.21.8 Giac [N/A]**

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)x} dx$$

input `integrate(1/x/(a+b*sech(d*x^2+c)),x, algorithm="giac")`output `integrate(1/((b*sech(d*x^2 + c) + a)*x), x)`**3.21.9 Mupad [N/A]**

Not integrable

Time = 2.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))} dx = \int \frac{1}{x \left( a + \frac{b}{\cosh(dx^2+c)} \right)} dx$$

input `int(1/(x*(a + b/cosh(c + d*x^2))),x)`output `int(1/(x*(a + b/cosh(c + d*x^2))), x)`



## 3.22 $\int \frac{a+b\operatorname{sech}(c+dx^2)}{x^2} dx$

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### 3.22.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx = -\frac{a}{x} + b\operatorname{Int}\left(\frac{\operatorname{sech}(c + dx^2)}{x^2}, x\right)$$

output `-a/x+b*Unintegrable(sech(d*x^2+c)/x^2,x)`

### 3.22.2 Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{a + b\operatorname{sech}(c + dx^2)}{x^2} dx$$

input `Integrate[(a + b*Sech[c + d*x^2])/x^2,x]`

output `Integrate[(a + b*Sech[c + d*x^2])/x^2, x]`

### 3.22.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx$$

↓ 2010

$$\int \left( \frac{a}{x^2} + \frac{b \operatorname{sech}(c + dx^2)}{x^2} \right) dx$$

↓ 2009

$$b \int \frac{\operatorname{sech}(dx^2 + c)}{x^2} dx - \frac{a}{x}$$

input `Int[(a + b*Sech[c + d*x^2])/x^2,x]`

output `$Aborted`

#### 3.22.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

**3.22.4 Maple [N/A] (verified)**

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a + b \operatorname{sech}(dx^2 + c)}{x^2} dx$$

input `int((a+b*sech(d*x^2+c))/x^2,x)`output `int((a+b*sech(d*x^2+c))/x^2,x)`**3.22.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{b \operatorname{sech}(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="fricas")`output `integral((b*sech(d*x^2 + c) + a)/x^2, x)`**3.22.6 Sympy [N/A]**

Not integrable

Time = 0.70 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx$$

input `integrate((a+b*sech(d*x**2+c))/x**2,x)`output `Integral((a + b*sech(c + d*x**2))/x**2, x)`

**3.22.7 Maxima [N/A]**

Not integrable

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{b \operatorname{sech}(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="maxima")`output `2*b*integrate(1/(x^2*(e^(d*x^2 + c) + e^(-d*x^2 - c))), x) - a/x`**3.22.8 Giac [N/A]**

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{b \operatorname{sech}(dx^2 + c) + a}{x^2} dx$$

input `integrate((a+b*sech(d*x^2+c))/x^2,x, algorithm="giac")`output `integrate((b*sech(d*x^2 + c) + a)/x^2, x)`**3.22.9 Mupad [N/A]**

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a + b \operatorname{sech}(c + dx^2)}{x^2} dx = \int \frac{a + \frac{b}{\cosh(dx^2+c)}}{x^2} dx$$

input `int((a + b/cosh(c + d*x^2))/x^2,x)`output `int((a + b/cosh(c + d*x^2))/x^2, x)`

---

3.22.  $\int \frac{a+b\operatorname{sech}(c+dx^2)}{x^2} dx$

$$3.23 \quad \int \frac{x^5}{(a+b\operatorname{sech}(c+dx^2))^2} dx$$

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### 3.23.1 Optimal result

Integrand size = 18, antiderivative size = 994

$$\begin{aligned}
\int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx &= \frac{b^2 x^4}{2a^2 (a^2 - b^2) d} + \frac{x^6}{6a^2} - \frac{b^2 x^2 \log\left(1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 (a^2 - b^2) d^2} \\
&+ \frac{b^3 x^4 \log\left(1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}}\right)}{2a^2 (-a^2 + b^2)^{3/2} d} - \frac{bx^4 \log\left(1 + \frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d} \\
&- \frac{b^2 x^2 \log\left(1 + \frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 (a^2 - b^2) d^2} - \frac{b^3 x^4 \log\left(1 + \frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}}\right)}{2a^2 (-a^2 + b^2)^{3/2} d} \\
&+ \frac{bx^4 \log\left(1 + \frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d} - \frac{b^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 (a^2 - b^2) d^3} \\
&+ \frac{b^3 x^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^2} \\
&- \frac{2bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^2} \\
&- \frac{b^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 (a^2 - b^2) d^3} \\
&- \frac{b^3 x^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^2} \\
&+ \frac{2bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^2} \\
&- \frac{b^3 \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^3} \\
&+ \frac{2b \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^2}}{b - \sqrt{-a^2 + b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^3} \\
&+ \frac{b^3 \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^3} \\
&- \frac{2b \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^2}}{b + \sqrt{-a^2 + b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^3} \\
&+ \frac{b^2 x^4 \sinh(c + dx^2)}{2a (a^2 - b^2) d (b + a \cosh(c + dx^2))}
\end{aligned}$$

---

3.23.  $\int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx$

output  $\frac{1}{2}b^2x^4/a^2/(a^2-b^2)/d+1/6x^6/a^2-b^2x^2\ln(1+a\exp(dx^2+c))/(b-(-a^2+b^2)^{(1/2)})/a^2/(a^2-b^2)/d^2+1/2b^3x^4\ln(1+a\exp(dx^2+c))/(b-(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d-b^2x^2\ln(1+a\exp(dx^2+c))/(b+(-a^2+b^2)^{(1/2)})/a^2/(a^2-b^2)/d^2-1/2b^3x^4\ln(1+a\exp(dx^2+c))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d-b^2\text{polylog}(2,-a\exp(dx^2+c))/(b-(-a^2+b^2)^{(1/2)})/a^2/(a^2-b^2)/d^3+b^3x^2\text{polylog}(2,-a\exp(dx^2+c))/(b-(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^2-b^2\text{polylog}(2,-a\exp(dx^2+c))/(b+(-a^2+b^2)^{(1/2)})/a^2/(a^2-b^2)/d^3-b^3x^2\text{polylog}(2,-a\exp(dx^2+c))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^2-b^3\text{polylog}(3,-a\exp(dx^2+c))/(b-(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^3+b^3\text{polylog}(3,-a\exp(dx^2+c))/(b+(-a^2+b^2)^{(1/2)})/a^2/(-a^2+b^2)^{(3/2)}/d^3+1/2b^2x^4\sinh(dx^2+c)/a/(a^2-b^2)/d/(b+a\cosh(dx^2+c))-bx^4\ln(1+a\exp(dx^2+c))/(b-(-a^2+b^2)^{(1/2)})/a^2/d/(-a^2+b^2)^{(1/2)}+bx^4\ln(1+a\exp(dx^2+c))/(b+(-a^2+b^2)^{(1/2)})/a^2/d/(-a^2+b^2)^{(1/2)}-2bx^2\text{polylog}(2,-a\exp(dx^2+c))/(b-(-a^2+b^2)^{(1/2)})/a^2/d^2/(-a^2+b^2)^{(1/2)}+2bx^2\text{polylog}(2,-a\exp(dx^2+c))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^2/(-a^2+b^2)^{(1/2)}+2b\text{polylog}(3,-a\exp(dx^2+c))/(b-(-a^2+b^2)^{(1/2)})/a^2/d^3/(-a^2+b^2)^{(1/2)}-2b\text{polylog}(3,-a\exp(dx^2+c))/(b+(-a^2+b^2)^{(1/2)})/a^2/d^3/(-a^2+b^2)^{(1/2)}$

### 3.23.2 Mathematica [A] (verified)

Time = 6.72 (sec) , antiderivative size = 1565, normalized size of antiderivative = 1.57

$$\int \frac{x^5}{(a + b\operatorname{sech}(c + dx^2))^2} dx = \text{Too large to display}$$

input `Integrate[x^5/(a + b*Sech[c + d*x^2])^2,x]`

output

```

((b + a*Cosh[c + d*x^2])*Sech[c + d*x^2]^2*(x^6*(b + a*Cosh[c + d*x^2]) -
(3*b*E^(2*c))*(b + a*Cosh[c + d*x^2])*(2*b*d^2*E^(2*c)*Sqrt[(-a^2 + b^2)*E^(
(2*c)]*x^4 - 2*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*x^
2))]/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])) - 2*b*d*E^(2*c)*Sqrt[(-a^2 + b^2
)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2
*c)])) - 2*a^2*d^2*E^c*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(-a^2
+ b^2)*E^(2*c)])) + b^2*d^2*E^c*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c -
Sqrt[(-a^2 + b^2)*E^(2*c)])) - 2*a^2*d^2*E^(3*c)*x^4*Log[1 + (a*E^(2*c + d
*x^2))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])) + b^2*d^2*E^(3*c)*x^4*Log[1 +
(a*E^(2*c + d*x^2))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])) - 2*b*d*Sqrt[(-
a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2 +
b^2)*E^(2*c)])) - 2*b*d*E^(2*c)*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*
E^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])) + 2*a^2*d^2*E^c*x^4
*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])) - b^2*d
^2*E^c*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]
)] + 2*a^2*d^2*E^(3*c)*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^c + Sqrt[(-a^2
+ b^2)*E^(2*c)])) - b^2*d^2*E^(3*c)*x^4*Log[1 + (a*E^(2*c + d*x^2))/(b*E^
c + Sqrt[(-a^2 + b^2)*E^(2*c)])) + 2*(1 + E^(2*c))*(-(b*Sqrt[(-a^2 + b^2)*
E^(2*c)]) - 2*a^2*d*E^c*x^2 + b^2*d*E^c*x^2)*PolyLog[2, -((a*E^(2*c + d*x^
2))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])))] - 2*(1 + E^(2*c))*(b*Sqrt[(-...

```

### 3.23.3 Rubi [A] (verified)

Time = 2.39 (sec) , antiderivative size = 990, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx \\
 & \quad \downarrow \text{5959} \\
 & \frac{1}{2} \int \frac{x^4}{(a + b \operatorname{sech}(dx^2 + c))^2} dx^2 \\
 & \quad \downarrow \text{3042} \\
 & \frac{1}{2} \int \frac{x^4}{(a + b \operatorname{csc}(idx^2 + ic + \frac{\pi}{2}))^2} dx^2 \\
 & \quad \downarrow \text{4679}
 \end{aligned}$$

---

3.23.  $\int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx$



$$\frac{1}{2} \int \left( -\frac{2bx^4}{a^2(b + a \cosh(dx^2 + c))} + \frac{x^4}{a^2} + \frac{b^2x^4}{a^2(b + a \cosh(dx^2 + c))^2} \right) dx^2$$

↓ 2009

$$\frac{1}{2} \left( \frac{x^6}{3a^2} - \frac{2b \log\left(\frac{e^{dx^2+c}a}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{a^2\sqrt{b^2-a^2}d} + \frac{b^3 \log\left(\frac{e^{dx^2+c}a}{b-\sqrt{b^2-a^2}} + 1\right) x^4}{a^2(b^2-a^2)^{3/2}d} + \frac{2b \log\left(\frac{e^{dx^2+c}a}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{a^2\sqrt{b^2-a^2}d} - \frac{b^3 \log\left(\frac{e^{dx^2+c}a}{b+\sqrt{b^2-a^2}} + 1\right) x^4}{a^2(b^2-a^2)^{3/2}d} \right)$$

input `Int[x^5/(a + b*Sech[c + d*x^2])^2,x]`

output

```
((b^2*x^4)/(a^2*(a^2 - b^2)*d) + x^6/(3*a^2) - (2*b^2*x^2*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]])/(a^2*(a^2 - b^2)*d^2) + (b^3*x^4*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]])/(a^2*(-a^2 + b^2)^(3/2)*d) - (2*b*x^4*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]])/(a^2*Sqrt[-a^2 + b^2]*d) - (2*b^2*x^2*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]])/(a^2*(a^2 - b^2)*d^2) - (b^3*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]])/(a^2*(-a^2 + b^2)^(3/2)*d) + (2*b*x^4*Log[1 + (a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]])/(a^2*Sqrt[-a^2 + b^2]*d) - (2*b^2*PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]])/(a^2*(a^2 - b^2)*d^3) + (2*b^3*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (4*b*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]])/(a^2*Sqrt[-a^2 + b^2]*d^2) - (2*b^2*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]])/(a^2*(a^2 - b^2)*d^3) - (2*b^3*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (4*b*x^2*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]])/(a^2*Sqrt[-a^2 + b^2]*d^2) - (2*b^3*PolyLog[3, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + (4*b*PolyLog[3, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (2*b^3*PolyLog[3, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - (4*b*PolyLog[3, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]])/(a^2*Sqrt[-a^2 + b^2]*d^3))...)
```

---

3.23.  $\int \frac{x^5}{(a+b\operatorname{sech}(c+dx^2))^2} dx$

### 3.23.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

### 3.23.4 Maple [F]

$$\int \frac{x^5}{(a + b \operatorname{sech}(dx^2 + c))^2} dx$$

input `int(x^5/(a+b*sech(d*x^2+c))^2,x)`

output `int(x^5/(a+b*sech(d*x^2+c))^2,x)`

### 3.23.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3918 vs.  $2(906) = 1812$ .

Time = 0.33 (sec) , antiderivative size = 3918, normalized size of antiderivative = 3.94

$$\int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \text{Too large to display}$$

input `integrate(x^5/(a+b*sech(d*x^2+c))^2,x, algorithm="fracas")`

---

3.23.  $\int \frac{x^5}{(a+b\operatorname{sech}(c+dx^2))^2} dx$

output

```

1/6*((a^5 - 2*a^3*b^2 + a*b^4)*d^3*x^6 - 6*(a^3*b^2 - a*b^4)*c^2 + ((a^5 -
2*a^3*b^2 + a*b^4)*d^3*x^6 + 6*(a^3*b^2 - a*b^4)*d^2*x^4 - 6*(a^3*b^2 - a
*b^4)*c^2)*cosh(d*x^2 + c)^2 + ((a^5 - 2*a^3*b^2 + a*b^4)*d^3*x^6 + 6*(a^3
*b^2 - a*b^4)*d^2*x^4 - 6*(a^3*b^2 - a*b^4)*c^2)*sinh(d*x^2 + c)^2 - 6*(2*
a^4*b - a^2*b^3 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b
^3)*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*cosh(d*x^2 + c) + 2*(2*a^3*b
^2 - a*b^4 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt(-(
a^2 - b^2)/a^2)*polylog(3, -(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*co
sh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) + 6*(2*a^4*b
- a^2*b^3 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*s
inh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*cosh(d*x^2 + c) + 2*(2*a^3*b^2 -
a*b^4 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt(-(a^2 -
b^2)/a^2)*polylog(3, -(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*
x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2))/a) + 2*((a^4*b - 2*a
^2*b^3 + b^5)*d^3*x^6 + 3*(a^2*b^3 - b^5)*d^2*x^4 - 6*(a^2*b^3 - b^5)*c^2)
*cosh(d*x^2 + c) - 6*(a^3*b^2 - a*b^4 + (a^3*b^2 - a*b^4)*cosh(d*x^2 + c)^
2 + (a^3*b^2 - a*b^4)*sinh(d*x^2 + c)^2 + 2*(a^2*b^3 - b^5)*cosh(d*x^2 + c
) + 2*(a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*cosh(d*x^2 + c))*sinh(d*x^2 + c)
- ((2*a^4*b - a^2*b^3)*d*x^2*cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*d*x^2
*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*d*x^2*cosh(d*x^2 + c) + (2*a...

```

### 3.23.6 Sympy [F]

$$\int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

input `integrate(x**5/(a+b*sech(d*x**2+c))**2,x)`

output `Integral(x**5/(a + b*sech(c + d*x**2))**2, x)`

**3.23.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^5/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

**3.23.8 Giac [F]**

$$\int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^5}{(b \operatorname{sech}(dx^2 + c) + a)^2} dx$$

input `integrate(x^5/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`

output `integrate(x^5/(b*sech(d*x^2 + c) + a)^2, x)`

**3.23.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x^5}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^5}{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2} dx$$

input `int(x^5/(a + b/cosh(c + d*x^2))^2,x)`

output `int(x^5/(a + b/cosh(c + d*x^2))^2, x)`

**3.24**  $\int \frac{x^4}{(a+b\operatorname{sech}(c+dx^2))^2} dx$

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**3.24.1 Optimal result**

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{(a + b\operatorname{sech}(c + dx^2))^2} dx = \operatorname{Int}\left(\frac{x^4}{(a + b\operatorname{sech}(c + dx^2))^2}, x\right)$$

output `Unintegrable(x^4/(a+b*sech(d*x^2+c))^2,x)`

**3.24.2 Mathematica [N/A]**

Not integrable

Time = 20.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a + b\operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^4}{(a + b\operatorname{sech}(c + dx^2))^2} dx$$

input `Integrate[x^4/(a + b*Sech[c + d*x^2])^2,x]`

output `Integrate[x^4/(a + b*Sech[c + d*x^2])^2, x]`

### 3.24.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

↓ 5961

$$\int \frac{x^4}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

input `Int[x^4/(a + b*Sech[c + d*x^2])^2,x]`

output `$Aborted`

#### 3.24.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.24.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a + b \operatorname{sech}(dx^2 + c))^2} dx$$

input `int(x^4/(a+b*sech(d*x^2+c))^2,x)`

output `int(x^4/(a+b*sech(d*x^2+c))^2,x)`

**3.24.5 Fricas [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^4}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^4}{(b \operatorname{sech}(dx^2 + c) + a)^2} dx$$

input `integrate(x^4/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")`output `integral(x^4/(b^2*sech(d*x^2 + c)^2 + 2*a*b*sech(d*x^2 + c) + a^2), x)`**3.24.6 Sympy [N/A]**

Not integrable

Time = 1.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^4}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

input `integrate(x**4/(a+b*sech(d*x**2+c))**2,x)`output `Integral(x**4/(a + b*sech(c + d*x**2))**2, x)`**3.24.7 Maxima [N/A]**

Not integrable

Time = 0.42 (sec) , antiderivative size = 312, normalized size of antiderivative = 17.33

$$\int \frac{x^4}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^4}{(b \operatorname{sech}(dx^2 + c) + a)^2} dx$$

input `integrate(x^4/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

output  $1/5*((a^3*d*e^{(2*c)} - a*b^2*d*e^{(2*c)})*x^5*e^{(2*d*x^2)} - 5*a*b^2*x^3 + (a^3*d - a*b^2*d)*x^5 - (5*b^3*x^3*e^c - 2*(a^2*b*d*e^c - b^3*d*e^c)*x^5)*e^{(d*x^2)})/(a^5*d - a^3*b^2*d + (a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*e^{(2*d*x^2)} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*e^{(d*x^2)}) - \text{integrate}(- (3*a*b^2*x^2 + (3*b^3*x^2*e^c - 2*(2*a^2*b*d*e^c - b^3*d*e^c)*x^4)*e^{(d*x^2)})/(a^5*d - a^3*b^2*d + (a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*e^{(2*d*x^2)} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*e^{(d*x^2)}), x)$

### 3.24.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^4}{(b \operatorname{sech}(dx^2 + c) + a)^2} dx$$

input `integrate(x^4/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`

output `integrate(x^4/(b*sech(d*x^2 + c) + a)^2, x)`

### 3.24.9 Mupad [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^4}{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2} dx$$

input `int(x^4/(a + b/cosh(c + d*x^2))^2,x)`

output `int(x^4/(a + b/cosh(c + d*x^2))^2, x)`



### 3.25 $\int \frac{x^3}{(a+b\operatorname{sech}(c+dx^2))^2} dx$

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#### 3.25.1 Optimal result

Integrand size = 18, antiderivative size = 555

$$\int \frac{x^3}{(a+b\operatorname{sech}(c+dx^2))^2} dx = \frac{x^4}{4a^2} + \frac{b^3 x^2 \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$

$$- \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^3 x^2 \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$

$$+ \frac{bx^2 \log\left(1 + \frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2 \log(b+a\cosh(c+dx^2))}{2a^2(a^2-b^2)d^2}$$

$$+ \frac{b^3 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} - \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$- \frac{b^3 \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} + \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^2}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$+ \frac{b^2 x^2 \sinh(c+dx^2)}{2a(a^2-b^2)d(b+a\cosh(c+dx^2))}$$

output  $\frac{1}{4}x^4/a^2 - 1/2b^2 \ln(b+a \cosh(dx^2+c))/a^2/(a^2-b^2)/d^2 + 1/2b^3x^2 \ln(1+a \exp(dx^2+c)/(b-(-a^2+b^2)^{1/2}))/a^2/(-a^2+b^2)^{3/2}/d - 1/2b^3x^2 \ln(1+a \exp(dx^2+c)/(b+(-a^2+b^2)^{1/2}))/a^2/(-a^2+b^2)^{3/2}/d + 1/2b^3 \operatorname{polylog}(2, -a \exp(dx^2+c)/(b-(-a^2+b^2)^{1/2}))/a^2/(-a^2+b^2)^{3/2}/d^2 - 1/2b^3 \operatorname{polylog}(2, -a \exp(dx^2+c)/(b+(-a^2+b^2)^{1/2}))/a^2/(-a^2+b^2)^{3/2}/d^2 + 1/2b^2x^2 \sinh(dx^2+c)/a/(a^2-b^2)/d/(b+a \cosh(dx^2+c)) - bx^2 \ln(1+a \exp(dx^2+c)/(b-(-a^2+b^2)^{1/2}))/a^2/d/(-a^2+b^2)^{1/2} + bx^2 \ln(1+a \exp(dx^2+c)/(b+(-a^2+b^2)^{1/2}))/a^2/d/(-a^2+b^2)^{1/2} - b \operatorname{polylog}(2, -a \exp(dx^2+c)/(b-(-a^2+b^2)^{1/2}))/a^2/d^2/(-a^2+b^2)^{1/2} + b \operatorname{polylog}(2, -a \exp(dx^2+c)/(b+(-a^2+b^2)^{1/2}))/a^2/d^2/(-a^2+b^2)^{1/2}$

### 3.25.2 Mathematica [A] (verified)

Time = 5.34 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

$$= \frac{(b + a \cosh(c + dx^2)) \operatorname{sech}^2(c + dx^2) \left( (-c + dx^2)(c + dx^2)(b + a \cosh(c + dx^2)) - \frac{2b(a^2 - b^2)(b + a \cosh(c + dx^2))}{\dots} \right)}{\dots}$$

input `Integrate[x^3/(a + b*Sech[c + d*x^2])^2,x]`

output  $((b + a \operatorname{Cosh}[c + d*x^2]) \operatorname{Sech}[c + d*x^2]^2 * ((-c + d*x^2) * (c + d*x^2) * (b + a \operatorname{Cosh}[c + d*x^2]) - (2*b*(a^2 - b^2) * (b + a \operatorname{Cosh}[c + d*x^2]) * (b \operatorname{Sqrt}[-(a^2 - b^2)^2] * (c + d*x^2) + 4*a^2 \operatorname{Sqrt}[-a^2 + b^2] * c \operatorname{ArcTan}[(b + a \operatorname{E}^{(c + d*x^2)})/\operatorname{Sqrt}[a^2 - b^2]] - 2*b^2 \operatorname{Sqrt}[-a^2 + b^2] * c \operatorname{ArcTan}[(b + a \operatorname{E}^{(c + d*x^2)})/\operatorname{Sqrt}[a^2 - b^2]]) - 2*a^2 \operatorname{Sqrt}[a^2 - b^2] * (c + d*x^2) * \operatorname{Log}[1 + (a \operatorname{E}^{(c + d*x^2)})/(b - \operatorname{Sqrt}[-a^2 + b^2])]) + b^2 \operatorname{Sqrt}[a^2 - b^2] * (c + d*x^2) * \operatorname{Log}[1 + (a \operatorname{E}^{(c + d*x^2)})/(b - \operatorname{Sqrt}[-a^2 + b^2])]) + 2*a^2 \operatorname{Sqrt}[a^2 - b^2] * (c + d*x^2) * \operatorname{Log}[1 + (a \operatorname{E}^{(c + d*x^2)})/(b + \operatorname{Sqrt}[-a^2 + b^2])]) - b^2 \operatorname{Sqrt}[a^2 - b^2] * (c + d*x^2) * \operatorname{Log}[1 + (a \operatorname{E}^{(c + d*x^2)})/(b + \operatorname{Sqrt}[-a^2 + b^2])]) - b \operatorname{Sqrt}[-(a^2 - b^2)^2] * \operatorname{Log}[a + 2*b \operatorname{E}^{(c + d*x^2)} + a \operatorname{E}^{(2*(c + d*x^2))}] + \operatorname{Sqrt}[a^2 - b^2] * (-2*a^2 + b^2) * \operatorname{PolyLog}[2, (a \operatorname{E}^{(c + d*x^2)})/(-b + \operatorname{Sqrt}[-a^2 + b^2])] + \operatorname{Sqrt}[a^2 - b^2] * (2*a^2 - b^2) * \operatorname{PolyLog}[2, -((a \operatorname{E}^{(c + d*x^2)})/(b + \operatorname{Sqrt}[-a^2 + b^2]))]) / (- (a^2 - b^2)^2)^{3/2} + (2*a*b^2*d*x^2 \operatorname{Sinh}[c + d*x^2]) / ((a - b) * (a + b))) / (4*a^2*d^2*(a + b \operatorname{Sech}[c + d*x^2])^2)$

3.25.  $\int \frac{x^3}{(a+b \operatorname{sech}(c+dx^2))^2} dx$

### 3.25.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^3}{(a + b \operatorname{sech}(c + dx^2))^2} dx \\
 & \quad \downarrow 5959 \\
 & \frac{1}{2} \int \frac{x^2}{(a + b \operatorname{sech}(dx^2 + c))^2} dx^2 \\
 & \quad \downarrow 3042 \\
 & \frac{1}{2} \int \frac{x^2}{(a + b \operatorname{csc}(idx^2 + ic + \frac{\pi}{2}))^2} dx^2 \\
 & \quad \downarrow 4679 \\
 & \frac{1}{2} \int \left( -\frac{2bx^2}{a^2(b + a \cosh(dx^2 + c))} + \frac{x^2}{a^2} + \frac{b^2x^2}{a^2(b + a \cosh(dx^2 + c))^2} \right) dx^2 \\
 & \quad \downarrow 2009 \\
 & \frac{1}{2} \left( -\frac{2b \operatorname{PolyLog}\left(2, -\frac{ae^{dx^2+c}}{b-\sqrt{b^2-a^2}}\right)}{a^2 d^2 \sqrt{b^2-a^2}} + \frac{2b \operatorname{PolyLog}\left(2, -\frac{ae^{dx^2+c}}{b+\sqrt{b^2-a^2}}\right)}{a^2 d^2 \sqrt{b^2-a^2}} - \frac{b^2 \log(a \cosh(c + dx^2) + b)}{a^2 d^2 (a^2 - b^2)} - \frac{2bx^2 \log\left(\frac{ae^{c+dx^2}}{b-\sqrt{b^2-a^2}}\right)}{a^2 d \sqrt{b^2-a^2}} \right)
 \end{aligned}$$

input `Int[x^3/(a + b*Sech[c + d*x^2])^2,x]`

```
output (x^4/(2*a^2) + (b^3*x^2*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2]])
/(a^2*(-a^2 + b^2)^(3/2)*d) - (2*b*x^2*Log[1 + (a*E^(c + d*x^2))/(b - Sqrt
[-a^2 + b^2]])/(a^2*Sqrt[-a^2 + b^2]*d) - (b^3*x^2*Log[1 + (a*E^(c + d*x^
2)))/(b + Sqrt[-a^2 + b^2]))/(a^2*(-a^2 + b^2)^(3/2)*d) + (2*b*x^2*Log[1 +
(a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]])/(a^2*Sqrt[-a^2 + b^2]*d) - (b^
2*Log[b + a*Cosh[c + d*x^2]])/(a^2*(a^2 - b^2)*d^2) + (b^3*PolyLog[2, -((a
*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2])))/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (
2*b*PolyLog[2, -((a*E^(c + d*x^2))/(b - Sqrt[-a^2 + b^2])))/(a^2*Sqrt[-a^
2 + b^2]*d^2) - (b^3*PolyLog[2, -((a*E^(c + d*x^2))/(b + Sqrt[-a^2 + b^2]
)))/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (2*b*PolyLog[2, -((a*E^(c + d*x^2))/(b
+ Sqrt[-a^2 + b^2])))/(a^2*Sqrt[-a^2 + b^2]*d^2) + (b^2*x^2*Sinh[c + d*x^
2])/(a*(a^2 - b^2)*d*(b + a*Cosh[c + d*x^2]))) / 2
```

### 3.25.3.1 Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

```
rule 4679 Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

### 3.25.4 Maple [F]

$$\int \frac{x^3}{(a + b \operatorname{sech}(dx^2 + c))^2} dx$$

input `int(x^3/(a+b*sech(d*x^2+c))^2,x)`

output `int(x^3/(a+b*sech(d*x^2+c))^2,x)`

### 3.25.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2473 vs.  $2(497) = 994$ .

Time = 0.31 (sec) , antiderivative size = 2473, normalized size of antiderivative = 4.46

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \text{Too large to display}$$

input `integrate(x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="fracas")`

output `1/4*((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4 + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4 + 4*(a^3*b^2 - a*b^4)*d*x^2 + 4*(a^3*b^2 - a*b^4)*c)*cosh(d*x^2 + c)^2 + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4 + 4*(a^3*b^2 - a*b^4)*d*x^2 + 4*(a^3*b^2 - a*b^4)*c)*sinh(d*x^2 + c)^2 + 2*(2*a^4*b - a^2*b^3 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*cosh(d*x^2 + c) + 2*(2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) - 2*(2*a^4*b - a^2*b^3 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*sinh(d*x^2 + c)^2 + 2*(2*a^3*b^2 - a*b^4)*cosh(d*x^2 + c) + 2*(2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*dilog(-(b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) - (a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2) + a)/a + 1) + 2*((2*a^4*b - a^2*b^3)*d*x^2 + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*cosh(d*x^2 + c)^2 + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*sinh(d*x^2 + c)^2 + (2*a^4*b - a^2*b^3)*c + 2*((2*a^3*b^2 - a*b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c)*cosh(d*x^2 + c) + 2*((2*a^3*b^2 - a*b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*cosh(d*x^2 + c))*sinh(d*x^2 + c))*sqrt(-(a^2 - b^2)/a^2)*log((b*cosh(d*x^2 + c) + b*sinh(d*x^2 + c) + (a*c...`

---

3.25.  $\int \frac{x^3}{(a+b\operatorname{sech}(c+dx^2))^2} dx$

### 3.25.6 Sympy [F]

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^3}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

input `integrate(x**3/(a+b*sech(d*x**2+c))**2,x)`

output `Integral(x**3/(a + b*sech(c + d*x**2))**2, x)`

### 3.25.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

### 3.25.8 Giac [F]

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^3}{(b \operatorname{sech}(dx^2 + c) + a)^2} dx$$

input `integrate(x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`

output `integrate(x^3/(b*sech(d*x^2 + c) + a)^2, x)`

**3.25.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^3}{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2} dx$$

input `int(x^3/(a + b/cosh(c + d*x^2))^2,x)`output `int(x^3/(a + b/cosh(c + d*x^2))^2, x)`

**3.26**  $\int \frac{x^2}{(a+b\operatorname{sech}(c+dx^2))^2} dx$

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**3.26.1 Optimal result**

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{(a + b\operatorname{sech}(c + dx^2))^2} dx = \operatorname{Int}\left(\frac{x^2}{(a + b\operatorname{sech}(c + dx^2))^2}, x\right)$$

output `Unintegrable(x^2/(a+b*sech(d*x^2+c))^2,x)`

**3.26.2 Mathematica [N/A]**

Not integrable

Time = 19.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a + b\operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^2}{(a + b\operatorname{sech}(c + dx^2))^2} dx$$

input `Integrate[x^2/(a + b*Sech[c + d*x^2])^2,x]`

output `Integrate[x^2/(a + b*Sech[c + d*x^2])^2, x]`



### 3.26.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

↓ 5961

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

input `Int[x^2/(a + b*Sech[c + d*x^2])^2,x]`

output `$Aborted`

#### 3.26.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.26.4 Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a + b \operatorname{sech}(dx^2 + c))^2} dx$$

input `int(x^2/(a+b*sech(d*x^2+c))^2,x)`

output `int(x^2/(a+b*sech(d*x^2+c))^2,x)`

**3.26.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^2}{(b \operatorname{sech}(dx^2 + c) + a)^2} dx$$

input `integrate(x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")`output `integral(x^2/(b^2*sech(d*x^2 + c)^2 + 2*a*b*sech(d*x^2 + c) + a^2), x)`**3.26.6 Sympy [N/A]**

Not integrable

Time = 0.81 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^2}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

input `integrate(x**2/(a+b*sech(d*x**2+c))**2,x)`output `Integral(x**2/(a + b*sech(c + d*x**2))**2, x)`**3.26.7 Maxima [N/A]**

Not integrable

Time = 0.43 (sec) , antiderivative size = 300, normalized size of antiderivative = 16.67

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^2}{(b \operatorname{sech}(dx^2 + c) + a)^2} dx$$

input `integrate(x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

output  $1/3*((a^3*d*e^{(2*c)} - a*b^2*d*e^{(2*c)})*x^3*e^{(2*d*x^2)} - 3*a*b^2*x + (a^3*d - a*b^2*d)*x^3 - (3*b^3*x*e^c - 2*(a^2*b*d*e^c - b^3*d*e^c)*x^3)*e^{(d*x^2)})/(a^5*d - a^3*b^2*d + (a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*e^{(2*d*x^2)} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*e^{(d*x^2)}) - \text{integrate}(-(a*b^2 + (b^3*e^c - 2*(2*a^2*b*d*e^c - b^3*d*e^c)*x^2)*e^{(d*x^2)})/(a^5*d - a^3*b^2*d + (a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*e^{(2*d*x^2)} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*e^{(d*x^2)}), x)$

### 3.26.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^2}{(b \operatorname{sech}(dx^2 + c) + a)^2} dx$$

input `integrate(x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`

output `integrate(x^2/(b*sech(d*x^2 + c) + a)^2, x)`

### 3.26.9 Mupad [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x^2}{\left(a + \frac{b}{\cosh(dx^2+c)}\right)^2} dx$$

input `int(x^2/(a + b/cosh(c + d*x^2))^2,x)`

output `int(x^2/(a + b/cosh(c + d*x^2))^2, x)`

**3.27**  $\int \frac{x}{(a+b\operatorname{sech}(c+dx^2))^2} dx$

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**3.27.1 Optimal result**

Integrand size = 16, antiderivative size = 123

$$\int \frac{x}{(a+b\operatorname{sech}(c+dx^2))^2} dx = \frac{x^2}{2a^2} - \frac{b(2a^2 - b^2) \arctan\left(\frac{\sqrt{a-b} \tanh(\frac{1}{2}(c+dx^2))}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2 \tanh(c+dx^2)}{2a(a^2 - b^2)d(a+b\operatorname{sech}(c+dx^2))}$$

output

```
1/2*x^2/a^2-b*(2*a^2-b^2)*arctan((a-b)^(1/2)*tanh(1/2*d*x^2+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*b^2*tanh(d*x^2+c)/a/(a^2-b^2)/d/(a+b*sech(d*x^2+c))
```

**3.27.2 Mathematica [A] (verified)**

Time = 0.86 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.79

$$\int \frac{x}{(a+b\operatorname{sech}(c+dx^2))^2} dx = \frac{a\left((a^2 - b^2)^{3/2}(c+dx^2) + (4a^2b - 2b^3) \arctan\left(\frac{(-a+b) \tanh(\frac{1}{2}(c+dx^2))}{\sqrt{a^2-b^2}}\right)\right) \cosh(c+dx^2) + b\left((a^2 - b^2)^{3/2}(c+dx^2) + (4a^2b - 2b^3) \arctan\left(\frac{(-a+b) \tanh(\frac{1}{2}(c+dx^2))}{\sqrt{a^2-b^2}}\right)\right)}{2a^2(a-b)(a+b)\sqrt{a^2 - b^2}d(b+a\cosh(c+dx^2))}$$

input

```
Integrate[x/(a + b*Sech[c + d*x^2])^2,x]
```

---

3.27.  $\int \frac{x}{(a+b\operatorname{sech}(c+dx^2))^2} dx$

output  $(a*((a^2 - b^2)^{(3/2)}*(c + d*x^2) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*x^2] + b*((a^2 - b^2)^{(3/2)}*(c + d*x^2) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x^2)/2])/Sqrt[a^2 - b^2]] + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*x^2]))/(2*a^2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d*(b + a*Cosh[c + d*x^2]))$

### 3.27.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$ , Rules used = {5959, 3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

$$\downarrow \text{5959}$$

$$\frac{1}{2} \int \frac{1}{(a + b \operatorname{sech}(dx^2 + c))^2} dx^2$$

$$\downarrow \text{3042}$$

$$\frac{1}{2} \int \frac{1}{(a + b \csc(idx^2 + ic + \frac{\pi}{2}))^2} dx^2$$

$$\downarrow \text{4272}$$

$$\frac{1}{2} \left( \frac{b^2 \tanh(c + dx^2)}{ad(a^2 - b^2)(a + b \operatorname{sech}(c + dx^2))} - \frac{\int \frac{a^2 - b \operatorname{sech}(dx^2 + c)a - b^2}{a + b \operatorname{sech}(dx^2 + c)} dx^2}{a(a^2 - b^2)} \right)$$

$$\downarrow \text{25}$$

$$\frac{1}{2} \left( \frac{\int \frac{a^2 - b \operatorname{sech}(dx^2 + c)a - b^2}{a + b \operatorname{sech}(dx^2 + c)} dx^2}{a(a^2 - b^2)} + \frac{b^2 \tanh(c + dx^2)}{ad(a^2 - b^2)(a + b \operatorname{sech}(c + dx^2))} \right)$$

$$\downarrow \text{3042}$$

$$\frac{1}{2} \left( \frac{b^2 \tanh(c + dx^2)}{ad(a^2 - b^2)(a + b \operatorname{sech}(c + dx^2))} + \frac{\int \frac{a^2 - b \csc(idx^2 + ic + \frac{\pi}{2})a - b^2}{a + b \csc(idx^2 + ic + \frac{\pi}{2})} dx^2}{a(a^2 - b^2)} \right)$$

---

3.27.  $\int \frac{x}{(a + b \operatorname{sech}(c + dx^2))^2} dx$

$$\begin{aligned}
& \downarrow 4407 \\
& \frac{1}{2} \left( \frac{x^2(a^2-b^2)}{a} - \frac{b(2a^2-b^2) \int \frac{\operatorname{sech}(dx^2+c)}{a+b\operatorname{sech}(dx^2+c)} dx^2}{a(a^2-b^2)} + \frac{b^2 \tanh(c+dx^2)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^2))} \right) \\
& \downarrow 3042 \\
& \frac{1}{2} \left( \frac{b^2 \tanh(c+dx^2)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^2))} + \frac{x^2(a^2-b^2)}{a} - \frac{b(2a^2-b^2) \int \frac{\csc(idx^2+ic+\frac{\pi}{2})}{a+b\csc(idx^2+ic+\frac{\pi}{2})} dx^2}{a(a^2-b^2)} \right) \\
& \downarrow 4318 \\
& \frac{1}{2} \left( \frac{x^2(a^2-b^2)}{a} - \frac{(2a^2-b^2) \int \frac{1}{\frac{a \cosh(\frac{1}{b}(dx^2+c))}{b} + 1} dx^2}{a(a^2-b^2)} + \frac{b^2 \tanh(c+dx^2)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^2))} \right) \\
& \downarrow 3042 \\
& \frac{1}{2} \left( \frac{b^2 \tanh(c+dx^2)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^2))} + \frac{x^2(a^2-b^2)}{a} - \frac{(2a^2-b^2) \int \frac{1}{\frac{a \sin(idx^2+ic+\frac{\pi}{2})}{b} + 1} dx^2}{a(a^2-b^2)} \right) \\
& \downarrow 3138 \\
& \frac{1}{2} \left( \frac{b^2 \tanh(c+dx^2)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^2))} + \frac{x^2(a^2-b^2)}{a} + \frac{2i(2a^2-b^2) \int \frac{1}{(1-\frac{a}{b})x^4+\frac{a+b}{b}} d(i \tanh(\frac{1}{2}(dx^2+c)))}{ad} \right) \\
& \downarrow 221 \\
& \frac{1}{2} \left( \frac{x^2(a^2-b^2)}{a} - \frac{2b(2a^2-b^2) \arctan\left(\frac{\sqrt{a-b} \tanh(\frac{1}{2}(c+dx^2))}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}} + \frac{b^2 \tanh(c+dx^2)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^2))} \right)
\end{aligned}$$

input `Int[x/(a + b*Sech[c + d*x^2])^2,x]`

```
output (((a^2 - b^2)*x^2)/a - (2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[(c + d
*x^2)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d)/(a*(a^2 - b^2)) + (
b^2*Tanh[c + d*x^2])/(a*(a^2 - b^2)*d*(a + b*Sech[c + d*x^2]))/2
```

### 3.27.3.1 Defintions of rubi rules used

```
rule 25 Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

```
rule 221 Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

```
rule 3138 Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

```
rule 4272 Int[(csc[(c_) + (d_)*(x_)]*(b_) + (a_))^(n_), x_Symbol] := Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2)) Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

```
rule 4318 Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbo
l] := Simp[1/b Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

```
rule 4407 Int[(csc[(e_) + (f_)*(x_)]*(d_) + (c_))/(csc[(e_) + (f_)*(x_)]*(b_) +
(a_)), x_Symbol] := Simp[c*(x/a), x] - Simp[(b*c - a*d)/a Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:= Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

### 3.27.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{\ln\left(1+\tanh\left(\frac{dx^2+c}{2}\right)\right)}{a^2} - \frac{2b \left( \frac{ab \tanh\left(\frac{dx^2+c}{2}\right)}{(a^2-b^2) \left(\tanh\left(\frac{dx^2+c}{2}\right)^2 a - \tanh\left(\frac{dx^2+c}{2}\right)^2 b + a+b}\right) + \frac{(2a^2-b^2) \arctan\left(\frac{(a-b) \tanh\left(\frac{dx^2+c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}} \right)}{a^2}$
default	$\frac{\ln\left(1+\tanh\left(\frac{dx^2+c}{2}\right)\right)}{a^2} - \frac{2b \left( \frac{ab \tanh\left(\frac{dx^2+c}{2}\right)}{(a^2-b^2) \left(\tanh\left(\frac{dx^2+c}{2}\right)^2 a - \tanh\left(\frac{dx^2+c}{2}\right)^2 b + a+b}\right) + \frac{(2a^2-b^2) \arctan\left(\frac{(a-b) \tanh\left(\frac{dx^2+c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}} \right)}{a^2}$
risch	$\frac{x^2}{2a^2} - \frac{b^2(e^{dx^2+cb+a})}{a^2(a^2-b^2)d(e^{2dx^2+2ca+2e^{dx^2+cb+a}})} - \frac{b \ln\left(\frac{e^{dx^2+c} + b\sqrt{-a^2+b^2+a^2-b^2}}{\sqrt{-a^2+b^2}a}\right)}{\sqrt{-a^2+b^2}(a+b)(a-b)d} + \frac{b^3 \ln\left(\frac{e^{dx^2+c} + b\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}}\right)}{2\sqrt{-a^2+b^2}(a+b)(a-b)}$

```
input int(x/(a+b*sech(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
output 1/2/d*(1/a^2*ln(1+tanh(1/2*d*x^2+1/2*c))-2/a^2*b*(-a*b/(a^2-b^2)*tanh(1/2*d*x^2+1/2*c)/(tanh(1/2*d*x^2+1/2*c)^2*a-tanh(1/2*d*x^2+1/2*c)^2*b+a+b)+(2*a^2-b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x^2+1/2*c)/((a+b)*(a-b))^(1/2)))-1/a^2*ln(tanh(1/2*d*x^2+1/2*c)-1))
```

3.27.  $\int \frac{x}{(a+b\operatorname{sech}(c+dx^2))^2} dx$



### 3.27.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs.  $2(110) = 220$ .

Time = 0.30 (sec) , antiderivative size = 1314, normalized size of antiderivative = 10.68

$$\int \frac{x}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \text{Too large to display}$$

```
input integrate(x/(a+b*sech(d*x^2+c))^2,x, algorithm="fracas")
```

```
output [1/2*((a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*cosh(d*x^2 + c)^2 + (a^5 - 2*a^3*b^2
+ a*b^4)*d*x^2*sinh(d*x^2 + c)^2 - 2*a^3*b^2 + 2*a*b^4 + (a^5 - 2*a^3*b^2
+ a*b^4)*d*x^2 - (2*a^3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x^2 + c)^2 +
(2*a^3*b - a*b^3)*sinh(d*x^2 + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x^2 + c)
+ 2*(2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cosh(d*x^2 + c))*sinh(d*x^2 + c)
)*sqrt(-a^2 + b^2)*log((a^2*cosh(d*x^2 + c)^2 + a^2*sinh(d*x^2 + c)^2 + 2*
a*b*cosh(d*x^2 + c) - a^2 + 2*b^2 + 2*(a^2*cosh(d*x^2 + c) + a*b)*sinh(d*x
^2 + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x^2 + c) + a*sinh(d*x^2 + c) + b))/
(a*cosh(d*x^2 + c)^2 + a*sinh(d*x^2 + c)^2 + 2*b*cosh(d*x^2 + c) + 2*(a*co
sh(d*x^2 + c) + b)*sinh(d*x^2 + c) + a)) - 2*(a^2*b^3 - b^5 - (a^4*b - 2*a
^2*b^3 + b^5)*d*x^2)*cosh(d*x^2 + c) - 2*(a^2*b^3 - b^5 - (a^5 - 2*a^3*b^2
+ a*b^4)*d*x^2*cosh(d*x^2 + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*x^2)*sinh(d*
x^2 + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x^2 + c)^2 + (a^7 - 2*a^5*
b^2 + a^3*b^4)*d*sinh(d*x^2 + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos
h(d*x^2 + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b
^4)*d*cosh(d*x^2 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x^2 + c)),
1/2*((a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*cosh(d*x^2 + c)^2 + (a^5 - 2*a^3*b^2
+ a*b^4)*d*x^2*sinh(d*x^2 + c)^2 - 2*a^3*b^2 + 2*a*b^4 + (a^5 - 2*a^3*b^2
+ a*b^4)*d*x^2 + 2*(2*a^3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x^2 + c)^2
+ (2*a^3*b - a*b^3)*sinh(d*x^2 + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x^2...
```

### 3.27.6 Sympy [F]

$$\int \frac{x}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{x}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

```
input integrate(x/(a+b*sech(d*x**2+c))**2,x)
```

```
output Integral(x/(a + b*sech(c + d*x**2))**2, x)
```

---

3.27.  $\int \frac{x}{(a+b\operatorname{sech}(c+dx^2))^2} dx$

### 3.27.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a + b \operatorname{sech}(c + dx^2))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f or more de`

### 3.27.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.20

$$\int \frac{x}{(a + b \operatorname{sech}(c + dx^2))^2} dx = -\frac{(2a^2b - b^3) \arctan\left(\frac{ae^{(dx^2+c)} + b}{\sqrt{a^2 - b^2}}\right)}{(a^4d - a^2b^2d)\sqrt{a^2 - b^2}} - \frac{b^3e^{(dx^2+c)} + ab^2}{(a^4d - a^2b^2d)(ae^{(2dx^2+2c)} + 2be^{(dx^2+c)} + a)} + \frac{dx^2 + c}{2a^2d}$$

input `integrate(x/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`

output `-(2*a^2*b - b^3)*arctan((a*e^(d*x^2 + c) + b)/sqrt(a^2 - b^2))/((a^4*d - a^2*b^2*d)*sqrt(a^2 - b^2)) - (b^3*e^(d*x^2 + c) + a*b^2)/((a^4*d - a^2*b^2*d)*(a*e^(2*d*x^2 + 2*c) + 2*b*e^(d*x^2 + c) + a)) + 1/2*(d*x^2 + c)/(a^2*d)`

**3.27.9 Mupad [B] (verification not implemented)**

Time = 2.58 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.57

$$\int \frac{x}{(a + b \operatorname{sech}(c + dx^2))^2} dx$$

$$= \frac{\frac{b^2}{d(a^2 - a^3)} + \frac{b^3 e^{d x^2 + c}}{a d (a^2 - a^3)}}{a + 2 b e^{d x^2 + c} + a e^{2 d x^2 + 2 c}} + \frac{x^2}{2 a^2}$$

$$+ \frac{b \ln \left( \frac{2 b x e^{d x^2 + c} (2 a^2 - b^2)}{a^3 (a^2 - b^2)} - \frac{2 b x (a + b e^{d x^2 + c}) (2 a^2 - b^2)}{a^3 (a + b)^{3/2} (b - a)^{3/2}} \right) (2 a^2 - b^2)}{2 a^2 d (a + b)^{3/2} (b - a)^{3/2}}$$

$$- \frac{b \ln \left( \frac{2 b x e^{d x^2 + c} (2 a^2 - b^2)}{a^3 (a^2 - b^2)} + \frac{2 b x (a + b e^{d x^2 + c}) (2 a^2 - b^2)}{a^3 (a + b)^{3/2} (b - a)^{3/2}} \right) (2 a^2 - b^2)}{2 a^2 d (a + b)^{3/2} (b - a)^{3/2}}$$

input `int(x/(a + b/cosh(c + d*x^2))^2,x)`

output

```
(b^2/(d*(a*b^2 - a^3)) + (b^3*exp(c + d*x^2))/(a*d*(a*b^2 - a^3)))/(a + 2*b*exp(c + d*x^2) + a*exp(2*c + 2*d*x^2)) + x^2/(2*a^2) + (b*log((2*b*x*exp(c + d*x^2)*(2*a^2 - b^2))/(a^3*(a^2 - b^2)) - (2*b*x*(a + b*exp(c + d*x^2)))*(2*a^2 - b^2))/(a^3*(a + b)^(3/2)*(b - a)^(3/2)))*(2*a^2 - b^2)/(2*a^2*d*(a + b)^(3/2)*(b - a)^(3/2)) - (b*log((2*b*x*exp(c + d*x^2)*(2*a^2 - b^2))/(a^3*(a^2 - b^2)) + (2*b*x*(a + b*exp(c + d*x^2)))*(2*a^2 - b^2))/(a^3*(a + b)^(3/2)*(b - a)^(3/2)))*(2*a^2 - b^2)/(2*a^2*d*(a + b)^(3/2)*(b - a)^(3/2))
```

**3.28** 
$$\int \frac{1}{x(a+b\mathbf{sech}(c+dx^2))^2} dx$$

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**3.28.1 Optimal result**

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x(a+b\mathbf{sech}(c+dx^2))^2} dx = \text{Int}\left(\frac{1}{x(a+b\mathbf{sech}(c+dx^2))^2}, x\right)$$

output `Unintegrable(1/x/(a+b*sech(d*x^2+c))^2,x)`

**3.28.2 Mathematica [N/A]**

Not integrable

Time = 53.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a+b\mathbf{sech}(c+dx^2))^2} dx = \int \frac{1}{x(a+b\mathbf{sech}(c+dx^2))^2} dx$$

input `Integrate[1/(x*(a + b*Sech[c + d*x^2])^2),x]`

output `Integrate[1/(x*(a + b*Sech[c + d*x^2])^2), x]`

### 3.28.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a + b \operatorname{sech}(c + dx^2))^2} dx$$

↓ 5961

$$\int \frac{1}{x (a + b \operatorname{sech}(c + dx^2))^2} dx$$

input `Int[1/(x*(a + b*Sech[c + d*x^2])^2),x]`

output `$Aborted`

#### 3.28.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.28.4 Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a + b \operatorname{sech}(dx^2 + c))^2} dx$$

input `int(1/x/(a+b*sech(d*x^2+c))^2,x)`

output `int(1/x/(a+b*sech(d*x^2+c))^2,x)`

---

3.28.  $\int \frac{1}{x(a+b\operatorname{sech}(c+dx^2))^2} dx$

**3.28.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x} dx$$

input `integrate(1/x/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")`output `integral(1/(b^2*x*sech(d*x^2 + c)^2 + 2*a*b*x*sech(d*x^2 + c) + a^2*x), x)`**3.28.6 Sympy [N/A]**

Not integrable

Time = 1.68 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))^2} dx$$

input `integrate(1/x/(a+b*sech(d*x**2+c))**2,x)`output `Integral(1/(x*(a + b*sech(c + d*x**2))**2), x)`**3.28.7 Maxima [N/A]**

Not integrable

Time = 0.42 (sec) , antiderivative size = 249, normalized size of antiderivative = 13.83

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x} dx$$

input `integrate(1/x/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")`

output  $-(b^3e^{(dx^2 + c)} + ab^2)/((a^5d e^{(2c)} - a^3b^2d e^{(2c)})x^2e^{(2dx^2)} + 2(a^4bd e^c - a^2b^3d e^c)x^2e^{(dx^2)} + (a^5d - a^3b^2d)x^2) + \log(x)/a^2 - \text{integrate}(2*(ab^2 + (b^3e^c + (2a^2bd e^c - b^3d e^c)x^2)*e^{(dx^2)})/((a^5d e^{(2c)} - a^3b^2d e^{(2c)})x^3e^{(2dx^2)} + 2(a^4bd e^c - a^2b^3d e^c)x^3e^{(dx^2)} + (a^5d - a^3b^2d)x^3), x)$

### 3.28.8 Giac [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x} dx$$

input `integrate(1/x/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`

output `integrate(1/((b*sech(d*x^2 + c) + a)^2*x), x)`

### 3.28.9 Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x(a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{x \left( a + \frac{b}{\cosh(dx^2+c)} \right)^2} dx$$

input `int(1/(x*(a + b/cosh(c + d*x^2))^2),x)`

output `int(1/(x*(a + b/cosh(c + d*x^2))^2), x)`

$$3.29 \quad \int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

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### 3.29.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx = \operatorname{Int}\left(\frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2}, x\right)$$

output `Unintegrable(1/x^2/(a+b*sech(d*x^2+c))^2,x)`

### 3.29.2 Mathematica [N/A]

Not integrable

Time = 26.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

input `Integrate[1/(x^2*(a + b*Sech[c + d*x^2])^2),x]`

output `Integrate[1/(x^2*(a + b*Sech[c + d*x^2])^2), x]`

---


$$3.29. \quad \int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx$$



**3.29.3 Rubi [N/A]**

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

↓ 5961

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

input `Int[1/(x^2*(a + b*Sech[c + d*x^2])^2),x]`

output `$Aborted`

**3.29.3.1 Defintions of rubi rules used**

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

**3.29.4 Maple [N/A] (verified)**

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(dx^2 + c))^2} dx$$

input `int(1/x^2/(a+b*sech(d*x^2+c))^2,x)`

output `int(1/x^2/(a+b*sech(d*x^2+c))^2,x)`

---

3.29.  $\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx$

**3.29.5 Fricas [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x^2} dx$$

```
input integrate(1/x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
```

```
output integral(1/(b^2*x^2*sech(d*x^2 + c)^2 + 2*a*b*x^2*sech(d*x^2 + c) + a^2*x^2), x)
```

**3.29.6 Sympy [N/A]**

Not integrable

Time = 1.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

```
input integrate(1/x**2/(a+b*sech(d*x**2+c))**2,x)
```

```
output Integral(1/(x**2*(a + b*sech(c + d*x**2))**2), x)
```

**3.29.7 Maxima [N/A]**

Not integrable

Time = 0.42 (sec) , antiderivative size = 318, normalized size of antiderivative = 17.67

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x^2} dx$$

```
input integrate(1/x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
```

output  $-\left(\left(a^3 d e^{2c} - a^2 b d e^{2c}\right) x^2 e^{2 d x^2} + a b^2 + \left(a^3 d - a^2 b\right) x^2 + \left(b^3 e^c + 2\left(a^2 b d e^c - b^3 d e^c\right) x^2\right) e^{d x^2}\right) / \left(\left(a^5 d e^{2c} - a^3 b^2 d e^{2c}\right) x^3 e^{2 d x^2} + 2\left(a^4 b d e^c - a^2 b^3 d e^c\right) x^3 e^{d x^2} + \left(a^5 d - a^3 b^2 d\right) x^3\right) - \text{integrate}\left(\left(3 a b^2 + \left(3 b^3 e^c + 2\left(2 a^2 b d e^c - b^3 d e^c\right) x^2\right) e^{d x^2}\right) / \left(\left(a^5 d e^{2c} - a^3 b^2 d e^{2c}\right) x^4 e^{2 d x^2} + 2\left(a^4 b d e^c - a^2 b^3 d e^c\right) x^4 e^{d x^2} + \left(a^5 d - a^3 b^2 d\right) x^4\right), x\right)$

### 3.29.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x^2} dx$$

input `integrate(1/x^2/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`

output `integrate(1/((b*sech(d*x^2 + c) + a)^2*x^2), x)`

### 3.29.9 Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{x^2 \left(a + \frac{b}{\cosh(dx^2 + c)}\right)^2} dx$$

input `int(1/(x^2*(a + b/cosh(c + d*x^2))^2),x)`

output `int(1/(x^2*(a + b/cosh(c + d*x^2))^2), x)`

**3.30** 
$$\int \frac{1}{x^3 (a+b\operatorname{sech}(c+dx^2))^2} dx$$

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**3.30.1 Optimal result**

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 (a + b\operatorname{sech}(c + dx^2))^2} dx = \operatorname{Int}\left(\frac{1}{x^3 (a + b\operatorname{sech}(c + dx^2))^2}, x\right)$$

output `Unintegrable(1/x^3/(a+b*sech(d*x^2+c))^2,x)`

**3.30.2 Mathematica [N/A]**

Not integrable

Time = 28.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{x^3 (a + b\operatorname{sech}(c + dx^2))^2} dx$$

input `Integrate[1/(x^3*(a + b*Sech[c + d*x^2])^2),x]`

output `Integrate[1/(x^3*(a + b*Sech[c + d*x^2])^2), x]`

**3.30.3 Rubi [N/A]**

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

↓ 5961

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

input `Int[1/(x^3*(a + b*Sech[c + d*x^2])^2),x]`

output `$Aborted`

**3.30.3.1 Defintions of rubi rules used**

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

**3.30.4 Maple [N/A] (verified)**

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(dx^2 + c))^2} dx$$

input `int(1/x^3/(a+b*sech(d*x^2+c))^2,x)`

output `int(1/x^3/(a+b*sech(d*x^2+c))^2,x)`

---

3.30.  $\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx$

**3.30.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x^3} dx$$

```
input integrate(1/x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="fricas")
```

```
output integral(1/(b^2*x^3*sech(d*x^2 + c)^2 + 2*a*b*x^3*sech(d*x^2 + c) + a^2*x^3), x)
```

**3.30.6 Sympy [N/A]**

Not integrable

Time = 1.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx$$

```
input integrate(1/x**3/(a+b*sech(d*x**2+c))**2,x)
```

```
output Integral(1/(x**3*(a + b*sech(c + d*x**2))**2), x)
```

**3.30.7 Maxima [N/A]**

Not integrable

Time = 0.41 (sec) , antiderivative size = 319, normalized size of antiderivative = 17.72

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x^3} dx$$

```
input integrate(1/x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="maxima")
```

output 
$$-1/2*((a^3*d*e^{(2*c)} - a*b^2*d*e^{(2*c)})*x^2*e^{(2*d*x^2)} + 2*a*b^2 + (a^3*d - a*b^2*d)*x^2 + 2*(b^3*e^c + (a^2*b*d*e^c - b^3*d*e^c)*x^2)*e^{(d*x^2)})/((a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*x^4*e^{(2*d*x^2)} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^4*e^{(d*x^2)} + (a^5*d - a^3*b^2*d)*x^4) - \text{integrate}(2*(2*a*b^2 + (2*b^3*e^c + (2*a^2*b*d*e^c - b^3*d*e^c)*x^2)*e^{(d*x^2)})/((a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*x^5*e^{(2*d*x^2)} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^5*e^{(d*x^2)} + (a^5*d - a^3*b^2*d)*x^5), x)$$

### 3.30.8 Giac [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{(b \operatorname{sech}(dx^2 + c) + a)^2 x^3} dx$$

input `integrate(1/x^3/(a+b*sech(d*x^2+c))^2,x, algorithm="giac")`

output `sage0*x`

### 3.30.9 Mupad [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a + b \operatorname{sech}(c + dx^2))^2} dx = \int \frac{1}{x^3 \left( a + \frac{b}{\cosh(dx^2+c)} \right)^2} dx$$

input `int(1/(x^3*(a + b/cosh(c + d*x^2))^2),x)`

output `int(1/(x^3*(a + b/cosh(c + d*x^2))^2), x)`

### 3.31 $\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx$

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#### 3.31.1 Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx = -\tanh\left(\frac{1}{x}\right)$$

output `-tanh(1/x)`

#### 3.31.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx = -\tanh\left(\frac{1}{x}\right)$$

input `Integrate[Sech[x^(-1)]^2/x^2,x]`

output `-Tanh[x^(-1)]`



**3.31.3 Rubi [A] (verified)**

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$ , Rules used = {5959, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx \\
 & \quad \downarrow \text{5959} \\
 & - \int \operatorname{sech}^2\left(\frac{1}{x}\right) d\frac{1}{x} \\
 & \quad \downarrow \text{3042} \\
 & - \int \csc\left(\frac{\pi}{2} + \frac{i}{x}\right)^2 d\frac{1}{x} \\
 & \quad \downarrow \text{4254} \\
 & -i \int 1d\left(-i \tanh\left(\frac{1}{x}\right)\right) \\
 & \quad \downarrow \text{24} \\
 & - \tanh\left(\frac{1}{x}\right)
 \end{aligned}$$

input `Int [Sech[x^(-1)]^2/x^2,x]`

output `-Tanh[x^(-1)]`

**3.31.3.1 Defintions of rubi rules used**

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

```
rule 4254 Int[csc[(c_.) + (d_.)*(x_)^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

### 3.31.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$-\tanh\left(\frac{1}{x}\right)$	7
default	$-\tanh\left(\frac{1}{x}\right)$	7
risch	$\frac{2}{e^{\frac{2}{x}} + 1}$	13
parallelrisch	$-\frac{2 \tanh\left(\frac{1}{2x}\right)}{1 + \tanh\left(\frac{1}{2x}\right)^2}$	21

```
input int(sech(1/x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
output -tanh(1/x)
```

### 3.31.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs.  $2(6) = 12$ .

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 4.67

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx = \frac{2}{\cosh\left(\frac{1}{x}\right)^2 + 2 \cosh\left(\frac{1}{x}\right) \sinh\left(\frac{1}{x}\right) + \sinh\left(\frac{1}{x}\right)^2 + 1}$$

```
input integrate(sech(1/x)^2/x^2,x, algorithm="fricas")
```

```
output 2/(cosh(1/x)^2 + 2*cosh(1/x)*sinh(1/x) + sinh(1/x)^2 + 1)
```

---

3.31.  $\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx$

**3.31.6 Sympy [F]**

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx = \int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx$$

input `integrate(sech(1/x)**2/x**2,x)`

output `Integral(sech(1/x)**2/x**2, x)`

**3.31.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx = \frac{2}{e^{\frac{2}{x}} + 1}$$

input `integrate(sech(1/x)^2/x^2,x, algorithm="maxima")`

output `2/(e^(2/x) + 1)`

**3.31.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx = \frac{2}{e^{\frac{2}{x}} + 1}$$

input `integrate(sech(1/x)^2/x^2,x, algorithm="giac")`

output `2/(e^(2/x) + 1)`

**3.31.9 Mupad [B] (verification not implemented)**

Time = 2.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{sech}^2\left(\frac{1}{x}\right)}{x^2} dx = \frac{2}{e^{2/x} + 1}$$

input `int(1/(x^2*cosh(1/x)^2),x)`

output `2/(exp(2/x) + 1)`

### 3.32 $\int x^3(a + b\operatorname{sech}(c + d\sqrt{x})) dx$

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### 3.32.1 Optimal result

Integrand size = 18, antiderivative size = 426

$$\begin{aligned}
 \int x^3 (a + b \operatorname{sech}(c + d\sqrt{x})) dx = & \frac{ax^4}{4} + \frac{4bx^{7/2} \arctan(e^{c+d\sqrt{x}})}{d} - \frac{14ibx^3 \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} \\
 & + \frac{14ibx^3 \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} \\
 & + \frac{84ibx^{5/2} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} \\
 & - \frac{84ibx^{5/2} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} \\
 & - \frac{420ibx^2 \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} \\
 & + \frac{420ibx^2 \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} \\
 & + \frac{1680ibx^{3/2} \operatorname{PolyLog}(5, -ie^{c+d\sqrt{x}})}{d^5} \\
 & - \frac{1680ibx^{3/2} \operatorname{PolyLog}(5, ie^{c+d\sqrt{x}})}{d^5} \\
 & - \frac{5040ibx \operatorname{PolyLog}(6, -ie^{c+d\sqrt{x}})}{d^6} \\
 & + \frac{5040ibx \operatorname{PolyLog}(6, ie^{c+d\sqrt{x}})}{d^6} \\
 & + \frac{10080ib\sqrt{x} \operatorname{PolyLog}(7, -ie^{c+d\sqrt{x}})}{d^7} \\
 & - \frac{10080ib\sqrt{x} \operatorname{PolyLog}(7, ie^{c+d\sqrt{x}})}{d^7} \\
 & - \frac{10080ib \operatorname{PolyLog}(8, -ie^{c+d\sqrt{x}})}{d^8} \\
 & + \frac{10080ib \operatorname{PolyLog}(8, ie^{c+d\sqrt{x}})}{d^8}
 \end{aligned}$$

output  $\frac{1}{4}ax^4 + 4bx^{7/2} \arctan(\exp(c+d\sqrt{x})) / d - 84I^2bx^{5/2} \operatorname{polylog}(3, I\exp(c+d\sqrt{x})) / d^3 + 1680I^2bx^{3/2} \operatorname{polylog}(5, -I\exp(c+d\sqrt{x})) / d^5 + 5040I^2bx \operatorname{polylog}(6, I\exp(c+d\sqrt{x})) / d^6 - 10080I^2b \operatorname{polylog}(8, -I\exp(c+d\sqrt{x})) / d^8 + 10080I^2b \operatorname{polylog}(8, I\exp(c+d\sqrt{x})) / d^8 + 14I^2bx^3 \operatorname{polylog}(2, I\exp(c+d\sqrt{x})) / d^2 + 84I^2bx^{5/2} \operatorname{polylog}(3, -I\exp(c+d\sqrt{x})) / d^3 - 420I^2bx^2 \operatorname{polylog}(4, -I\exp(c+d\sqrt{x})) / d^4 + 420I^2bx^2 \operatorname{polylog}(4, I\exp(c+d\sqrt{x})) / d^4 - 14I^2bx^3 \operatorname{polylog}(2, -I\exp(c+d\sqrt{x})) / d^2 + 10080I^2b \operatorname{polylog}(7, -I\exp(c+d\sqrt{x})) x^{1/2} / d^7 - 1680I^2bx^{3/2} \operatorname{polylog}(5, I\exp(c+d\sqrt{x})) / d^5 - 10080I^2b \operatorname{polylog}(7, I\exp(c+d\sqrt{x})) x^{1/2} / d^7 - 5040I^2bx \operatorname{polylog}(6, -I\exp(c+d\sqrt{x})) / d^6$

### 3.32.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.97

$$\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x})) dx = \frac{ax^4}{4} + \frac{2ib(d^7 x^{7/2} \log(1 - ie^{c+d\sqrt{x}}) - d^7 x^{7/2} \log(1 + ie^{c+d\sqrt{x}}) - 7d^6 x^3 \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}}) + 7d^6 x^3 \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}}))}{d^8}$$

input `Integrate[x^3*(a + b*Sech[c + d*Sqrt[x]]),x]`

output  $(ax^4)/4 + ((2I)^2 b (d^7 x^{7/2} \operatorname{Log}[1 - I E^{(c + d\sqrt{x})}] - d^7 x^{7/2} \operatorname{Log}[1 + I E^{(c + d\sqrt{x})}] - 7d^6 x^3 \operatorname{PolyLog}[2, (-I) E^{(c + d\sqrt{x})}] + 7d^6 x^3 \operatorname{PolyLog}[2, I E^{(c + d\sqrt{x})}] + 42d^5 x^{5/2} \operatorname{PolyLog}[3, (-I) E^{(c + d\sqrt{x})}] - 42d^5 x^{5/2} \operatorname{PolyLog}[3, I E^{(c + d\sqrt{x})}] - 210d^4 x^2 \operatorname{PolyLog}[4, (-I) E^{(c + d\sqrt{x})}] + 210d^4 x^2 \operatorname{PolyLog}[4, I E^{(c + d\sqrt{x})}] + 840d^3 x^{3/2} \operatorname{PolyLog}[5, (-I) E^{(c + d\sqrt{x})}] - 840d^3 x^{3/2} \operatorname{PolyLog}[5, I E^{(c + d\sqrt{x})}] - 2520d^2 x \operatorname{PolyLog}[6, (-I) E^{(c + d\sqrt{x})}] + 2520d^2 x \operatorname{PolyLog}[6, I E^{(c + d\sqrt{x})}] + 5040d \operatorname{Sqrt}[x] \operatorname{PolyLog}[7, (-I) E^{(c + d\sqrt{x})}] - 5040d \operatorname{Sqrt}[x] \operatorname{PolyLog}[7, I E^{(c + d\sqrt{x})}] - 5040 \operatorname{PolyLog}[8, (-I) E^{(c + d\sqrt{x})}] + 5040 \operatorname{PolyLog}[8, I E^{(c + d\sqrt{x})}])) / d^8$

### 3.32.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^3 (a + b \operatorname{sech}(c + d\sqrt{x})) \, dx \\
 & \quad \downarrow \text{2010} \\
 & \int (ax^3 + bx^3 \operatorname{sech}(c + d\sqrt{x})) \, dx \\
 & \quad \downarrow \text{2009} \\
 & \frac{ax^4}{4} + \frac{4bx^{7/2} \arctan(e^{c+d\sqrt{x}})}{d} - \frac{10080ib \operatorname{PolyLog}(8, -ie^{c+d\sqrt{x}})}{d^8} + \\
 & \frac{10080ib \operatorname{PolyLog}(8, ie^{c+d\sqrt{x}})}{d^8} + \frac{10080ib\sqrt{x} \operatorname{PolyLog}(7, -ie^{c+d\sqrt{x}})}{d^7} - \\
 & \frac{10080ib\sqrt{x} \operatorname{PolyLog}(7, ie^{c+d\sqrt{x}})}{d^7} - \frac{5040ibx \operatorname{PolyLog}(6, -ie^{c+d\sqrt{x}})}{d^6} + \\
 & \frac{5040ibx \operatorname{PolyLog}(6, ie^{c+d\sqrt{x}})}{d^6} + \frac{1680ibx^{3/2} \operatorname{PolyLog}(5, -ie^{c+d\sqrt{x}})}{d^5} - \\
 & \frac{1680ibx^{3/2} \operatorname{PolyLog}(5, ie^{c+d\sqrt{x}})}{d^5} - \frac{420ibx^2 \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} + \\
 & \frac{420ibx^2 \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} + \frac{84ibx^{5/2} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \\
 & \frac{84ibx^{5/2} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} - \frac{14ibx^3 \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{14ibx^3 \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2}
 \end{aligned}$$

input `Int[x^3*(a + b*Sech[c + d*sqrt[x]]),x]`



```
output (a*x^4)/4 + (4*b*x^(7/2)*ArcTan[E^(c + d*Sqrt[x])])/d - ((14*I)*b*x^3*Poly
Log[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((14*I)*b*x^3*PolyLog[2, I*E^(c + d*
Sqrt[x])])/d^2 + ((84*I)*b*x^(5/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3
- ((84*I)*b*x^(5/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 - ((420*I)*b*x^2
*PolyLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((420*I)*b*x^2*PolyLog[4, I*E^(
c + d*Sqrt[x])])/d^4 + ((1680*I)*b*x^(3/2)*PolyLog[5, (-I)*E^(c + d*Sqrt[x
])])/d^5 - ((1680*I)*b*x^(3/2)*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 - ((50
40*I)*b*x*PolyLog[6, (-I)*E^(c + d*Sqrt[x])])/d^6 + ((5040*I)*b*x*PolyLog[
6, I*E^(c + d*Sqrt[x])])/d^6 + ((10080*I)*b*Sqrt[x]*PolyLog[7, (-I)*E^(c +
d*Sqrt[x])])/d^7 - ((10080*I)*b*Sqrt[x]*PolyLog[7, I*E^(c + d*Sqrt[x])])/
d^7 - ((10080*I)*b*PolyLog[8, (-I)*E^(c + d*Sqrt[x])])/d^8 + ((10080*I)*b*
PolyLog[8, I*E^(c + d*Sqrt[x])])/d^8
```

### 3.32.3.1 Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2010 Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

### 3.32.4 Maple [F]

$$\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

```
input int(x^3*(a+b*sech(c+d*x^(1/2))),x)
```

```
output int(x^3*(a+b*sech(c+d*x^(1/2))),x)
```

**3.32.5 Fricas [F]**

$$\int x^3(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)x^3 dx$$

input `integrate(x^3*(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(b*x^3*sech(d*sqrt(x) + c) + a*x^3, x)`

**3.32.6 Sympy [F]**

$$\int x^3(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int x^3(a + b\operatorname{sech}(c + d\sqrt{x})) dx$$

input `integrate(x**3*(a+b*sech(c+d*x**(1/2))),x)`

output `Integral(x**3*(a + b*sech(c + d*sqrt(x))), x)`

**3.32.7 Maxima [F]**

$$\int x^3(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)x^3 dx$$

input `integrate(x^3*(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

output `1/4*a*x^4 + 2*b*integrate(x^3*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2*c) + 1), x)`

**3.32.8 Giac [F]**

$$\int x^3(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)x^3 dx$$

input `integrate(x^3*(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)*x^3, x)`

**3.32.9 Mupad [F(-1)]**

Timed out.

$$\int x^3(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int x^3 \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right) dx$$

input `int(x^3*(a + b/cosh(c + d*x^(1/2))),x)`

output `int(x^3*(a + b/cosh(c + d*x^(1/2))), x)`

### 3.33 $\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x})) dx$

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#### 3.33.1 Optimal result

Integrand size = 18, antiderivative size = 310

$$\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x})) dx = \frac{ax^3}{3} + \frac{4bx^{5/2} \arctan(e^{c+d\sqrt{x}})}{d} - \frac{10ibx^2 \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{10ibx^2 \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} + \frac{40ibx^{3/2} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{40ibx^{3/2} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} - \frac{120ibx \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} + \frac{120ibx \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} + \frac{240ib\sqrt{x} \operatorname{PolyLog}(5, -ie^{c+d\sqrt{x}})}{d^5} - \frac{240ib\sqrt{x} \operatorname{PolyLog}(5, ie^{c+d\sqrt{x}})}{d^5} - \frac{240ib \operatorname{PolyLog}(6, -ie^{c+d\sqrt{x}})}{d^6} + \frac{240ib \operatorname{PolyLog}(6, ie^{c+d\sqrt{x}})}{d^6}$$

output  $\frac{1}{3}ax^3 + 4bx^{5/2} \arctan(\exp(c+d\sqrt{x})) / d - 10I^2bx^2 \operatorname{polylog}(2, -I\exp(c+d\sqrt{x})) / d^2 + 10I^2bx^2 \operatorname{polylog}(2, I\exp(c+d\sqrt{x})) / d^2 + 40I^3bx^{3/2} \operatorname{polylog}(3, -I\exp(c+d\sqrt{x})) / d^3 - 40I^3bx^{3/2} \operatorname{polylog}(3, I\exp(c+d\sqrt{x})) / d^3 - 120I^4bx \operatorname{polylog}(4, -I\exp(c+d\sqrt{x})) / d^4 + 120I^4bx \operatorname{polylog}(4, I\exp(c+d\sqrt{x})) / d^4 - 240I^5b \operatorname{polylog}(6, -I\exp(c+d\sqrt{x})) / d^6 + 240I^5b \operatorname{polylog}(6, I\exp(c+d\sqrt{x})) / d^6 + 240I^6b \operatorname{polylog}(5, -I\exp(c+d\sqrt{x})) x^{1/2} / d^5 - 240I^6b \operatorname{polylog}(5, I\exp(c+d\sqrt{x})) x^{1/2} / d^5$

### 3.33.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00

$$\int x^2(a + b \operatorname{sech}(c + d\sqrt{x})) dx = \frac{ax^3}{3} + \frac{2ib(d^5x^{5/2} \log(1 - ie^{c+d\sqrt{x}}) - d^5x^{5/2} \log(1 + ie^{c+d\sqrt{x}}) - 5d^4x^2 \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}}) + 5d^4x^2 \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}}))}{d^6}$$

input `Integrate[x^2*(a + b*Sech[c + d*Sqrt[x]]),x]`

output  $(ax^3)/3 + ((2I)*b*(d^5x^{5/2}*\operatorname{Log}[1 - I*E^{(c + d*\operatorname{Sqrt}[x])}] - d^5x^{5/2}*\operatorname{Log}[1 + I*E^{(c + d*\operatorname{Sqrt}[x])}] - 5*d^4*x^2*\operatorname{PolyLog}[2, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}] + 5*d^4*x^2*\operatorname{PolyLog}[2, I*E^{(c + d*\operatorname{Sqrt}[x])}] + 20*d^3*x^{3/2}*\operatorname{PolyLog}[3, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}] - 20*d^3*x^{3/2}*\operatorname{PolyLog}[3, I*E^{(c + d*\operatorname{Sqrt}[x])}] - 60*d^2*x*\operatorname{PolyLog}[4, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}] + 60*d^2*x*\operatorname{PolyLog}[4, I*E^{(c + d*\operatorname{Sqrt}[x])}] + 120*d*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[5, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}] - 120*d*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[5, I*E^{(c + d*\operatorname{Sqrt}[x])}] - 120*\operatorname{PolyLog}[6, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}] + 120*\operatorname{PolyLog}[6, I*E^{(c + d*\operatorname{Sqrt}[x])}]))/d^6$

### 3.33.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

$$\begin{array}{c}
 \downarrow \text{2010} \\
 \int (ax^2 + bx^2 \operatorname{sech}(c + d\sqrt{x})) dx \\
 \downarrow \text{2009} \\
 \frac{ax^3}{3} + \frac{4bx^{5/2} \arctan(e^{c+d\sqrt{x}})}{d} - \frac{240ib \operatorname{PolyLog}(6, -ie^{c+d\sqrt{x}})}{d^6} + \frac{240ib \operatorname{PolyLog}(6, ie^{c+d\sqrt{x}})}{d^6} + \\
 \frac{240ib\sqrt{x} \operatorname{PolyLog}(5, -ie^{c+d\sqrt{x}})}{d^5} - \frac{240ib\sqrt{x} \operatorname{PolyLog}(5, ie^{c+d\sqrt{x}})}{d^5} - \\
 \frac{120ibx \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} + \frac{120ibx \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} + \frac{40ibx^{3/2} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} \\
 \frac{40ibx^{3/2} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} - \frac{10ibx^2 \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{10ibx^2 \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2}
 \end{array}$$

input `Int[x^2*(a + b*Sech[c + d*Sqrt[x]]),x]`

output `(a*x^3)/3 + (4*b*x^(5/2)*ArcTan[E^(c + d*Sqrt[x])])/d - ((10*I)*b*x^2*PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((10*I)*b*x^2*PolyLog[2, I*E^(c + d*Sqrt[x])])/d^2 + ((40*I)*b*x^(3/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((40*I)*b*x^(3/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 - ((120*I)*b*x*PolyLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((120*I)*b*x*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 + ((240*I)*b*Sqrt[x]*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 - ((240*I)*b*Sqrt[x]*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 - ((240*I)*b*PolyLog[6, (-I)*E^(c + d*Sqrt[x])])/d^6 + ((240*I)*b*PolyLog[6, I*E^(c + d*Sqrt[x])])/d^6`

### 3.33.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

**3.33.4 Maple [F]**

$$\int x^2(a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

input `int(x^2*(a+b*sech(c+d*x^(1/2))),x)`

output `int(x^2*(a+b*sech(c+d*x^(1/2))),x)`

**3.33.5 Fricas [F]**

$$\int x^2(a + b \operatorname{sech}(c + d\sqrt{x})) dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)x^2 dx$$

input `integrate(x^2*(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(b*x^2*sech(d*sqrt(x) + c) + a*x^2, x)`

**3.33.6 Sympy [F]**

$$\int x^2(a + b \operatorname{sech}(c + d\sqrt{x})) dx = \int x^2(a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

input `integrate(x**2*(a+b*sech(c+d*x**(1/2))),x)`

output `Integral(x**2*(a + b*sech(c + d*sqrt(x))), x)`

**3.33.7 Maxima [F]**

$$\int x^2(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)x^2 dx$$

input `integrate(x^2*(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

output `1/3*a*x^3 + 2*b*integrate(x^2*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2*c) + 1), x)`

**3.33.8 Giac [F]**

$$\int x^2(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)x^2 dx$$

input `integrate(x^2*(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)*x^2, x)`

**3.33.9 Mupad [F(-1)]**

Timed out.

$$\int x^2(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int x^2 \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right) dx$$

input `int(x^2*(a + b/cosh(c + d*x^(1/2))),x)`

output `int(x^2*(a + b/cosh(c + d*x^(1/2))), x)`



### 3.34 $\int x(a + b\operatorname{sech}(c + d\sqrt{x})) dx$

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#### 3.34.1 Optimal result

Integrand size = 16, antiderivative size = 194

$$\int x(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \frac{ax^2}{2} + \frac{4bx^{3/2} \arctan(e^{c+d\sqrt{x}})}{d} - \frac{6ibx \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{6ibx \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} + \frac{12ib\sqrt{x} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{12ib\sqrt{x} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} - \frac{12ib \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} + \frac{12ib \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4}$$

output  $\frac{1}{2}ax^2 + 4bx^{3/2} \arctan(\exp(c+d*x^{1/2}))/d - 6I*b*x*\operatorname{polylog}(2, -I*\exp(c+d*x^{1/2}))/d^2 + 6I*b*x*\operatorname{polylog}(2, I*\exp(c+d*x^{1/2}))/d^2 - 12I*b*\operatorname{polylog}(4, -I*\exp(c+d*x^{1/2}))/d^4 + 12I*b*\operatorname{polylog}(4, I*\exp(c+d*x^{1/2}))/d^4 + 12I*b*\operatorname{polylog}(3, -I*\exp(c+d*x^{1/2}))*x^{1/2}/d^3 - 12I*b*\operatorname{polylog}(3, I*\exp(c+d*x^{1/2}))*x^{1/2}/d^3$

### 3.34.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.07

$$\int x(a + b \operatorname{sech}(c + d\sqrt{x})) dx = \frac{ax^2}{2} + \frac{2ib(d^3x^{3/2} \log(1 - ie^{c+d\sqrt{x}}) - d^3x^{3/2} \log(1 + ie^{c+d\sqrt{x}}) - 3d^2x \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}}) + 3d^2x \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}}) - 6d\sqrt{x} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}}) + 6d\sqrt{x} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}}) - 6 \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}}) + 6 \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}}))}{d^4}$$

input `Integrate[x*(a + b*Sech[c + d*Sqrt[x]]),x]`

output  $(a*x^2)/2 + ((2*I)*b*(d^3*x^(3/2)*\operatorname{Log}[1 - I*E^(c + d*Sqrt[x])] - d^3*x^(3/2)*\operatorname{Log}[1 + I*E^(c + d*Sqrt[x])] - 3*d^2*x*\operatorname{PolyLog}[2, (-I)*E^(c + d*Sqrt[x])] + 3*d^2*x*\operatorname{PolyLog}[2, I*E^(c + d*Sqrt[x])] + 6*d*Sqrt[x]*\operatorname{PolyLog}[3, (-I)*E^(c + d*Sqrt[x])] - 6*d*Sqrt[x]*\operatorname{PolyLog}[3, I*E^(c + d*Sqrt[x])] - 6*\operatorname{PolyLog}[4, (-I)*E^(c + d*Sqrt[x])] + 6*\operatorname{PolyLog}[4, I*E^(c + d*Sqrt[x])])/d^4$

### 3.34.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int x(a + b \operatorname{sech}(c + d\sqrt{x})) dx \\ & \quad \downarrow \text{2010} \\ & \int (ax + bx \operatorname{sech}(c + d\sqrt{x})) dx \\ & \quad \downarrow \text{2009} \\ & \frac{ax^2}{2} + \frac{4bx^{3/2} \arctan(e^{c+d\sqrt{x}})}{d} - \frac{12ib \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} + \frac{12ib \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} + \\ & \frac{12ib\sqrt{x} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{12ib\sqrt{x} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} - \frac{6ibx \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \\ & \frac{6ibx \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} \end{aligned}$$

input `Int[x*(a + b*Sech[c + d*Sqrt[x]]),x]`

output `(a*x^2)/2 + (4*b*x^(3/2)*ArcTan[E^(c + d*Sqrt[x])])/d - ((6*I)*b*x*PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((6*I)*b*x*PolyLog[2, I*E^(c + d*Sqrt[x])])/d^2 + ((12*I)*b*Sqrt[x]*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((12*I)*b*Sqrt[x]*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 - ((12*I)*b*PolyLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((12*I)*b*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4`

### 3.34.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

### 3.34.4 Maple [F]

$$\int x(a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

input `int(x*(a+b*sech(c+d*x^(1/2))),x)`

output `int(x*(a+b*sech(c+d*x^(1/2))),x)`

### 3.34.5 Fracas [F]

$$\int x(a + b \operatorname{sech}(c + d\sqrt{x})) dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)x dx$$

input `integrate(x*(a+b*sech(c+d*x^(1/2))),x, algorithm="fracas")`

output `integral(b*x*sech(d*sqrt(x) + c) + a*x, x)`

**3.34.6 Sympy [F]**

$$\int x(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int x(a + b\operatorname{sech}(c + d\sqrt{x})) dx$$

input `integrate(x*(a+b*sech(c+d*x**(1/2))),x)`

output `Integral(x*(a + b*sech(c + d*sqrt(x))), x)`

**3.34.7 Maxima [F]**

$$\int x(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)x dx$$

input `integrate(x*(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

output `1/2*a*x^2 + 2*b*integrate(x*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2*c) + 1), x)`

**3.34.8 Giac [F]**

$$\int x(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)x dx$$

input `integrate(x*(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)*x, x)`

**3.34.9 Mupad [F(-1)]**

Timed out.

$$\int x(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int x \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right) dx$$

input `int(x*(a + b/cosh(c + d*x^(1/2))),x)`output `int(x*(a + b/cosh(c + d*x^(1/2))), x)`

### 3.35 $\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x} dx$

3.35.1	Optimal result	221
3.35.2	Mathematica [N/A]	221
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3.35.7	Maxima [N/A]	224
3.35.8	Giac [N/A]	224
3.35.9	Mupad [N/A]	224

#### 3.35.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x} dx = a \log(x) + b \operatorname{Int}\left(\frac{\operatorname{sech}(c + d\sqrt{x})}{x}, x\right)$$

output `a*ln(x)+b*Unintegrable(sech(c+d*x^(1/2))/x,x)`

#### 3.35.2 Mathematica [N/A]

Not integrable

Time = 11.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x} dx = \int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x} dx$$

input `Integrate[(a + b*Sech[c + d*Sqrt[x]])/x,x]`

output `Integrate[(a + b*Sech[c + d*Sqrt[x]])/x, x]`

### 3.35.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x} dx$$

↓ 2010

$$\int \left( \frac{a}{x} + \frac{b \operatorname{sech}(c + d\sqrt{x})}{x} \right) dx$$

↓ 2009

$$b \int \frac{\operatorname{sech}(c + d\sqrt{x})}{x} dx + a \log(x)$$

input `Int[(a + b*Sech[c + d*Sqrt[x]])/x,x]`

output `$Aborted`

#### 3.35.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

**3.35.4 Maple [N/A] (verified)**

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x} dx$$

input `int((a+b*sech(c+d*x^(1/2)))/x,x)`output `int((a+b*sech(c+d*x^(1/2)))/x,x)`**3.35.5 Fricas [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x,x, algorithm="fricas")`output `integral((b*sech(d*sqrt(x) + c) + a)/x, x)`**3.35.6 Sympy [N/A]**

Not integrable

Time = 2.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x} dx = \int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x} dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))/x,x)`output `Integral((a + b*sech(c + d*sqrt(x)))/x, x)`



**3.35.7 Maxima [N/A]**

Not integrable

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x,x, algorithm="maxima")`output `2*b*integrate(e^(d*sqrt(x) + c)/(x*e^(2*d*sqrt(x) + 2*c) + x), x) + a*log(x)`**3.35.8 Giac [N/A]**

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x,x, algorithm="giac")`output `integrate((b*sech(d*sqrt(x) + c) + a)/x, x)`**3.35.9 Mupad [N/A]**

Not integrable

Time = 2.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x} dx = \int \frac{a + \frac{b}{\cosh(c + d\sqrt{x})}}{x} dx$$

input `int((a + b/cosh(c + d*x^(1/2)))/x,x)`output `int((a + b/cosh(c + d*x^(1/2)))/x, x)`

---

3.35.  $\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x} dx$

### 3.36 $\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^2} dx$

3.36.1	Optimal result	225
3.36.2	Mathematica [N/A]	225
3.36.3	Rubi [N/A]	226
3.36.4	Maple [N/A] (verified)	227
3.36.5	Fricas [N/A]	227
3.36.6	Sympy [N/A]	227
3.36.7	Maxima [N/A]	228
3.36.8	Giac [N/A]	228
3.36.9	Mupad [N/A]	228

#### 3.36.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx = -\frac{a}{x} + b\operatorname{Int}\left(\frac{\operatorname{sech}(c + d\sqrt{x})}{x^2}, x\right)$$

output `-a/x+b*Unintegrable(sech(c+d*x^(1/2))/x^2,x)`

#### 3.36.2 Mathematica [N/A]

Not integrable

Time = 10.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

input `Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^2,x]`

output `Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^2, x]`

**3.36.3 Rubi [N/A]**

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

↓ 2010

$$\int \left( \frac{a}{x^2} + \frac{b \operatorname{sech}(c + d\sqrt{x})}{x^2} \right) dx$$

↓ 2009

$$b \int \frac{\operatorname{sech}(c + d\sqrt{x})}{x^2} dx - \frac{a}{x}$$

input `Int[(a + b*Sech[c + d*Sqrt[x]])/x^2,x]`

output `$Aborted`

**3.36.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

**3.36.4 Maple [N/A] (verified)**

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

input `int((a+b*sech(c+d*x^(1/2)))/x^2,x)`output `int((a+b*sech(c+d*x^(1/2)))/x^2,x)`**3.36.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="fricas")`output `integral((b*sech(d*sqrt(x) + c) + a)/x^2, x)`**3.36.6 Sympy [N/A]**

Not integrable

Time = 1.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))/x**2,x)`output `Integral((a + b*sech(c + d*sqrt(x)))/x**2, x)`

**3.36.7 Maxima [N/A]**

Not integrable

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="maxima")`

output `2*b*integrate(e^(d*sqrt(x) + c)/(x^2*e^(2*d*sqrt(x) + 2*c) + x^2), x) - a/x`

**3.36.8 Giac [N/A]**

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)/x^2, x)`

**3.36.9 Mupad [N/A]**

Not integrable

Time = 2.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{a + \frac{b}{\cosh(c + d\sqrt{x})}}{x^2} dx$$

input `int((a + b/cosh(c + d*x^(1/2)))/x^2,x)`

output `int((a + b/cosh(c + d*x^(1/2)))/x^2, x)`

---

3.36.  $\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^2} dx$

### 3.37 $\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$

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### 3.37.1 Optimal result

Integrand size = 20, antiderivative size = 677

$$\begin{aligned}
 \int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx &= \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \arctan(e^{c+d\sqrt{x}})}{d} \\
 &\quad - \frac{14b^2 x^3 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} \\
 &\quad - \frac{28iabx^3 \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} \\
 &\quad + \frac{28iabx^3 \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} \\
 &\quad - \frac{42b^2 x^{5/2} \operatorname{PolyLog}(2, -e^{2(c+d\sqrt{x})})}{d^3} \\
 &\quad + \frac{168iabx^{5/2} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} \\
 &\quad - \frac{168iabx^{5/2} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} \\
 &\quad + \frac{105b^2 x^2 \operatorname{PolyLog}(3, -e^{2(c+d\sqrt{x})})}{d^4} \\
 &\quad - \frac{840iabx^2 \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} \\
 &\quad + \frac{840iabx^2 \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} \\
 &\quad - \frac{210b^2 x^{3/2} \operatorname{PolyLog}(4, -e^{2(c+d\sqrt{x})})}{d^5} \\
 &\quad + \frac{3360iabx^{3/2} \operatorname{PolyLog}(5, -ie^{c+d\sqrt{x}})}{d^5} \\
 &\quad - \frac{3360iabx^{3/2} \operatorname{PolyLog}(5, ie^{c+d\sqrt{x}})}{d^5} \\
 &\quad + \frac{315b^2 x \operatorname{PolyLog}(5, -e^{2(c+d\sqrt{x})})}{d^6} \\
 &\quad - \frac{10080iabx \operatorname{PolyLog}(6, -ie^{c+d\sqrt{x}})}{d^6} \\
 &\quad + \frac{10080iabx \operatorname{PolyLog}(6, ie^{c+d\sqrt{x}})}{d^6} \\
 &\quad - \frac{315b^2 \sqrt{x} \operatorname{PolyLog}(6, -e^{2(c+d\sqrt{x})})}{d^7} \\
 &\quad + \frac{20160iab\sqrt{x} \operatorname{PolyLog}(7, -ie^{c+d\sqrt{x}})}{d^7} \\
 &\quad - \frac{20160iab\sqrt{x} \operatorname{PolyLog}(7, ie^{c+d\sqrt{x}})}{d^7} \\
 \hline
 3.37. \quad \int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx &= \frac{d^7}{315b^2 \operatorname{PolyLog}(7, -e^{2(c+d\sqrt{x})})} \\
 &\quad + \frac{d^7}{20160iab\sqrt{x} \operatorname{PolyLog}(7, ie^{c+d\sqrt{x}})} \\
 &\quad - \frac{d^7}{20160iab\sqrt{x} \operatorname{PolyLog}(7, -ie^{c+d\sqrt{x}})} \\
 &\quad + \frac{d^6}{10080iabx \operatorname{PolyLog}(6, ie^{c+d\sqrt{x}})} \\
 &\quad - \frac{d^6}{10080iabx \operatorname{PolyLog}(6, -ie^{c+d\sqrt{x}})} \\
 &\quad + \frac{d^6}{315b^2 x \operatorname{PolyLog}(5, -e^{2(c+d\sqrt{x})})} \\
 &\quad - \frac{d^5}{3360iabx^{3/2} \operatorname{PolyLog}(5, -ie^{c+d\sqrt{x}})} \\
 &\quad + \frac{d^5}{3360iabx^{3/2} \operatorname{PolyLog}(5, ie^{c+d\sqrt{x}})} \\
 &\quad - \frac{d^5}{210b^2 x^{3/2} \operatorname{PolyLog}(4, -e^{2(c+d\sqrt{x})})} \\
 &\quad + \frac{d^4}{840iabx^2 \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})} \\
 &\quad - \frac{d^4}{840iabx^2 \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})} \\
 &\quad + \frac{d^4}{105b^2 x^2 \operatorname{PolyLog}(3, -e^{2(c+d\sqrt{x})})} \\
 &\quad - \frac{d^3}{168iabx^{5/2} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})} \\
 &\quad + \frac{d^3}{168iabx^{5/2} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})} \\
 &\quad - \frac{d^3}{42b^2 x^{5/2} \operatorname{PolyLog}(2, -e^{2(c+d\sqrt{x})})} \\
 &\quad + \frac{d^2}{28iabx^3 \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})} \\
 &\quad - \frac{d^2}{28iabx^3 \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})} \\
 &\quad - \frac{d^2}{14b^2 x^3 \log(1 + e^{2(c+d\sqrt{x})})} \\
 &\quad + \frac{2b^2 x^{7/2}}{d} + \frac{a^2 x^4}{4} + \frac{8abx^{7/2} \arctan(e^{c+d\sqrt{x}})}{d}
 \end{aligned}$$

output 
$$\begin{aligned} & 315/2*b^2*polylog(7,-exp(2*c+2*d*x^(1/2)))/d^8+2*b^2*x^(7/2)/d+8*a*b*x^(7/2)*arctan(exp(c+d*x^(1/2)))/d-20160*I*a*b*polylog(8,-I*exp(c+d*x^(1/2)))/d^8+168*I*a*b*x^(5/2)*polylog(3,-I*exp(c+d*x^(1/2)))/d^3+28*I*a*b*x^3*polylog(2,I*exp(c+d*x^(1/2)))/d^2+840*I*a*b*x^2*polylog(4,I*exp(c+d*x^(1/2)))/d^4+3360*I*a*b*x^(3/2)*polylog(5,-I*exp(c+d*x^(1/2)))/d^5+10080*I*a*b*x*polylog(6,I*exp(c+d*x^(1/2)))/d^6+20160*I*a*b*polylog(7,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^7+1/4*a^2*x^4-14*b^2*x^3*ln(1+exp(2*c+2*d*x^(1/2)))/d^2-42*b^2*x^(5/2)*polylog(2,-exp(2*c+2*d*x^(1/2)))/d^3+105*b^2*x^2*polylog(3,-exp(2*c+2*d*x^(1/2)))/d^4-210*b^2*x^(3/2)*polylog(4,-exp(2*c+2*d*x^(1/2)))/d^5+315*b^2*x*polylog(5,-exp(2*c+2*d*x^(1/2)))/d^6-315*b^2*polylog(6,-exp(2*c+2*d*x^(1/2)))*x^(1/2)/d^7+2*b^2*x^(7/2)*tanh(c+d*x^(1/2))/d+20160*I*a*b*polylog(8,I*exp(c+d*x^(1/2)))/d^8-28*I*a*b*x^3*polylog(2,-I*exp(c+d*x^(1/2)))/d^2-168*I*a*b*x^(5/2)*polylog(3,I*exp(c+d*x^(1/2)))/d^3-840*I*a*b*x^2*polylog(4,-I*exp(c+d*x^(1/2)))/d^4-3360*I*a*b*x^(3/2)*polylog(5,I*exp(c+d*x^(1/2)))/d^5-10080*I*a*b*x*polylog(6,-I*exp(c+d*x^(1/2)))/d^6-20160*I*a*b*polylog(7,I*exp(c+d*x^(1/2)))*x^(1/2)/d^7 \end{aligned}$$

### 3.37.2 Mathematica [A] (verified)

Time = 8.60 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.10

$$\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

$$= \frac{\cosh(c + d\sqrt{x}) (a + b \operatorname{sech}(c + d\sqrt{x}))^2 \left( \frac{16b^2 e^{2c} x^{7/2} \cosh(c + d\sqrt{x})}{d(1 + e^{2c})} + a^2 x^4 \cosh(c + d\sqrt{x}) \right) + \frac{2ib \cosh(c + d\sqrt{x}) (8ad^7)}{d(1 + e^{2c})}}{d(1 + e^{2c})}$$

input `Integrate[x^3*(a + b*Sech[c + d*Sqrt[x]])^2,x]`



output

```
(Cosh[c + d*Sqrt[x]]*(a + b*Sech[c + d*Sqrt[x]])^2*((16*b^2*E^(2*c)*x^(7/2)
)*Cosh[c + d*Sqrt[x]])/(d*(1 + E^(2*c))) + a^2*x^4*Cosh[c + d*Sqrt[x]] + (
(2*I)*b*Cosh[c + d*Sqrt[x]]*(8*a*d^7*x^(7/2)*Log[1 - I*E^(c + d*Sqrt[x])]
- 8*a*d^7*x^(7/2)*Log[1 + I*E^(c + d*Sqrt[x])] + (28*I)*b*d^6*x^3*Log[1 +
E^(2*(c + d*Sqrt[x]))] - 56*a*d^6*x^3*PolyLog[2, (-I)*E^(c + d*Sqrt[x])] +
56*a*d^6*x^3*PolyLog[2, I*E^(c + d*Sqrt[x])] + (84*I)*b*d^5*x^(5/2)*PolyL
og[2, -E^(2*(c + d*Sqrt[x]))] + 336*a*d^5*x^(5/2)*PolyLog[3, (-I)*E^(c + d
*Sqrt[x])] - 336*a*d^5*x^(5/2)*PolyLog[3, I*E^(c + d*Sqrt[x])] - (210*I)*b
*d^4*x^2*PolyLog[3, -E^(2*(c + d*Sqrt[x]))] - 1680*a*d^4*x^2*PolyLog[4, (-
I)*E^(c + d*Sqrt[x])] + 1680*a*d^4*x^2*PolyLog[4, I*E^(c + d*Sqrt[x])] + (
420*I)*b*d^3*x^(3/2)*PolyLog[4, -E^(2*(c + d*Sqrt[x]))] + 6720*a*d^3*x^(3/
2)*PolyLog[5, (-I)*E^(c + d*Sqrt[x])] - 6720*a*d^3*x^(3/2)*PolyLog[5, I*E^
(c + d*Sqrt[x])] - (630*I)*b*d^2*x*PolyLog[5, -E^(2*(c + d*Sqrt[x]))] - 20
160*a*d^2*x*PolyLog[6, (-I)*E^(c + d*Sqrt[x])] + 20160*a*d^2*x*PolyLog[6,
I*E^(c + d*Sqrt[x])] + (630*I)*b*d*Sqrt[x]*PolyLog[6, -E^(2*(c + d*Sqrt[x]
))] + 40320*a*d*Sqrt[x]*PolyLog[7, (-I)*E^(c + d*Sqrt[x])] - 40320*a*d*Sqr
t[x]*PolyLog[7, I*E^(c + d*Sqrt[x])] - (315*I)*b*PolyLog[7, -E^(2*(c + d*S
qrt[x]))] - 40320*a*PolyLog[8, (-I)*E^(c + d*Sqrt[x])] + 40320*a*PolyLog[8
, I*E^(c + d*Sqrt[x])])]/d^8 + (8*b^2*x^(7/2)*Sech[c]*Sinh[d*Sqrt[x]])/d)
/(4*(b + a*Cosh[c + d*Sqrt[x]])^2)
```

### 3.37.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {5959, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

$$\downarrow 5959$$

$$2 \int x^{7/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 d\sqrt{x}$$

$$\downarrow 3042$$

$$2 \int x^{7/2} \left( a + b \operatorname{csc} \left( ic + id\sqrt{x} + \frac{\pi}{2} \right) \right)^2 d\sqrt{x}$$

$$\downarrow 4678$$

$$2 \int \left( a^2 x^{7/2} + b^2 \operatorname{sech}^2(c + d\sqrt{x}) x^{7/2} + 2ab \operatorname{sech}(c + d\sqrt{x}) x^{7/2} \right) d\sqrt{x}$$

↓ 2009

$$2 \left( \frac{a^2 x^4}{8} + \frac{4abx^{7/2} \arctan\left(e^{c+d\sqrt{x}}\right)}{d} - \frac{10080iab \operatorname{PolyLog}\left(8, -ie^{c+d\sqrt{x}}\right)}{d^8} + \frac{10080iab \operatorname{PolyLog}\left(8, ie^{c+d\sqrt{x}}\right)}{d^8} + \frac{10080iab \operatorname{PolyLog}\left(8, -ie^{c+d\sqrt{x}}\right)}{d^8} + \frac{10080iab \operatorname{PolyLog}\left(8, ie^{c+d\sqrt{x}}\right)}{d^8} \right)$$

input `Int[x^3*(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output

```
2*((b^2*x^(7/2))/d + (a^2*x^4)/8 + (4*a*b*x^(7/2)*ArcTan[E^(c + d*Sqrt[x])
])/d - (7*b^2*x^3*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((14*I)*a*b*x^3*Po
lyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((14*I)*a*b*x^3*PolyLog[2, I*E^(c
+ d*Sqrt[x])])/d^2 - (21*b^2*x^(5/2)*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d
^3 + ((84*I)*a*b*x^(5/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((84*I
)*a*b*x^(5/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (105*b^2*x^2*PolyLog[3
, -E^(2*(c + d*Sqrt[x]))])/(2*d^4) - ((420*I)*a*b*x^2*PolyLog[4, (-I)*E^(c
+ d*Sqrt[x])])/d^4 + ((420*I)*a*b*x^2*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^
4 - (105*b^2*x^(3/2)*PolyLog[4, -E^(2*(c + d*Sqrt[x]))])/d^5 + ((1680*I)*a
*b*x^(3/2)*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 - ((1680*I)*a*b*x^(3/2)
*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 + (315*b^2*x*PolyLog[5, -E^(2*(c + d
*Sqrt[x]))])/(2*d^6) - ((5040*I)*a*b*x*PolyLog[6, (-I)*E^(c + d*Sqrt[x])])
/d^6 + ((5040*I)*a*b*x*PolyLog[6, I*E^(c + d*Sqrt[x])])/d^6 - (315*b^2*Sqr
t[x]*PolyLog[6, -E^(2*(c + d*Sqrt[x]))])/(2*d^7) + ((10080*I)*a*b*Sqrt[x]*
PolyLog[7, (-I)*E^(c + d*Sqrt[x])])/d^7 - ((10080*I)*a*b*Sqrt[x]*PolyLog[7
, I*E^(c + d*Sqrt[x])])/d^7 + (315*b^2*PolyLog[7, -E^(2*(c + d*Sqrt[x]))])
/(4*d^8) - ((10080*I)*a*b*PolyLog[8, (-I)*E^(c + d*Sqrt[x])])/d^8 + ((1008
0*I)*a*b*PolyLog[8, I*E^(c + d*Sqrt[x])])/d^8 + (b^2*x^(7/2)*Tanh[c + d*Sq
rt[x]])/d
```

### 3.37.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

---

3.37.  $\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

### 3.37.4 Maple [F]

$$\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

input `int(x^3*(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(x^3*(a+b*sech(c+d*x^(1/2)))^2,x)`

### 3.37.5 Fracas [F]

$$\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^3 dx$$

input `integrate(x^3*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(b^2*x^3*sech(d*sqrt(x) + c)^2 + 2*a*b*x^3*sech(d*sqrt(x) + c) + a^2*x^3, x)`

**3.37.6 Sympy [F]**

$$\int x^3(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int x^3(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

input `integrate(x**3*(a+b*sech(c+d*x**(1/2)))**2,x)`

output `Integral(x**3*(a + b*sech(c + d*sqrt(x)))**2, x)`

**3.37.7 Maxima [F]**

$$\int x^3(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^3 dx$$

input `integrate(x^3*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `1/4*(a^2*d*x^4*e^(2*d*sqrt(x) + 2*c) + a^2*d*x^4 - 16*b^2*x^(7/2))/(d*e^(2*d*sqrt(x) + 2*c) + d) + integrate(2*(2*a*b*d*x^3*e^(d*sqrt(x) + c) + 7*b^2*x^(5/2))/(d*e^(2*d*sqrt(x) + 2*c) + d), x)`

**3.37.8 Giac [F]**

$$\int x^3(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^3 dx$$

input `integrate(x^3*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)^2*x^3, x)`

**3.37.9 Mupad [F(-1)]**

Timed out.

$$\int x^3 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int x^3 \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2 dx$$

input `int(x^3*(a + b/cosh(c + d*x^(1/2)))^2,x)`output `int(x^3*(a + b/cosh(c + d*x^(1/2)))^2, x)`

### 3.38 $\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$

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3.38.8	Giac [F]	243
3.38.9	Mupad [F(-1)]	243

### 3.38.1 Optimal result

Integrand size = 20, antiderivative size = 497

$$\begin{aligned}
 \int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = & \frac{2b^2 x^{5/2}}{d} + \frac{a^2 x^3}{3} + \frac{8abx^{5/2} \arctan(e^{c+d\sqrt{x}})}{d} \\
 & - \frac{10b^2 x^2 \log(1 + e^{2(c+d\sqrt{x})})}{d^2} \\
 & - \frac{20iabx^2 \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} \\
 & + \frac{20iabx^2 \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} \\
 & - \frac{20b^2 x^{3/2} \operatorname{PolyLog}(2, -e^{2(c+d\sqrt{x})})}{d^3} \\
 & + \frac{80iabx^{3/2} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} \\
 & - \frac{80iabx^{3/2} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} \\
 & + \frac{30b^2 x \operatorname{PolyLog}(3, -e^{2(c+d\sqrt{x})})}{d^4} \\
 & - \frac{240iabx \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} \\
 & + \frac{240iabx \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} \\
 & - \frac{30b^2 \sqrt{x} \operatorname{PolyLog}(4, -e^{2(c+d\sqrt{x})})}{d^5} \\
 & + \frac{480iab\sqrt{x} \operatorname{PolyLog}(5, -ie^{c+d\sqrt{x}})}{d^5} \\
 & - \frac{480iab\sqrt{x} \operatorname{PolyLog}(5, ie^{c+d\sqrt{x}})}{d^5} \\
 & + \frac{15b^2 \operatorname{PolyLog}(5, -e^{2(c+d\sqrt{x})})}{d^6} \\
 & - \frac{480iab \operatorname{PolyLog}(6, -ie^{c+d\sqrt{x}})}{d^6} \\
 & + \frac{480iab \operatorname{PolyLog}(6, ie^{c+d\sqrt{x}})}{d^6} + \frac{2b^2 x^{5/2} \tanh(c + d\sqrt{x})}{d}
 \end{aligned}$$

output  $2*b^2*x^{(5/2)}/d+1/3*a^2*x^3+8*a*b*x^{(5/2)}*\arctan(\exp(c+d*x^{(1/2)}))/d-10*b^2*x^2*\ln(1+\exp(2*c+2*d*x^{(1/2)}))/d^2-20*I*a*b*x^2*\text{polylog}(2,-I*\exp(c+d*x^{(1/2)}))/d^2+480*I*a*b*\text{polylog}(6,I*\exp(c+d*x^{(1/2)}))/d^6-20*b^2*x^{(3/2)}*\text{polylog}(2,-\exp(2*c+2*d*x^{(1/2)}))/d^3-480*I*a*b*\text{polylog}(6,-I*\exp(c+d*x^{(1/2)}))/d^6-480*I*a*b*\text{polylog}(5,I*\exp(c+d*x^{(1/2)}))*x^{(1/2)}/d^5+30*b^2*x*\text{polylog}(3,-\exp(2*c+2*d*x^{(1/2)}))/d^4+80*I*a*b*x^{(3/2)}*\text{polylog}(3,-I*\exp(c+d*x^{(1/2)}))/d^3+20*I*a*b*x^2*\text{polylog}(2,I*\exp(c+d*x^{(1/2)}))/d^2+15*b^2*\text{polylog}(5,-\exp(2*c+2*d*x^{(1/2)}))/d^6-80*I*a*b*x^{(3/2)}*\text{polylog}(3,I*\exp(c+d*x^{(1/2)}))/d^3+240*I*a*b*x*\text{polylog}(4,I*\exp(c+d*x^{(1/2)}))/d^4-30*b^2*\text{polylog}(4,-\exp(2*c+2*d*x^{(1/2)}))*x^{(1/2)}/d^5+480*I*a*b*\text{polylog}(5,-I*\exp(c+d*x^{(1/2)}))*x^{(1/2)}/d^5-240*I*a*b*x*\text{polylog}(4,-I*\exp(c+d*x^{(1/2)}))/d^4+2*b^2*x^{(5/2)}*\tanh(c+d*x^{(1/2)})/d$

### 3.38.2 Mathematica [A] (verified)

Time = 7.13 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.17

$$\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

$$= \frac{\cosh(c + d\sqrt{x}) (a + b \operatorname{sech}(c + d\sqrt{x}))^2 \left( \frac{12b^2 e^{2c} x^{5/2} \cosh(c + d\sqrt{x})}{d(1+e^{2c})} + a^2 x^3 \cosh(c + d\sqrt{x}) + \frac{3ib \cosh(c + d\sqrt{x}) (4ad^5)}{d(1+e^{2c})} \right)}{d^6 + (6b^2 x^{5/2} \operatorname{sech}[c] \operatorname{Sinh}[d\sqrt{x}])/d) / (3(b + a \cosh[c + d\sqrt{x}]))^2}$$

input `Integrate[x^2*(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output  $(\operatorname{Cosh}[c + d*\operatorname{Sqrt}[x]]*(a + b*\operatorname{Sech}[c + d*\operatorname{Sqrt}[x]])^2*((12*b^2*E^{(2*c)}*x^{(5/2)})*\operatorname{Cosh}[c + d*\operatorname{Sqrt}[x]])/(d*(1 + E^{(2*c)})) + a^2*x^3*\operatorname{Cosh}[c + d*\operatorname{Sqrt}[x]] + ((3*I)*b*\operatorname{Cosh}[c + d*\operatorname{Sqrt}[x]]*(4*a*d^5*x^{(5/2)}*\operatorname{Log}[1 - I*E^{(c + d*\operatorname{Sqrt}[x])}] - 4*a*d^5*x^{(5/2)}*\operatorname{Log}[1 + I*E^{(c + d*\operatorname{Sqrt}[x])}] + (10*I)*b*d^4*x^2*\operatorname{Log}[1 + E^{(2*(c + d*\operatorname{Sqrt}[x])}]) - 20*a*d^4*x^2*\operatorname{PolyLog}[2, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}] + 20*a*d^4*x^2*\operatorname{PolyLog}[2, I*E^{(c + d*\operatorname{Sqrt}[x])}] + (20*I)*b*d^3*x^{(3/2)}*\operatorname{PolyLog}[2, -E^{(2*(c + d*\operatorname{Sqrt}[x])}]) + 80*a*d^3*x^{(3/2)}*\operatorname{PolyLog}[3, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}] - 80*a*d^3*x^{(3/2)}*\operatorname{PolyLog}[3, I*E^{(c + d*\operatorname{Sqrt}[x])}] - (30*I)*b*d^2*x*\operatorname{PolyLog}[3, -E^{(2*(c + d*\operatorname{Sqrt}[x])}]) - 240*a*d^2*x*\operatorname{PolyLog}[4, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}] + 240*a*d^2*x*\operatorname{PolyLog}[4, I*E^{(c + d*\operatorname{Sqrt}[x])}] + (30*I)*b*d*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[4, -E^{(2*(c + d*\operatorname{Sqrt}[x])}]) + 480*a*d*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[5, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}] - 480*a*d*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[5, I*E^{(c + d*\operatorname{Sqrt}[x])}] - (15*I)*b*\operatorname{PolyLog}[5, -E^{(2*(c + d*\operatorname{Sqrt}[x])}]) - 480*a*\operatorname{PolyLog}[6, (-I)*E^{(c + d*\operatorname{Sqrt}[x])}] + 480*a*\operatorname{PolyLog}[6, I*E^{(c + d*\operatorname{Sqrt}[x])}]))/d^6 + (6*b^2*x^{(5/2)}*\operatorname{Sech}[c]*\operatorname{Sinh}[d*\operatorname{Sqrt}[x]])/d)/(3*(b + a*\operatorname{Cosh}[c + d*\operatorname{Sqrt}[x]]))^2$

$$3.38. \int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$



**3.38.3 Rubi [A] (verified)**

Time = 0.87 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {5959, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx \\
 & \quad \downarrow \text{5959} \\
 & 2 \int x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int x^{5/2} \left( a + b \csc \left( ic + id\sqrt{x} + \frac{\pi}{2} \right) \right)^2 d\sqrt{x} \\
 & \quad \downarrow \text{4678} \\
 & 2 \int \left( a^2 x^{5/2} + b^2 \operatorname{sech}^2(c + d\sqrt{x}) x^{5/2} + 2ab \operatorname{sech}(c + d\sqrt{x}) x^{5/2} \right) d\sqrt{x} \\
 & \quad \downarrow \text{2009} \\
 & 2 \left( \frac{a^2 x^3}{6} + \frac{4abx^{5/2} \arctan(e^{c+d\sqrt{x}})}{d} - \frac{240iab \operatorname{PolyLog}(6, -ie^{c+d\sqrt{x}})}{d^6} + \frac{240iab \operatorname{PolyLog}(6, ie^{c+d\sqrt{x}})}{d^6} + \frac{240iab\sqrt{x}}{d^6} \right)
 \end{aligned}$$

input `Int[x^2*(a + b*Sech[c + d*Sqrt[x]])^2,x]`

```

output 2*((b^2*x^(5/2))/d + (a^2*x^3)/6 + (4*a*b*x^(5/2)*ArcTan[E^(c + d*Sqrt[x])
])/d - (5*b^2*x^2*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((10*I)*a*b*x^2*Po
lyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((10*I)*a*b*x^2*PolyLog[2, I*E^(c
+ d*Sqrt[x])])/d^2 - (10*b^2*x^(3/2)*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d
^3 + ((40*I)*a*b*x^(3/2)*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((40*I)
*a*b*x^(3/2)*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (15*b^2*x*PolyLog[3, -
E^(2*(c + d*Sqrt[x]))])/d^4 - ((120*I)*a*b*x*PolyLog[4, (-I)*E^(c + d*Sqrt
[x])])/d^4 + ((120*I)*a*b*x*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 - (15*b^2
*Sqrt[x]*PolyLog[4, -E^(2*(c + d*Sqrt[x]))])/d^5 + ((240*I)*a*b*Sqrt[x]*Po
lyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 - ((240*I)*a*b*Sqrt[x]*PolyLog[5, I*
E^(c + d*Sqrt[x])])/d^5 + (15*b^2*PolyLog[5, -E^(2*(c + d*Sqrt[x]))])/(2*d
^6) - ((240*I)*a*b*PolyLog[6, (-I)*E^(c + d*Sqrt[x])])/d^6 + ((240*I)*a*b*
PolyLog[6, I*E^(c + d*Sqrt[x])])/d^6 + (b^2*x^(5/2)*Tanh[c + d*Sqrt[x]])/d
)

```

### 3.38.3.1 Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

```
rule 4678 Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

**3.38.4 Maple [F]**

$$\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

input `int(x^2*(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(x^2*(a+b*sech(c+d*x^(1/2)))^2,x)`

**3.38.5 Fricas [F]**

$$\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^2 dx$$

input `integrate(x^2*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(b^2*x^2*sech(d*sqrt(x) + c)^2 + 2*a*b*x^2*sech(d*sqrt(x) + c) + a^2*x^2, x)`

**3.38.6 Sympy [F]**

$$\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

input `integrate(x**2*(a+b*sech(c+d*x**(1/2)))**2,x)`

output `Integral(x**2*(a + b*sech(c + d*sqrt(x)))**2, x)`

**3.38.7 Maxima [F]**

$$\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^2 dx$$

input `integrate(x^2*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `1/3*(a^2*d*x^3*e^(2*d*sqrt(x) + 2*c) + a^2*d*x^3 - 12*b^2*x^(5/2))/(d*e^(2*d*sqrt(x) + 2*c) + d) + integrate(2*(2*a*b*d*x^2*e^(d*sqrt(x) + c) + 5*b^2*x^(3/2))/(d*e^(2*d*sqrt(x) + 2*c) + d), x)`

**3.38.8 Giac [F]**

$$\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^2 dx$$

input `integrate(x^2*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)^2*x^2, x)`

**3.38.9 Mupad [F(-1)]**

Timed out.

$$\int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int x^2 \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2 dx$$

input `int(x^2*(a + b/cosh(c + d*x^(1/2)))^2,x)`

output `int(x^2*(a + b/cosh(c + d*x^(1/2)))^2, x)`

### 3.39 $\int x(a + b\operatorname{sech}(c + d\sqrt{x}))^2 dx$

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#### 3.39.1 Optimal result

Integrand size = 18, antiderivative size = 319

$$\int x(a + b\operatorname{sech}(c + d\sqrt{x}))^2 dx = \frac{2b^2x^{3/2}}{d} + \frac{a^2x^2}{2} + \frac{8abx^{3/2} \arctan(e^{c+d\sqrt{x}})}{d} - \frac{6b^2x \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{12iabx \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{12iabx \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} - \frac{6b^2\sqrt{x} \operatorname{PolyLog}(2, -e^{2(c+d\sqrt{x})})}{d^3} + \frac{24iab\sqrt{x} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{24iab\sqrt{x} \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} + \frac{3b^2 \operatorname{PolyLog}(3, -e^{2(c+d\sqrt{x})})}{d^4} - \frac{24iab \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} + \frac{24iab \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} + \frac{2b^2x^{3/2} \tanh(c + d\sqrt{x})}{d}$$

output  $2*b^2*x^{(3/2)}/d+1/2*a^2*x^2+8*a*b*x^{(3/2)}*\arctan(\exp(c+d*x^{(1/2)}))/d-6*b^2*x*\ln(1+\exp(2*c+2*d*x^{(1/2)}))/d^2-12*I*a*b*x*\text{polylog}(2,-I*\exp(c+d*x^{(1/2)}))/d^2+12*I*a*b*x*\text{polylog}(2,I*\exp(c+d*x^{(1/2)}))/d^2+3*b^2*\text{polylog}(3,-\exp(2*c+2*d*x^{(1/2)}))/d^4-24*I*a*b*\text{polylog}(4,-I*\exp(c+d*x^{(1/2)}))/d^4+24*I*a*b*\text{polylog}(4,I*\exp(c+d*x^{(1/2)}))/d^4-6*b^2*\text{polylog}(2,-\exp(2*c+2*d*x^{(1/2)}))*x^{(1/2)}/d^3+24*I*a*b*\text{polylog}(3,-I*\exp(c+d*x^{(1/2)}))*x^{(1/2)}/d^3-24*I*a*b*\text{polylog}(3,I*\exp(c+d*x^{(1/2)}))*x^{(1/2)}/d^3+2*b^2*x^{(3/2)}*\tanh(c+d*x^{(1/2)})/d$

### 3.39.2 Mathematica [A] (warning: unable to verify)

Time = 5.82 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.46

$$\int x(a + b\operatorname{sech}(c + d\sqrt{x}))^2 dx$$

$$= \frac{\cosh(c + d\sqrt{x}) (a + b\operatorname{sech}(c + d\sqrt{x}))^2 \left( a^2 x^2 \cosh(c + d\sqrt{x}) + \frac{2b \cosh(c + d\sqrt{x}) \left( 4be^{2c} x^{3/2} + \frac{i(1+e^{2c}) (12ibd^2 x \log(1 - Ie^{c+d\sqrt{x}}))}{d} \right)}{d} \right)}{d^3}$$

input `Integrate[x*(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output  $(\operatorname{Cosh}[c + d\sqrt{x}])*(a + b\operatorname{Sech}[c + d\sqrt{x}])^2*(a^2*x^2*\operatorname{Cosh}[c + d\sqrt{x}] + (2*b*\operatorname{Cosh}[c + d\sqrt{x}])*(4*b*E^{(2*c)}*x^{(3/2)} + (I*(1 + E^{(2*c)}))*(12*I)*b*d^2*x*\operatorname{Log}[1 - I*E^{(c + d\sqrt{x})}] + 4*a*d^3*x^{(3/2)}*\operatorname{Log}[1 - I*E^{(c + d\sqrt{x})}] + (12*I)*b*d^2*x*\operatorname{Log}[1 + I*E^{(c + d\sqrt{x})}] - 4*a*d^3*x^{(3/2)}*\operatorname{Log}[1 + I*E^{(c + d\sqrt{x})}] - (6*I)*b*d^2*x*\operatorname{Log}[1 + E^{(2*(c + d\sqrt{x})})]) - 12*((-I)*b*d*\sqrt{x} + a*d^2*x)*\operatorname{PolyLog}[2, (-I)*E^{(c + d\sqrt{x})}]) + 12*(I*b*d*\sqrt{x} + a*d^2*x)*\operatorname{PolyLog}[2, I*E^{(c + d\sqrt{x})}] + 24*a*d*\sqrt{x}*\operatorname{PolyLog}[3, (-I)*E^{(c + d\sqrt{x})}] - 24*a*d*\sqrt{x}*\operatorname{PolyLog}[3, I*E^{(c + d\sqrt{x})}] - (3*I)*b*\operatorname{PolyLog}[3, -E^{(2*(c + d\sqrt{x})})]) - 24*a*\operatorname{PolyLog}[4, (-I)*E^{(c + d\sqrt{x})}] + 24*a*\operatorname{PolyLog}[4, I*E^{(c + d\sqrt{x})}]))/d^3)/(d*(1 + E^{(2*c)})) + (4*b^2*x^{(3/2)}*\operatorname{Sech}[c]*\operatorname{Sinh}[d*\sqrt{x}])/d)/(2*(b + a*\operatorname{Cosh}[c + d*\sqrt{x}])^2)$

### 3.39.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {5959, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x(a + b\operatorname{sech}(c + d\sqrt{x}))^2 dx \\
 & \quad \downarrow \text{5959} \\
 & 2 \int x^{3/2}(a + b\operatorname{sech}(c + d\sqrt{x}))^2 d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int x^{3/2} \left( a + b \csc \left( ic + id\sqrt{x} + \frac{\pi}{2} \right) \right)^2 d\sqrt{x} \\
 & \quad \downarrow \text{4678} \\
 & 2 \int \left( x^{3/2}a^2 + 2bx^{3/2}\operatorname{sech}(c + d\sqrt{x})a + b^2x^{3/2}\operatorname{sech}^2(c + d\sqrt{x}) \right) d\sqrt{x} \\
 & \quad \downarrow \text{2009} \\
 & 2 \left( \frac{a^2x^2}{4} + \frac{4abx^{3/2} \arctan(e^{c+d\sqrt{x}})}{d} - \frac{12iab \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} + \frac{12iab \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} + \frac{12iab\sqrt{x} \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} + \frac{12iab\sqrt{x} \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} \right)
 \end{aligned}$$

input `Int[x*(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output `2*((b^2*x^(3/2))/d + (a^2*x^2)/4 + (4*a*b*x^(3/2)*ArcTan[E^(c + d*Sqrt[x])])/d - (3*b^2*x*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((6*I)*a*b*x*PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((6*I)*a*b*x*PolyLog[2, I*E^(c + d*Sqrt[x])])/d^2 - (3*b^2*Sqrt[x]*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d^3 + ((12*I)*a*b*Sqrt[x]*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((12*I)*a*b*Sqrt[x]*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (3*b^2*PolyLog[3, -E^(2*(c + d*Sqrt[x]))])/(2*d^4) - ((12*I)*a*b*PolyLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((12*I)*a*b*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 + (b^2*x^(3/2)*Tanh[c + d*Sqrt[x]])/d)`

## 3.39.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

## 3.39.4 Maple [F]

$$\int x(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

input `int(x*(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(x*(a+b*sech(c+d*x^(1/2)))^2,x)`

## 3.39.5 Fracas [F]

$$\int x(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x dx$$

input `integrate(x*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fracas")`

output `integral(b^2*x*sech(d*sqrt(x) + c)^2 + 2*a*b*x*sech(d*sqrt(x) + c) + a^2*x, x)`



**3.39.6 Sympy [F]**

$$\int x(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int x(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

input `integrate(x*(a+b*sech(c+d*x**(1/2)))**2,x)`

output `Integral(x*(a + b*sech(c + d*sqrt(x)))**2, x)`

**3.39.7 Maxima [F]**

$$\int x(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x dx$$

input `integrate(x*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `1/2*(a^2*d*x^2*e^(2*d*sqrt(x) + 2*c) + a^2*d*x^2 - 8*b^2*x^(3/2))/(d*e^(2*d*sqrt(x) + 2*c) + d) + integrate(2*(2*a*b*d*x*e^(d*sqrt(x) + c) + 3*b^2*sqrt(x))/(d*e^(2*d*sqrt(x) + 2*c) + d), x)`

**3.39.8 Giac [F]**

$$\int x(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x dx$$

input `integrate(x*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)^2*x, x)`

**3.39.9 Mupad [F(-1)]**

Timed out.

$$\int x(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int x \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2 dx$$

input `int(x*(a + b/cosh(c + d*x^(1/2)))^2,x)`output `int(x*(a + b/cosh(c + d*x^(1/2)))^2, x)`

**3.40** 
$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x} dx$$

3.40.1	Optimal result	250
3.40.2	Mathematica [N/A]	250
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3.40.4	Maple [N/A] (verified)	251
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3.40.7	Maxima [N/A]	252
3.40.8	Giac [N/A]	253
3.40.9	Mupad [N/A]	253

**3.40.1 Optimal result**

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x} dx = \operatorname{Int}\left(\frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x}, x\right)$$

output `Unintegrable((a+b*sech(c+d*x^(1/2)))^2/x,x)`

**3.40.2 Mathematica [N/A]**

Not integrable

Time = 98.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x} dx = \int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x} dx$$

input `Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x,x]`

output `Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x, x]`

---

3.40. 
$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x} dx$$

**3.40.3 Rubi [N/A]**

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x} dx$$

↓ 5961

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x} dx$$

input `Int[(a + b*Sech[c + d*Sqrt[x]])^2/x,x]`

output `$Aborted`

**3.40.3.1 Defintions of rubi rules used**

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

**3.40.4 Maple [N/A] (verified)**

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x} dx$$

input `int((a+b*sech(c+d*x^(1/2)))^2/x,x)`

output `int((a+b*sech(c+d*x^(1/2)))^2/x,x)`

---

3.40.  $\int \frac{(a+b \operatorname{sech}(c+d\sqrt{x}))^2}{x} dx$

**3.40.5 Fracas [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x} dx = \int \frac{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2}{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x,x, algorithm="fricas")`output `integral((b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2)/x, x)`**3.40.6 Sympy [N/A]**

Not integrable

Time = 13.81 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x} dx = \int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x} dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))**2/x,x)`output `Integral((a + b*sech(c + d*sqrt(x)))**2/x, x)`**3.40.7 Maxima [N/A]**

Not integrable

Time = 0.57 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.30

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x} dx = \int \frac{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2}{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x,x, algorithm="maxima")`output `a^2*log(x) - 4*b^2*sqrt(x)/(d*x*e^(2*d*sqrt(x) + 2*c) + d*x) + integrate(2*(2*a*b*d*x*e^(d*sqrt(x) + c) - b^2*sqrt(x))/(d*x^2*e^(2*d*sqrt(x) + 2*c) + d*x^2), x)`

---

3.40.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x} dx$

**3.40.8 Giac [N/A]**

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x} dx = \int \frac{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2}{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x,x, algorithm="giac")`output `integrate((b*sech(d*sqrt(x) + c) + a)^2/x, x)`**3.40.9 Mupad [N/A]**

Not integrable

Time = 2.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x} dx = \int \frac{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2}{x} dx$$

input `int((a + b/cosh(c + d*x^(1/2)))^2/x,x)`output `int((a + b/cosh(c + d*x^(1/2)))^2/x, x)`

**3.41**  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^2} dx$

3.41.1	Optimal result	254
3.41.2	Mathematica [N/A]	254
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3.41.4	Maple [N/A] (verified)	255
3.41.5	Fricas [N/A]	256
3.41.6	Sympy [N/A]	256
3.41.7	Maxima [N/A]	256
3.41.8	Giac [N/A]	257
3.41.9	Mupad [N/A]	257

**3.41.1 Optimal result**

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx = \operatorname{Int}\left(\frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^2}, x\right)$$

output `Unintegrable((a+b*sech(c+d*x^(1/2)))^2/x^2,x)`

**3.41.2 Mathematica [N/A]**

Not integrable

Time = 37.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx$$

input `Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^2,x]`

output `Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^2, x]`

---

3.41.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^2} dx$

### 3.41.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx$$

↓ 5961

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx$$

input `Int[(a + b*Sech[c + d*Sqrt[x]])^2/x^2,x]`

output `$Aborted`

#### 3.41.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.41.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx$$

input `int((a+b*sech(c+d*x^(1/2)))^2/x^2,x)`

output `int((a+b*sech(c+d*x^(1/2)))^2/x^2,x)`

---

3.41.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^2} dx$



**3.41.5 Fracas [N/A]**

Not integrable

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x^2,x, algorithm="fricas")`output `integral((b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2)/x^2, x)`**3.41.6 Sympy [N/A]**

Not integrable

Time = 2.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))**2/x**2,x)`output `Integral((a + b*sech(c + d*sqrt(x)))**2/x**2, x)`**3.41.7 Maxima [N/A]**

Not integrable

Time = 0.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.50

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x^2,x, algorithm="maxima")`output `-(a^2*d*x*e^(2*d*sqrt(x) + 2*c) + a^2*d*x + 4*b^2*sqrt(x))/(d*x^2*e^(2*d*sqrt(x) + 2*c) + d*x^2) + integrate(2*(2*a*b*d*x*e^(d*sqrt(x) + c) - 3*b^2*sqrt(x))/(d*x^3*e^(2*d*sqrt(x) + 2*c) + d*x^3), x)`

---

3.41.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^2} dx$

**3.41.8 Giac [N/A]**

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2}{x^2} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x^2,x, algorithm="giac")`output `integrate((b*sech(d*sqrt(x) + c) + a)^2/x^2, x)`**3.41.9 Mupad [N/A]**

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^2} dx = \int \frac{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2}{x^2} dx$$

input `int((a + b/cosh(c + d*x^(1/2)))^2/x^2,x)`output `int((a + b/cosh(c + d*x^(1/2)))^2/x^2, x)`

$$3.42 \quad \int \frac{x^3}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$$

3.42.1	Optimal result	259
3.42.2	Mathematica [A] (verified)	260
3.42.3	Rubi [A] (verified)	261
3.42.4	Maple [F]	263
3.42.5	Fricas [F]	263
3.42.6	Sympy [F]	264
3.42.7	Maxima [F(-2)]	264
3.42.8	Giac [F]	264
3.42.9	Mupad [F(-1)]	265

### 3.42.1 Optimal result

Integrand size = 20, antiderivative size = 961

$$\begin{aligned}
 \int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx &= \frac{x^4}{4a} - \frac{2bx^{7/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{7/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
 &\quad - \frac{14bx^3 \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
 &\quad + \frac{14bx^3 \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
 &\quad + \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
 &\quad - \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
 &\quad - \frac{420bx^2 \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
 &\quad + \frac{420bx^2 \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
 &\quad + \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
 &\quad - \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
 &\quad - \frac{5040bx \operatorname{PolyLog}\left(6, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} \\
 &\quad + \frac{5040bx \operatorname{PolyLog}\left(6, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} \\
 &\quad + \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} \\
 &\quad - \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} \\
 &\quad - \frac{10080b \operatorname{PolyLog}\left(8, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8} \\
 &\quad + \frac{10080b \operatorname{PolyLog}\left(8, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8}
 \end{aligned}$$

---

3.42.  $\int \frac{x^3}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$

output

$$\begin{aligned} & \frac{1}{4}x^4/a-2bx^{7/2}*\ln(1+a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d/(- \\ & a^2+b^2)^{1/2}+2*b*x^{7/2}*\ln(1+a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a \\ & /d/(-a^2+b^2)^{1/2}-14*b*x^3*polylog(2,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2} \\ & (1/2))/a/d^2/(-a^2+b^2)^{1/2}+14*b*x^3*polylog(2,-a*\exp(c+d*x^{1/2}))/b+(- \\ & a^2+b^2)^{1/2}))/a/d^2/(-a^2+b^2)^{1/2}+84*b*x^{5/2}*polylog(3,-a*\exp(c+d* \\ & x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d^3/(-a^2+b^2)^{1/2}-84*b*x^{5/2}*polylog \\ & (3,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d^3/(-a^2+b^2)^{1/2}-420*b* \\ & x^2*polylog(4,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d^4/(-a^2+b^2)^{1/2} \\ & +420*b*x^2*polylog(4,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d^4/(- \\ & a^2+b^2)^{1/2}+1680*b*x^{3/2}*polylog(5,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2} \\ & (1/2))/a/d^5/(-a^2+b^2)^{1/2}-1680*b*x^{3/2}*polylog(5,-a*\exp(c+d*x^{1/2} \\ & ))/b+(-a^2+b^2)^{1/2}))/a/d^5/(-a^2+b^2)^{1/2}-5040*b*x*polylog(6,-a*\exp( \\ & c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d^6/(-a^2+b^2)^{1/2}+5040*b*x*polylog \\ & (6,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d^6/(-a^2+b^2)^{1/2}-10080* \\ & b*polylog(8,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d^8/(-a^2+b^2)^{1/2} \\ & +10080*b*polylog(8,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d^8/(-a^2 \\ & +b^2)^{1/2}+10080*b*polylog(7,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))*x^{ \\ & (1/2)}/a/d^7/(-a^2+b^2)^{1/2}-10080*b*polylog(7,-a*\exp(c+d*x^{1/2}))/b+(-a^ \\ & 2+b^2)^{1/2}))*x^{1/2}}/a/d^7/(-a^2+b^2)^{1/2} \end{aligned}$$

### 3.42.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 721, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

$$= \frac{\sqrt{-a^2 + b^2} d^8 x^4 - 8bd^7 x^{7/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right) + 8bd^7 x^{7/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right) - 56bd^6 x^3 \operatorname{PolyLog}\left(2, \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right) - 56bd^6 x^3 \operatorname{PolyLog}\left(2, \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{d^8}$$

input `Integrate[x^3/(a + b*Sech[c + d*Sqrt[x]]),x]`

output

```
(Sqrt[-a^2 + b^2]*d^8*x^4 - 8*b*d^7*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/
(b - Sqrt[-a^2 + b^2])] + 8*b*d^7*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b
+ Sqrt[-a^2 + b^2])] - 56*b*d^6*x^3*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b
+ Sqrt[-a^2 + b^2])] + 56*b*d^6*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b
+ Sqrt[-a^2 + b^2]))] + 336*b*d^5*x^(5/2)*PolyLog[3, (a*E^(c + d*Sqrt[x]))
/(-b + Sqrt[-a^2 + b^2])] - 336*b*d^5*x^(5/2)*PolyLog[3, -((a*E^(c + d*Sqr
t[x]))/(b + Sqrt[-a^2 + b^2]))] - 1680*b*d^4*x^2*PolyLog[4, (a*E^(c + d*Sq
rt[x]))/(-b + Sqrt[-a^2 + b^2])] + 1680*b*d^4*x^2*PolyLog[4, -((a*E^(c + d
*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))] + 6720*b*d^3*x^(3/2)*PolyLog[5, (a*E^(
c + d*Sqrt[x]))/(-b + Sqrt[-a^2 + b^2])] - 6720*b*d^3*x^(3/2)*PolyLog[5, -
((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))] - 20160*b*d^2*x*PolyLog[6,
(a*E^(c + d*Sqrt[x]))/(-b + Sqrt[-a^2 + b^2])] + 20160*b*d^2*x*PolyLog[6,
-((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))] + 40320*b*d*Sqrt[x]*Poly
Log[7, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[-a^2 + b^2])] - 40320*b*d*Sqrt[x]*
PolyLog[7, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))] - 40320*b*Poly
Log[8, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[-a^2 + b^2])] + 40320*b*PolyLog[8,
-((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(4*a*Sqrt[-a^2 + b^2]*d
^8)
```

### 3.42.3 Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 962, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

$$\downarrow 5959$$

$$2 \int \frac{x^{7/2}}{a + b \operatorname{sech}(c + d\sqrt{x})} d\sqrt{x}$$

$$\downarrow 3042$$

$$2 \int \frac{x^{7/2}}{a + b \csc\left(ic + id\sqrt{x} + \frac{\pi}{2}\right)} d\sqrt{x}$$

$$\downarrow 4679$$

---

3.42.  $\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$

$$2 \int \left( \frac{x^{7/2}}{a} - \frac{bx^{7/2}}{a(b + a \cosh(c + d\sqrt{x}))} \right) d\sqrt{x}$$

↓ 2009

$$2 \left( \frac{x^4}{8a} - \frac{b \log \left( \frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}} + 1 \right) x^{7/2}}{a\sqrt{b^2-a^2}d} + \frac{b \log \left( \frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}} + 1 \right) x^{7/2}}{a\sqrt{b^2-a^2}d} - \frac{7b \operatorname{PolyLog} \left( 2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}} \right) x^3}{a\sqrt{b^2-a^2}d^2} + \frac{7b \operatorname{PolyLog} \left( 2, \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}} \right) x^3}{a\sqrt{b^2-a^2}d^2} \right)$$

input `Int[x^3/(a + b*Sech[c + d*Sqrt[x]]),x]`

output

```
2*(x^4/(8*a) - (b*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (b*x^(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (7*b*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (7*b*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (42*b*x^(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (42*b*x^(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (210*b*x^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (210*b*x^2*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (840*b*x^(3/2)*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) - (840*b*x^(3/2)*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) - (2520*b*x*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6) + (2520*b*x*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^6) + (5040*b*Sqrt[x]*PolyLog[7, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^7) - (5040*b*Sqrt[x]*PolyLog[7, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^7) - (5040*b*PolyLog[8, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sq...
```

### 3.42.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

---

3.42.  $\int \frac{x^3}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

### 3.42.4 Maple [F]

$$\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

input `int(x^3/(a+b*sech(c+d*x^(1/2))),x)`

output `int(x^3/(a+b*sech(c+d*x^(1/2))),x)`

### 3.42.5 Fracas [F]

$$\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^3}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

input `integrate(x^3/(a+b*sech(c+d*x^(1/2))),x, algorithm="fracas")`

output `integral(x^3/(b*sech(d*sqrt(x) + c) + a), x)`



### 3.42.6 Sympy [F]

$$\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

input `integrate(x**3/(a+b*sech(c+d*x**(1/2))),x)`

output `Integral(x**3/(a + b*sech(c + d*sqrt(x))), x)`

### 3.42.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \text{Exception raised: ValueError}$$

input `integrate(x^3/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

### 3.42.8 Giac [F]

$$\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^3}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

input `integrate(x^3/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate(x^3/(b*sech(d*sqrt(x) + c) + a), x)`

**3.42.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x^3}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^3}{a + \frac{b}{\cosh(c + d\sqrt{x})}} dx$$

input `int(x^3/(a + b/cosh(c + d*x^(1/2))),x)`output `int(x^3/(a + b/cosh(c + d*x^(1/2))), x)`

$$3.43 \quad \int \frac{x^2}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$$

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### 3.43.1 Optimal result

Integrand size = 20, antiderivative size = 721

$$\begin{aligned}
 \int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx &= \frac{x^3}{3a} - \frac{2bx^{5/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx^{5/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
 &\quad - \frac{10bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
 &\quad + \frac{10bx^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
 &\quad + \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
 &\quad - \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
 &\quad - \frac{120bx \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
 &\quad + \frac{120bx \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
 &\quad + \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
 &\quad - \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
 &\quad - \frac{240b \operatorname{PolyLog}\left(6, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} \\
 &\quad + \frac{240b \operatorname{PolyLog}\left(6, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}
 \end{aligned}$$

output  $\frac{1}{3}x^3/a-2bx^{5/2}*\ln(1+a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d/(-a^2+b^2)^{1/2}+2*b*x^{5/2}*\ln(1+a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d/(-a^2+b^2)^{1/2}-10*b*x^2*polylog(2,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d^2/(-a^2+b^2)^{1/2}+10*b*x^2*polylog(2,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d^2/(-a^2+b^2)^{1/2}+40*b*x^{3/2}*polylog(3,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d^3/(-a^2+b^2)^{1/2}-40*b*x^{3/2}*polylog(3,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d^3/(-a^2+b^2)^{1/2}-120*b*x*polylog(4,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d^4/(-a^2+b^2)^{1/2}+120*b*x*polylog(4,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d^4/(-a^2+b^2)^{1/2}-240*b*polylog(6,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d^6/(-a^2+b^2)^{1/2}+240*b*polylog(6,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d^6/(-a^2+b^2)^{1/2}+240*b*polylog(5,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))*x^{1/2}/a/d^5/(-a^2+b^2)^{1/2}-240*b*polylog(5,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))*x^{1/2}/a/d^5/(-a^2+b^2)^{1/2}$

### 3.43.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 547, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

$$= \frac{\sqrt{-a^2 + b^2} d^6 x^3 - 6bd^5 x^{5/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right) + 6bd^5 x^{5/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right) - 30bd^4 x^2 \operatorname{PolyLog}\left(2, \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right) - 30bd^4 x^2 \operatorname{PolyLog}\left(2, \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{\dots}$$

input `Integrate[x^2/(a + b*Sech[c + d*Sqrt[x]]),x]`

output  $(\operatorname{Sqrt}[-a^2 + b^2]*d^6*x^3 - 6*b*d^5*x^{5/2}*\operatorname{Log}[1 + (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b - \operatorname{Sqrt}[-a^2 + b^2])]) + 6*b*d^5*x^{5/2}*\operatorname{Log}[1 + (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b + \operatorname{Sqrt}[-a^2 + b^2])]) - 30*b*d^4*x^2*\operatorname{PolyLog}[2, (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(-b + \operatorname{Sqrt}[-a^2 + b^2])]) + 30*b*d^4*x^2*\operatorname{PolyLog}[2, -((a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b + \operatorname{Sqrt}[-a^2 + b^2]))]) + 120*b*d^3*x^{3/2}*\operatorname{PolyLog}[3, (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(-b + \operatorname{Sqrt}[-a^2 + b^2])]) - 120*b*d^3*x^{3/2}*\operatorname{PolyLog}[3, -((a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b + \operatorname{Sqrt}[-a^2 + b^2]))]) - 360*b*d^2*x*\operatorname{PolyLog}[4, (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(-b + \operatorname{Sqrt}[-a^2 + b^2])]) + 360*b*d^2*x*\operatorname{PolyLog}[4, -((a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b + \operatorname{Sqrt}[-a^2 + b^2]))]) + 720*b*d*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[5, (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(-b + \operatorname{Sqrt}[-a^2 + b^2])]) - 720*b*d*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[5, -((a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b + \operatorname{Sqrt}[-a^2 + b^2]))]) - 720*b*\operatorname{PolyLog}[6, (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(-b + \operatorname{Sqrt}[-a^2 + b^2])]) + 720*b*\operatorname{PolyLog}[6, -((a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b + \operatorname{Sqrt}[-a^2 + b^2]))])]/(3*a*\operatorname{Sqrt}[-a^2 + b^2]*d^6)$

3.43.  $\int \frac{x^2}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$

### 3.43.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx \\
 & \quad \downarrow \text{5959} \\
 & 2 \int \frac{x^{5/2}}{a + b \operatorname{sech}(c + d\sqrt{x})} d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int \frac{x^{5/2}}{a + b \operatorname{csc}\left(ic + id\sqrt{x} + \frac{\pi}{2}\right)} d\sqrt{x} \\
 & \quad \downarrow \text{4679} \\
 & 2 \int \left( \frac{x^{5/2}}{a} - \frac{bx^{5/2}}{a(b + a \cosh(c + d\sqrt{x}))} \right) d\sqrt{x} \\
 & \quad \downarrow \text{2009} \\
 & 2 \left( -\frac{120b \operatorname{PolyLog}\left(6, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^6\sqrt{b^2-a^2}} + \frac{120b \operatorname{PolyLog}\left(6, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^6\sqrt{b^2-a^2}} + \frac{120b\sqrt{x} \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{120b\sqrt{x} \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} \right)
 \end{aligned}$$

input `Int[x^2/(a + b*Sech[c + d*Sqrt[x]]),x]`

```

output 2*(x^3/(6*a) - (b*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b
^2])])/(a*Sqrt[-a^2 + b^2]*d) + (b*x^(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(
b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (5*b*x^2*PolyLog[2, -((a*
E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])]))/(a*Sqrt[-a^2 + b^2]*d^2) + (5
*b*x^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])]))/(a*Sqr
t[-a^2 + b^2]*d^2) + (20*b*x^(3/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b -
Sqrt[-a^2 + b^2])]))/(a*Sqrt[-a^2 + b^2]*d^3) - (20*b*x^(3/2)*PolyLog[3,
-((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])]))/(a*Sqrt[-a^2 + b^2]*d^3)
- (60*b*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])]))/(a
*Sqrt[-a^2 + b^2]*d^4) + (60*b*x*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + S
qrt[-a^2 + b^2])]))/(a*Sqrt[-a^2 + b^2]*d^4) + (120*b*Sqrt[x]*PolyLog[5, -
((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])]))/(a*Sqrt[-a^2 + b^2]*d^5)
- (120*b*Sqrt[x]*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]
)))/(a*Sqrt[-a^2 + b^2]*d^5) - (120*b*PolyLog[6, -((a*E^(c + d*Sqrt[x]))/(
b - Sqrt[-a^2 + b^2])]))/(a*Sqrt[-a^2 + b^2]*d^6) + (120*b*PolyLog[6, -((a
*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])]))/(a*Sqrt[-a^2 + b^2]*d^6))

```

### 3.43.3.1 Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

```
rule 4679 Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbo
l] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x]
)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m
+ 1)/n], 0] && IntegerQ[p]
```

**3.43.4 Maple [F]**

$$\int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

input `int(x^2/(a+b*sech(c+d*x^(1/2))),x)`

output `int(x^2/(a+b*sech(c+d*x^(1/2))),x)`

**3.43.5 Fricas [F]**

$$\int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^2}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

input `integrate(x^2/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(x^2/(b*sech(d*sqrt(x) + c) + a), x)`

**3.43.6 Sympy [F]**

$$\int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

input `integrate(x**2/(a+b*sech(c+d*x**(1/2))),x)`

output `Integral(x**2/(a + b*sech(c + d*sqrt(x))), x)`



**3.43.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \text{Exception raised: ValueError}$$

input `integrate(x^2/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

**3.43.8 Giac [F]**

$$\int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^2}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

input `integrate(x^2/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate(x^2/(b*sech(d*sqrt(x) + c) + a), x)`

**3.43.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^2}{a + \frac{b}{\cosh(c + d\sqrt{x})}} dx$$

input `int(x^2/(a + b/cosh(c + d*x^(1/2))),x)`

output `int(x^2/(a + b/cosh(c + d*x^(1/2))), x)`

### 3.44 $\int \frac{x}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$

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3.44.9	Mupad [F(-1)]	278

#### 3.44.1 Optimal result

Integrand size = 18, antiderivative size = 481

$$\int \frac{x}{a+b\operatorname{sech}(c+d\sqrt{x})} dx = \frac{x^2}{2a} - \frac{2bx^{3/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$

$$+ \frac{2bx^{3/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{6bx \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$+ \frac{6bx \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$+ \frac{12b\sqrt{x} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$- \frac{12b\sqrt{x} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$- \frac{12b \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$

$$+ \frac{12b \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$

output  $\frac{1}{2}x^2/a-2bx^{3/2}*\ln(1+a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d/(-a^2+b^2)^{1/2}+2*b*x^{3/2}*\ln(1+a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d/(-a^2+b^2)^{1/2}-6*b*x*polylog(2,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d^2/(-a^2+b^2)^{1/2}+6*b*x*polylog(2,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d^2/(-a^2+b^2)^{1/2}-12*b*polylog(4,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))/a/d^4/(-a^2+b^2)^{1/2}+12*b*polylog(4,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))/a/d^4/(-a^2+b^2)^{1/2}+12*b*polylog(3,-a*\exp(c+d*x^{1/2}))/b-(-a^2+b^2)^{1/2}))*x^{1/2}/a/d^3/(-a^2+b^2)^{1/2}-12*b*polylog(3,-a*\exp(c+d*x^{1/2}))/b+(-a^2+b^2)^{1/2}))*x^{1/2}/a/d^3/(-a^2+b^2)^{1/2}$

### 3.44.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.78

$$\int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

$$= \frac{\sqrt{-a^2 + b^2} d^4 x^2 - 4bd^3 x^{3/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right) + 4bd^3 x^{3/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right) - 12bd^2 x \operatorname{PolyLog}\left(2, \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right) - 12bd^2 x \operatorname{PolyLog}\left(2, \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2 + b^2} d^4}$$

input `Integrate[x/(a + b*Sech[c + d*Sqrt[x]]),x]`

output  $(\operatorname{Sqrt}[-a^2 + b^2]*d^4*x^2 - 4*b*d^3*x^{3/2}*\operatorname{Log}[1 + (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b - \operatorname{Sqrt}[-a^2 + b^2])] + 4*b*d^3*x^{3/2}*\operatorname{Log}[1 + (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b + \operatorname{Sqrt}[-a^2 + b^2])] - 12*b*d^2*x*\operatorname{PolyLog}[2, (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(-b + \operatorname{Sqrt}[-a^2 + b^2])] + 12*b*d^2*x*\operatorname{PolyLog}[2, -((a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b + \operatorname{Sqrt}[-a^2 + b^2]))] + 24*b*d*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[3, (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(-b + \operatorname{Sqrt}[-a^2 + b^2])] - 24*b*d*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[3, -((a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b + \operatorname{Sqrt}[-a^2 + b^2]))] - 24*b*\operatorname{PolyLog}[4, (a*E^{(c + d*\operatorname{Sqrt}[x]))}/(-b + \operatorname{Sqrt}[-a^2 + b^2])] + 24*b*\operatorname{PolyLog}[4, -((a*E^{(c + d*\operatorname{Sqrt}[x]))}/(b + \operatorname{Sqrt}[-a^2 + b^2]))])]/(2*a*\operatorname{Sqrt}[-a^2 + b^2]*d^4)$

**3.44.3 Rubi [A] (verified)**

Time = 1.10 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx \\
 & \quad \downarrow \text{5959} \\
 & 2 \int \frac{x^{3/2}}{a + b \operatorname{sech}(c + d\sqrt{x})} d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int \frac{x^{3/2}}{a + b \csc\left(ic + id\sqrt{x} + \frac{\pi}{2}\right)} d\sqrt{x} \\
 & \quad \downarrow \text{4679} \\
 & 2 \int \left( \frac{x^{3/2}}{a} - \frac{bx^{3/2}}{a(b + a \cosh(c + d\sqrt{x}))} \right) d\sqrt{x} \\
 & \quad \downarrow \text{2009} \\
 & 2 \left( -\frac{6b \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{6b \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{6b\sqrt{x} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{6b\sqrt{x} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} \right)
 \end{aligned}$$

input `Int[x/(a + b*Sech[c + d*Sqrt[x]]),x]`

```
output 2*(x^2/(4*a) - (b*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (b*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (3*b*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (3*b*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (6*b*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (6*b*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (6*b*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (6*b*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4))
```

### 3.44.3.1 Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

```
rule 4679 Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]
```

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

**3.44.4 Maple [F]**

$$\int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

input `int(x/(a+b*sech(c+d*x^(1/2))),x)`

output `int(x/(a+b*sech(c+d*x^(1/2))),x)`

**3.44.5 Fricas [F]**

$$\int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

input `integrate(x/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(x/(b*sech(d*sqrt(x) + c) + a), x)`

**3.44.6 Sympy [F]**

$$\int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

input `integrate(x/(a+b*sech(c+d*x**(1/2))),x)`

output `Integral(x/(a + b*sech(c + d*sqrt(x))), x)`

**3.44.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \text{Exception raised: ValueError}$$

input `integrate(x/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

**3.44.8 Giac [F]**

$$\int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

input `integrate(x/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate(x/(b*sech(d*sqrt(x) + c) + a), x)`

**3.44.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x}{a + \frac{b}{\cosh(c + d\sqrt{x})}} dx$$

input `int(x/(a + b/cosh(c + d*x^(1/2))),x)`

output `int(x/(a + b/cosh(c + d*x^(1/2))), x)`

**3.45** 
$$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

3.45.1	Optimal result	279
3.45.2	Mathematica [N/A]	279
3.45.3	Rubi [N/A]	280
3.45.4	Maple [N/A] (verified)	280
3.45.5	Fricas [N/A]	281
3.45.6	Sympy [N/A]	281
3.45.7	Maxima [N/A]	281
3.45.8	Giac [N/A]	282
3.45.9	Mupad [N/A]	282

**3.45.1 Optimal result**

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))} dx = \operatorname{Int}\left(\frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))}, x\right)$$

output `Unintegrable(1/x/(a+b*sech(c+d*x^(1/2))),x)`

**3.45.2 Mathematica [N/A]**

Not integrable

Time = 5.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))} dx = \int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

input `Integrate[1/(x*(a + b*Sech[c + d*Sqrt[x]])),x]`

output `Integrate[1/(x*(a + b*Sech[c + d*Sqrt[x]])), x]`



### 3.45.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a + b \operatorname{sech} (c + d \sqrt{x}))} dx$$

↓ 5961

$$\int \frac{1}{x (a + b \operatorname{sech} (c + d \sqrt{x}))} dx$$

input `Int[1/(x*(a + b*Sech[c + d*Sqrt[x]])),x]`

output `$Aborted`

#### 3.45.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.45.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x (a + b \operatorname{sech} (c + d \sqrt{x}))} dx$$

input `int(1/x/(a+b*sech(c+d*x^(1/2))),x)`

output `int(1/x/(a+b*sech(c+d*x^(1/2))),x)`

**3.45.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)x} dx$$

input `integrate(1/x/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")`output `integral(1/(b*x*sech(d*sqrt(x) + c) + a*x), x)`**3.45.6 Sympy [N/A]**

Not integrable

Time = 2.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{x(a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

input `integrate(1/x/(a+b*sech(c+d*x**(1/2))),x)`output `Integral(1/(x*(a + b*sech(c + d*sqrt(x)))), x)`**3.45.7 Maxima [N/A]**

Not integrable

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{1}{x(a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)x} dx$$

input `integrate(1/x/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`output `-2*b*integrate(e^(d*sqrt(x) + c)/(a^2*x*e^(2*d*sqrt(x) + 2*c) + 2*a*b*x*e^(d*sqrt(x) + c) + a^2*x), x) + log(x)/a`

---

3.45.  $\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$

**3.45.8 Giac [N/A]**

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)x} dx$$

input `integrate(1/x/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`output `integrate(1/((b*sech(d*sqrt(x) + c) + a)*x), x)`**3.45.9 Mupad [N/A]**

Not integrable

Time = 2.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{x \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)} dx$$

input `int(1/(x*(a + b/cosh(c + d*x^(1/2))))),x)`output `int(1/(x*(a + b/cosh(c + d*x^(1/2))))), x)`

### 3.46 $\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^2} dx$

3.46.1	Optimal result	283
3.46.2	Mathematica [N/A]	283
3.46.3	Rubi [N/A]	284
3.46.4	Maple [N/A] (verified)	285
3.46.5	Fricas [N/A]	285
3.46.6	Sympy [N/A]	285
3.46.7	Maxima [N/A]	286
3.46.8	Giac [N/A]	286
3.46.9	Mupad [N/A]	286

#### 3.46.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx = -\frac{a}{x} + b\operatorname{Int}\left(\frac{\operatorname{sech}(c + d\sqrt{x})}{x^2}, x\right)$$

output `-a/x+b*Unintegrable(sech(c+d*x^(1/2))/x^2,x)`

#### 3.46.2 Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

input `Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^2,x]`

output `Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^2, x]`

**3.46.3 Rubi [N/A]**

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

↓ 2010

$$\int \left( \frac{a}{x^2} + \frac{b \operatorname{sech}(c + d\sqrt{x})}{x^2} \right) dx$$

↓ 2009

$$b \int \frac{\operatorname{sech}(c + d\sqrt{x})}{x^2} dx - \frac{a}{x}$$

input `Int[(a + b*Sech[c + d*Sqrt[x]])/x^2,x]`

output `$Aborted`

**3.46.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

**3.46.4 Maple [N/A] (verified)**

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

input `int((a+b*sech(c+d*x^(1/2)))/x^2,x)`output `int((a+b*sech(c+d*x^(1/2)))/x^2,x)`**3.46.5 Fricas [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="fricas")`output `integral((b*sech(d*sqrt(x) + c) + a)/x^2, x)`**3.46.6 Sympy [N/A]**

Not integrable

Time = 1.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))/x**2,x)`output `Integral((a + b*sech(c + d*sqrt(x)))/x**2, x)`

**3.46.7 Maxima [N/A]**

Not integrable

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="maxima")`output `2*b*integrate(e^(d*sqrt(x) + c)/(x^2*e^(2*d*sqrt(x) + 2*c) + x^2), x) - a/x`**3.46.8 Giac [N/A]**

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^2} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^2,x, algorithm="giac")`output `integrate((b*sech(d*sqrt(x) + c) + a)/x^2, x)`**3.46.9 Mupad [N/A]**

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^2} dx = \int \frac{a + \frac{b}{\cosh(c + d\sqrt{x})}}{x^2} dx$$

input `int((a + b/cosh(c + d*x^(1/2)))/x^2,x)`output `int((a + b/cosh(c + d*x^(1/2)))/x^2, x)`

---

3.46.  $\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^2} dx$

$$3.47 \quad \int \frac{x^3}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

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### 3.47.1 Optimal result

Integrand size = 20, antiderivative size = 2851

$$\int \frac{x^3}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx = \text{Too large to display}$$

output

```

10080*b^2*polylog(7,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2
)/d^8+10080*b^2*polylog(7,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a
^2-b^2)/d^8+10080*b^3*polylog(8,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/
a^2/(-a^2+b^2)^(3/2)/d^8-10080*b^3*polylog(8,-a*exp(c+d*x^(1/2))/(b+(-a^2+
b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^8+2*b^2*x^(7/2)*sinh(c+d*x^(1/2))/a/(a
^2-b^2)/d/(b+a*cosh(c+d*x^(1/2)))-20160*b*polylog(8,-a*exp(c+d*x^(1/2))/(b
-(-a^2+b^2)^(1/2)))/a^2/d^8/(-a^2+b^2)^(1/2)+20160*b*polylog(8,-a*exp(c+d*
x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^8/(-a^2+b^2)^(1/2)+2*b^2*x^(7/2)/a^2/
(a^2-b^2)/d-4*b*x^(7/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/
d/(-a^2+b^2)^(1/2)+4*b*x^(7/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)
))/a^2/d/(-a^2+b^2)^(1/2)-28*b*x^3*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+
b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+28*b*x^3*polylog(2,-a*exp(c+d*x^(1/2)
))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+168*b*x^(5/2)*polylog(3,
-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-168*b*x
^(5/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b
^2)^(1/2)-840*b*x^2*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^
2/d^4/(-a^2+b^2)^(1/2)+840*b*x^2*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^
2)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1/2)+3360*b*x^(3/2)*polylog(5,-a*exp(c+d*x^
(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)-3360*b*x^(3/2)*polyl
og(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)...
    
```

---

3.47.  $\int \frac{x^3}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$



### 3.47.2 Mathematica [A] (verified)

Time = 10.37 (sec) , antiderivative size = 3035, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Result too large to show}$$

input `Integrate[x^3/(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output

```
(x^4*(b + a*Cosh[c + d*Sqrt[x]])^2*Sech[c + d*Sqrt[x]]^2)/(4*a^2*(a + b*Sech[c + d*Sqrt[x]])^2) + (2*b*E^c*(b + a*Cosh[c + d*Sqrt[x]])^2*(2*b*E^c*x^(7/2) - ((1 + E^(2*c))*(7*b*d^6*Sqrt[(-a^2 + b^2)*E^(2*c)])*x^3*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 2*a^2*d^7*E^c*x^(7/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) - b^2*d^7*E^c*x^(7/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 7*b*d^6*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^3*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 2*a^2*d^7*E^c*x^(7/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) + b^2*d^7*E^c*x^(7/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 7*d^5*(6*b*Sqrt[(-a^2 + b^2)*E^(2*c)] + 2*a^2*d*E^c*Sqrt[x] - b^2*d*E^c*Sqrt[x])*x^(5/2)*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))]) + 7*d^5*(6*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x^(5/2)*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))]) - 210*b*d^4*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))]) - 84*a^2*d^5*E^c*x^(5/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))]) + 42*b^2*d^5*E^c*x^(5/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))]) - 210*b*d^4*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*PolyLog[3, -((a...
```

### 3.47.3 Rubi [A] (verified)

Time = 4.38 (sec) , antiderivative size = 2850, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

---

3.47.  $\int \frac{x^3}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$

$$\begin{aligned}
& \int \frac{x^3}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx \\
& \quad \downarrow \text{5959} \\
& 2 \int \frac{x^{7/2}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} d\sqrt{x} \\
& \quad \downarrow \text{3042} \\
& 2 \int \frac{x^{7/2}}{(a + b \csc(ic + id\sqrt{x} + \frac{\pi}{2}))^2} d\sqrt{x} \\
& \quad \downarrow \text{4679} \\
& 2 \int \left( -\frac{2bx^{7/2}}{a^2(b + a \cosh(c + d\sqrt{x}))} + \frac{x^{7/2}}{a^2} + \frac{b^2 x^{7/2}}{a^2(b + a \cosh(c + d\sqrt{x}))^2} \right) d\sqrt{x} \\
& \quad \downarrow \text{2009} \\
& 2 \left( \frac{x^4}{8a^2} - \frac{2b \log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a^2 \sqrt{b^2-a^2} d} + \frac{b^3 \log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a^2 (b^2-a^2)^{3/2} d} + \frac{2b \log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a^2 \sqrt{b^2-a^2} d} - \frac{b^3 \log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}} + 1\right) x^{7/2}}{a^2 (b^2-a^2)^{3/2} d} \right)
\end{aligned}$$

input `Int[x^3/(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output  $2*((b^2*x^{(7/2)})/(a^2*(a^2 - b^2)*d) + x^4/(8*a^2) - (7*b^2*x^3*Log[1 + (a *E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (b^3*x^{(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{(3/2)*d} - (2*b*x^{(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (7*b^2*x^3*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) - (b^3*x^{(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{(3/2)*d} + (2*b*x^{(7/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (42*b^2*x^{(5/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])})/(a^2*(a^2 - b^2)*d^3) + (7*b^3*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])})/(a^2*(-a^2 + b^2)^{(3/2)*d^2} - (14*b*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])})/(a^2*Sqrt[-a^2 + b^2]*d^2) - (42*b^2*x^{(5/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])})/(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^3) - (7*b^3*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])})/(a^2*(-a^2 + b^2)^{(3/2)*d^2} + (14*b*x^3*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])})/(a^2*Sqrt[-a^2 + b^2]*d^2) + (210*b^2*x^2*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])})/(a^2*(a^2 - b^2)*d^4) - (42*b^3*x^{(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])})/(a^2*(-a^2 + b^2)^{(3/2)*d^3} + (84*b*x^{(5/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]...$

### 3.47.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear Q[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

$$3.47. \int \frac{x^3}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**3.47.4 Maple [F]**

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `int(x^3/(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(x^3/(a+b*sech(c+d*x^(1/2)))^2,x)`

**3.47.5 Fricas [F]**

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^3}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^3/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(x^3/(b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2), x)`

**3.47.6 Sympy [F]**

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^3}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `integrate(x**3/(a+b*sech(c+d*x**(1/2)))**2,x)`

output `Integral(x**3/(a + b*sech(c + d*sqrt(x)))**2, x)`

**3.47.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^3/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

**3.47.8 Giac [F]**

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^3}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^3/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `sage0*x`

**3.47.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x^3}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^3}{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2} dx$$

input `int(x^3/(a + b/cosh(c + d*x^(1/2)))^2,x)`

output `int(x^3/(a + b/cosh(c + d*x^(1/2)))^2, x)`

**3.48** 
$$\int \frac{x^2}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

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**3.48.1 Optimal result**

Integrand size = 20, antiderivative size = 2123

$$\int \frac{x^2}{(a + b\operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

output

```
2*b^2*x^(5/2)*sinh(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cosh(c+d*x^(1/2)))+240*
b^2*polylog(5,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^6+
240*b^2*polylog(5,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/
d^6+240*b^3*polylog(6,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+
b^2)^(3/2)/d^6-240*b^3*polylog(6,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/
a^2/(-a^2+b^2)^(3/2)/d^6-480*b*polylog(6,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)
)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)+480*b*polylog(6,-a*exp(c+d*x^(1/2))/(b+
(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)+2*b^2*x^(5/2)/a^2/(a^2-b^2)/d-
10*b^2*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^2
+2*b^3*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)
)^(3/2)/d-10*b^2*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^
2-b^2)/d^2-2*b^3*x^(5/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2
/(-a^2+b^2)^(3/2)/d-40*b^2*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+
b^2)^(1/2)))/a^2/(a^2-b^2)/d^3+10*b^3*x^2*polylog(2,-a*exp(c+d*x^(1/2))/(b
-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2-40*b^2*x^(3/2)*polylog(2,-a*
exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-10*b^3*x^2*polylog
(2,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+120*
b^2*x*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a^2-b^2)/d^
4-40*b^3*x^(3/2)*polylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(
-a^2+b^2)^(3/2)/d^3+120*b^2*x*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^...
```

### 3.48.2 Mathematica [A] (verified)

Time = 9.72 (sec) , antiderivative size = 2247, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Result too large to show}$$

input `Integrate[x^2/(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output `(x^3*(b + a*Cosh[c + d*Sqrt[x]])^2*Sech[c + d*Sqrt[x]]^2)/(3*a^2*(a + b*Sech[c + d*Sqrt[x]])^2) + (2*b*E^c*(b + a*Cosh[c + d*Sqrt[x]])^2*(2*b*E^c*x^(5/2) - ((1 + E^(2*c))*(5*b*d^4*Sqrt[(-a^2 + b^2)*E^(2*c)])*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*a^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) - b^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + 5*b*d^4*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] - 2*a^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + b^2*d^5*E^c*x^(5/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + 5*d^3*(4*b*Sqrt[(-a^2 + b^2)*E^(2*c)] + 2*a^2*d*E^c*Sqrt[x] - b^2*d*E^c*Sqrt[x])*x^(3/2)*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 5*d^3*(4*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*x^(3/2)*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 60*b*d^2*Sqrt[(-a^2 + b^2)*E^(2*c)]*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 40*a^2*d^3*E^c*x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) + 20*b^2*d^3*E^c*x^(3/2)*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 60*b*d^2*Sqrt[(-a^2 + b^2)*E^(2*c)]*x*PolyLog[3, -((a*E^(2*...`

### 3.48.3 Rubi [A] (verified)

Time = 3.31 (sec) , antiderivative size = 2122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

---

3.48.  $\int \frac{x^2}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$

$$\begin{aligned}
& \int \frac{x^2}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx \\
& \quad \downarrow \text{5959} \\
& 2 \int \frac{x^{5/2}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} d\sqrt{x} \\
& \quad \downarrow \text{3042} \\
& 2 \int \frac{x^{5/2}}{(a + b \csc(ic + id\sqrt{x} + \frac{\pi}{2}))^2} d\sqrt{x} \\
& \quad \downarrow \text{4679} \\
& 2 \int \left( -\frac{2bx^{5/2}}{a^2 (b + a \cosh(c + d\sqrt{x}))} + \frac{x^{5/2}}{a^2} + \frac{b^2 x^{5/2}}{a^2 (b + a \cosh(c + d\sqrt{x}))^2} \right) d\sqrt{x} \\
& \quad \downarrow \text{2009} \\
& 2 \left( \frac{x^{5/2} \log\left(\frac{e^{c+d\sqrt{x}} a}{b - \sqrt{b^2 - a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{x^{5/2} \log\left(\frac{e^{c+d\sqrt{x}} a}{b + \sqrt{b^2 - a^2}} + 1\right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{5x^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{b^2 - a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{5x^2 \operatorname{PolyLog}\left(2, \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{b^2 - a^2}}\right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} \right)
\end{aligned}$$

input `Int[x^2/(a + b*Sech[c + d*Sqrt[x]])^2,x]`



output  $2*((b^2*x^{(5/2)})/(a^2*(a^2 - b^2)*d) + x^3/(6*a^2) - (5*b^2*x^2*Log[1 + (a *E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (b^3*x^{(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{(3/2)*d} - (2*b*x^{(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (5*b^2*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) - (b^3*x^{(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^{(3/2)*d} + (2*b*x^{(5/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (20*b^2*x^{(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])})/(a^2*(a^2 - b^2)*d^3) + (5*b^3*x^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])})/(a^2*(-a^2 + b^2)^{(3/2)*d^2} - (10*b*x^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])})/(a^2*Sqrt[-a^2 + b^2]*d^2) - (20*b^2*x^{(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])})/(a^2*(a^2 - b^2)*d^3) - (5*b^3*x^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])})/(a^2*(-a^2 + b^2)^{(3/2)*d^2} + (10*b*x^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])})/(a^2*Sqrt[-a^2 + b^2]*d^2) + (60*b^2*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])})/(a^2*(a^2 - b^2)*d^4) - (20*b^3*x^{(3/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])})/(a^2*(-a^2 + b^2)^{(3/2)*d^3} + (40*b*x^{(3/2)*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/...$

### 3.48.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

$$3.48. \int \frac{x^2}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**3.48.4 Maple [F]**

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `int(x^2/(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(x^2/(a+b*sech(c+d*x^(1/2)))^2,x)`

**3.48.5 Fricas [F]**

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^2}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(x^2/(b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2), x)`

**3.48.6 Sympy [F]**

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^2}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `integrate(x**2/(a+b*sech(c+d*x**(1/2)))**2,x)`

output `Integral(x**2/(a + b*sech(c + d*sqrt(x)))**2, x)`

**3.48.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

```
input integrate(x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
output Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more
details)Is
```

**3.48.8 Giac [F]**

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^2}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2} dx$$

```
input integrate(x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
```

```
output integrate(x^2/(b*sech(d*sqrt(x) + c) + a)^2, x)
```

**3.48.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x^2}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^2}{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2} dx$$

```
input int(x^2/(a + b/cosh(c + d*x^(1/2)))^2,x)
```

```
output int(x^2/(a + b/cosh(c + d*x^(1/2)))^2, x)
```

$$3.49 \quad \int \frac{x}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

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### 3.49.1 Optimal result

Integrand size = 18, antiderivative size = 1395

$$\begin{aligned}
 \int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx &= \frac{2b^2 x^{3/2}}{a^2 (a^2 - b^2) d} + \frac{x^2}{2a^2} - \frac{6b^2 x \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 (a^2 - b^2) d^2} \\
 &+ \frac{2b^3 x^{3/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d} \\
 &- \frac{4bx^{3/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d} - \frac{6b^2 x \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 (a^2 - b^2) d^2} \\
 &- \frac{2b^3 x^{3/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d} \\
 &+ \frac{4bx^{3/2} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d} \\
 &- \frac{12b^2 \sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 (a^2 - b^2) d^3} \\
 &+ \frac{6b^3 x \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^2} \\
 &- \frac{12bx \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^2} \\
 &- \frac{12b^2 \sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 (a^2 - b^2) d^3} \\
 &- \frac{6b^3 x \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^2} \\
 &+ \frac{12bx \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^2} \\
 &+ \frac{12b^2 \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 (a^2 - b^2) d^4} \\
 &- \frac{12b^3 \sqrt{x} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^3} \\
 &+ \frac{24b \sqrt{x} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^3} \\
 &+ \frac{12b^2 \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 (a^2 - b^2) d^4}
 \end{aligned}$$

---


$$\begin{aligned}
 3.49. \quad \int \frac{x}{(a+b \operatorname{sech}(c+d\sqrt{x}))^2} dx &+ \frac{12b^3 \sqrt{x} \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^3}
 \end{aligned}$$

output  $2*b^2*x^{(3/2)}*\sinh(c+d*x^{(1/2)})/a/(a^2-b^2)/d/(b+a*\cosh(c+d*x^{(1/2)}))+1/2*x^2/a^2-12*b^3*polylog(4,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^4-24*b*polylog(4,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/d^4/(-a^2+b^2)^{(1/2)}+24*b*polylog(4,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/d^4/(-a^2+b^2)^{(1/2)}+12*b^2*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(a^2-b^2)/d^4+12*b^2*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(a^2-b^2)/d^4+12*b^3*polylog(4,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^4+2*b^2*x^{(3/2)}/a^2/(a^2-b^2)/d-12*b^3*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/(-a^2+b^2)^{(3/2)}/d^3+12*b^3*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/(-a^2+b^2)^{(3/2)}/d^3+24*b*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/d^3/(-a^2+b^2)^{(1/2)}-24*b*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a^2/d^3/(-a^2+b^2)^{(1/2)}-6*b^2*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(a^2-b^2)/d^2+2*b^3*x^{(3/2)}*\ln(1+a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d-6*b^2*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(a^2-b^2)/d^2-2*b^3*x^{(3/2)}*\ln(1+a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d+6*b^3*x*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^2-6*b^3*x*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^2-4*b*x^{(3/2)}*\ln(1+a*\exp(c+d*x^{(1/2)}))...$

### 3.49.2 Mathematica [A] (verified)

Time = 8.95 (sec) , antiderivative size = 1393, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

input `Integrate[x/(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output

```

((b + a*Cosh[c + d*Sqrt[x]])*Sech[c + d*Sqrt[x]]^2*(x^2*(b + a*Cosh[c + d*
Sqrt[x]]) + (4*b*E^c*(b + a*Cosh[c + d*Sqrt[x]])*(2*b*E^c*x^(3/2) + ((1 +
E^(2*c))*(-3*b*d^2*Sqrt[(-a^2 + b^2)*E^(2*c)])*x*Log[1 + (a*E^(2*c + d*Sqrt
[x]))]/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])) - 2*a^2*d^3*E^c*x^(3/2)*Log[1
+ (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])) + b^2*d^3*
E^c*x^(3/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(
2*c)])) - 3*b*d^2*Sqrt[(-a^2 + b^2)*E^(2*c)]*x*Log[1 + (a*E^(2*c + d*Sqrt[
x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])) + 2*a^2*d^3*E^c*x^(3/2)*Log[1 +
(a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])) - b^2*d^3*E
^c*x^(3/2)*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2
*c)])) + 3*d*(-2*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqrt[x] + b^2*
d*E^c*Sqrt[x])*Sqrt[x]*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[
(-a^2 + b^2)*E^(2*c)])) - 3*d*(2*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*a^2*d*E
^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*Sqrt[x]*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]
))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])) + 6*b*Sqrt[(-a^2 + b^2)*E^(2*c)]
*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]
))] + 12*a^2*d*E^c*Sqrt[x]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sq
rt[(-a^2 + b^2)*E^(2*c)])) - 6*b^2*d*E^c*Sqrt[x]*PolyLog[3, -((a*E^(2*c +
d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])) + 6*b*Sqrt[(-a^2 + b^2
)*E^(2*c)]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b...

```

### 3.49.3 Rubi [A] (verified)

Time = 2.59 (sec) , antiderivative size = 1394, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx \\
 & \quad \downarrow \text{5959} \\
 & 2 \int \frac{x^{3/2}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int \frac{x^{3/2}}{(a + b \operatorname{csc}(ic + id\sqrt{x} + \frac{\pi}{2}))^2} d\sqrt{x}
 \end{aligned}$$

---

3.49.  $\int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$

$$\begin{array}{c}
\downarrow 4679 \\
2 \int \left( \frac{x^{3/2} b^2}{a^2 (b + a \cosh(c + d\sqrt{x}))^2} - \frac{2x^{3/2} b}{a^2 (b + a \cosh(c + d\sqrt{x}))} + \frac{x^{3/2}}{a^2} \right) d\sqrt{x} \\
\downarrow 2009 \\
2 \left( \frac{x^{3/2} \log \left( \frac{e^{c+d\sqrt{x}} a}{b - \sqrt{b^2 - a^2}} + 1 \right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{x^{3/2} \log \left( \frac{e^{c+d\sqrt{x}} a}{b + \sqrt{b^2 - a^2}} + 1 \right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{3x \operatorname{PolyLog} \left( 2, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{b^2 - a^2}} \right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{3x \operatorname{PolyLog} \left( 2, -\frac{ae^{c+d\sqrt{x}}}{b + \sqrt{b^2 - a^2}} \right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} \right)
\end{array}$$

input `Int[x/(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output

```

2*((b^2*x^(3/2))/(a^2*(a^2 - b^2)*d) + x^2/(4*a^2) - (3*b^2*x*Log[1 + (a*E
^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (b^3*x^
(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 +
b^2)^(3/2)*d) - (2*b*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2
+ b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - (3*b^2*x*Log[1 + (a*E^(c + d*Sqrt[x])
))/(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) - (b^3*x^(3/2)*Log[1 + (a
*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) +
(2*b*x^(3/2)*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*S
qrt[-a^2 + b^2]*d) - (6*b^2*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b
- Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) + (3*b^3*x*PolyLog[2, -((a*E
(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) -
(6*b*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*S
qrt[-a^2 + b^2]*d^2) - (6*b^2*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(
b + Sqrt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^3) - (3*b^3*x*PolyLog[2, -((a*
E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2)
+ (6*b*x*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2
*Sqrt[-a^2 + b^2]*d^2) + (6*b^2*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sq
rt[-a^2 + b^2]))])/(a^2*(a^2 - b^2)*d^4) - (6*b^3*Sqrt[x]*PolyLog[3, -((a*
E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3)
+ (12*b*Sqrt[x]*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]...

```



## 3.49.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

## 3.49.4 Maple [F]

$$\int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `int(x/(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(x/(a+b*sech(c+d*x^(1/2)))^2,x)`

## 3.49.5 Fracas [F]

$$\int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fracas")`

output `integral(x/(b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2), x)`

---

3.49.  $\int \frac{x}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$

**3.49.6 Sympy [F]**

$$\int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `integrate(x/(a+b*sech(c+d*x**(1/2))))**2,x`

output `Integral(x/(a + b*sech(c + d*sqrt(x))))**2, x`

**3.49.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

**3.49.8 Giac [F]**

$$\int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(x/(b*sech(d*sqrt(x) + c) + a)^2, x)`

**3.49.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x}{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2} dx$$

input `int(x/(a + b/cosh(c + d*x^(1/2)))^2,x)`output `int(x/(a + b/cosh(c + d*x^(1/2)))^2, x)`

$$3.50 \quad \int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

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### 3.50.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx = \operatorname{Int}\left(\frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2}, x\right)$$

output `Unintegrable(1/x/(a+b*sech(c+d*x^(1/2)))^2,x)`

### 3.50.2 Mathematica [N/A]

Not integrable

Time = 179.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx = \int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

input `Integrate[1/(x*(a + b*Sech[c + d*Sqrt[x]])^2),x]`

output `Integrate[1/(x*(a + b*Sech[c + d*Sqrt[x]])^2), x]`

---

3.50.  $\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$

### 3.50.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

↓ 5961

$$\int \frac{1}{x (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `Int[1/(x*(a + b*Sech[c + d*Sqrt[x]])^2),x]`

output `$Aborted`

#### 3.50.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.50.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `int(1/x/(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(1/x/(a+b*sech(c+d*x^(1/2)))^2,x)`

---

3.50.  $\int \frac{1}{x (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$

**3.50.5 Fracas [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{1}{x (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x} dx$$

input `integrate(1/x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`output `integral(1/(b^2*x*sech(d*sqrt(x) + c)^2 + 2*a*b*x*sech(d*sqrt(x) + c) + a^2*x), x)`**3.50.6 Sympy [N/A]**

Not integrable

Time = 4.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{x (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `integrate(1/x/(a+b*sech(c+d*x**(1/2)))**2,x)`output `Integral(1/(x*(a + b*sech(c + d*sqrt(x)))**2), x)`**3.50.7 Maxima [N/A]**

Not integrable

Time = 0.98 (sec) , antiderivative size = 253, normalized size of antiderivative = 12.65

$$\int \frac{1}{x (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x} dx$$

input `integrate(1/x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

---

3.50.  $\int \frac{1}{x(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$

output 
$$-4*(b^3*\sqrt{x}*e^{(d*\sqrt{x} + c) + a*b^2*\sqrt{x}})/((a^5*d*e^{(2*c) - a^3*b^2*d*e^{(2*c)}})*x*e^{(2*d*\sqrt{x})} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x*e^{(d*\sqrt{x})} + (a^5*d - a^3*b^2*d)*x) + \log(x)/a^2 - \text{integrate}(2*(a*b^2*\sqrt{x} + (b^3*\sqrt{x}*e^c + (2*a^2*b*d*e^c - b^3*d*e^c)*x)*e^{(d*\sqrt{x})})/((a^5*d*e^{(2*c) - a^3*b^2*d*e^{(2*c)}})*x^2*e^{(2*d*\sqrt{x})} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^2*e^{(d*\sqrt{x})} + (a^5*d - a^3*b^2*d)*x^2), x)$$

### 3.50.8 Giac [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a + b\operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b\operatorname{sech}(d\sqrt{x} + c) + a)^2 x} dx$$

input `integrate(1/x/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(1/((b*sech(d*sqrt(x) + c) + a)^2*x), x)`

### 3.50.9 Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x(a + b\operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{x\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2} dx$$

input `int(1/(x*(a + b/cosh(c + d*x^(1/2))))^2),x)`

output `int(1/(x*(a + b/cosh(c + d*x^(1/2))))^2), x)`

**3.51**  $\int \frac{1}{x^2 (a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$

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**3.51.1 Optimal result**

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x^2 (a + b\operatorname{sech}(c + d\sqrt{x}))^2} dx = \operatorname{Int}\left(\frac{1}{x^2 (a + b\operatorname{sech}(c + d\sqrt{x}))^2}, x\right)$$

output `Unintegrable(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x)`

**3.51.2 Mathematica [N/A]**

Not integrable

Time = 91.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 (a + b\operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^2 (a + b\operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `Integrate[1/(x^2*(a + b*Sech[c + d*Sqrt[x]])^2),x]`

output `Integrate[1/(x^2*(a + b*Sech[c + d*Sqrt[x]])^2), x]`



### 3.51.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

↓ 5961

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `Int[1/(x^2*(a + b*Sech[c + d*Sqrt[x]])^2),x]`

output `$Aborted`

#### 3.51.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.51.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `int(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x)`

---

3.51.  $\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$

**3.51.5 Fricas [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^2} dx$$

input `integrate(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`output `integral(1/(b^2*x^2*sech(d*sqrt(x) + c)^2 + 2*a*b*x^2*sech(d*sqrt(x) + c) + a^2*x^2), x)`**3.51.6 Sympy [N/A]**

Not integrable

Time = 9.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `integrate(1/x**2/(a+b*sech(c+d*x**(1/2)))**2,x)`output `Integral(1/(x**2*(a + b*sech(c + d*sqrt(x)))**2), x)`**3.51.7 Maxima [N/A]**

Not integrable

Time = 1.16 (sec) , antiderivative size = 324, normalized size of antiderivative = 16.20

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^2} dx$$

input `integrate(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

---

3.51.  $\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$

output  $-(4*a*b^2*\sqrt{x} + (a^3*d*e^{(2*c)} - a*b^2*d*e^{(2*c)})*x*e^{(2*d*\sqrt{x})} + (a^3*d - a*b^2*d)*x + 2*(2*b^3*\sqrt{x}*e^c + (a^2*b*d*e^c - b^3*d*e^c)*x)*e^{(d*\sqrt{x})})/((a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*x^2*e^{(2*d*\sqrt{x})} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^2*e^{(d*\sqrt{x})} + (a^5*d - a^3*b^2*d)*x^2) - \text{integrate}(2*(3*a*b^2*\sqrt{x} + (3*b^3*\sqrt{x}*e^c + (2*a^2*b*d*e^c - b^3*d*e^c)*x)*e^{(d*\sqrt{x})})/((a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*x^3*e^{(2*d*\sqrt{x})} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^3*e^{(d*\sqrt{x})} + (a^5*d - a^3*b^2*d)*x^3), x)$

### 3.51.8 Giac [N/A]

Not integrable

Time = 3.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^2} dx$$

input `integrate(1/x^2/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `sage0*x`

### 3.51.9 Mupad [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^2 \left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2} dx$$

input `int(1/(x^2*(a + b/cosh(c + d*x^(1/2))))^2),x`

output `int(1/(x^2*(a + b/cosh(c + d*x^(1/2))))^2), x`

### 3.52 $\int x^{3/2}(a + b\operatorname{sech}(c + d\sqrt{x})) dx$

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#### 3.52.1 Optimal result

Integrand size = 20, antiderivative size = 254

$$\begin{aligned} \int x^{3/2}(a + b\operatorname{sech}(c + d\sqrt{x})) dx = & \frac{2}{5}ax^{5/2} + \frac{4bx^2 \arctan(e^{c+d\sqrt{x}})}{d} \\ & - \frac{8ibx^{3/2} \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{8ibx^{3/2} \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} \\ & + \frac{24ibx \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{24ibx \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} \\ & - \frac{48ib\sqrt{x} \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} + \frac{48ib\sqrt{x} \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} \\ & + \frac{48ib \operatorname{PolyLog}(5, -ie^{c+d\sqrt{x}})}{d^5} - \frac{48ib \operatorname{PolyLog}(5, ie^{c+d\sqrt{x}})}{d^5} \end{aligned}$$

```
output 2/5*a*x^(5/2)+4*b*x^2*arctan(exp(c+d*x^(1/2)))/d-8*I*b*x^(3/2)*polylog(2,-
I*exp(c+d*x^(1/2)))/d^2+8*I*b*x^(3/2)*polylog(2,I*exp(c+d*x^(1/2)))/d^2+24
*I*b*x*polylog(3,-I*exp(c+d*x^(1/2)))/d^3-24*I*b*x*polylog(3,I*exp(c+d*x^(
1/2)))/d^3+48*I*b*polylog(5,-I*exp(c+d*x^(1/2)))/d^5-48*I*b*polylog(5,I*ex
p(c+d*x^(1/2)))/d^5-48*I*b*polylog(4,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^4+48*I
*b*polylog(4,I*exp(c+d*x^(1/2)))*x^(1/2)/d^4
```

### 3.52.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.13

$$\int x^{3/2}(a + b \operatorname{sech}(c + d\sqrt{x})) dx = \frac{2(ad^5x^{5/2} + 5ibd^4x^2 \log(1 - ie^{c+d\sqrt{x}}) - 5ibd^4x^2 \log(1 + ie^{c+d\sqrt{x}}) - 20ibd^3x^{3/2} \operatorname{PolyLog}[2, (-I)E^{c+d\sqrt{x}}] + (20I)b*d^3*x^{3/2}*\operatorname{PolyLog}[2, I*E^{c+d\sqrt{x}}]) + (60I)*b*d^2*x*\operatorname{PolyLog}[3, (-I)E^{c+d\sqrt{x}}] - (60I)*b*d^2*x*\operatorname{PolyLog}[3, I*E^{c+d\sqrt{x}}] - (120I)*b*d*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[4, (-I)E^{c+d\sqrt{x}}] + (120I)*b*d*\operatorname{Sqrt}[x]*\operatorname{PolyLog}[4, I*E^{c+d\sqrt{x}}] + (120I)*b*\operatorname{PolyLog}[5, (-I)E^{c+d\sqrt{x}}] - (120I)*b*\operatorname{PolyLog}[5, I*E^{c+d\sqrt{x}}])}{5*d^5}$$

input `Integrate[x^(3/2)*(a + b*Sech[c + d*Sqrt[x]]),x]`

output `(2*(a*d^5*x^(5/2) + (5*I)*b*d^4*x^2*Log[1 - I*E^(c + d*Sqrt[x])] - (5*I)*b*d^4*x^2*Log[1 + I*E^(c + d*Sqrt[x])] - (20*I)*b*d^3*x^(3/2)*PolyLog[2, (-I)*E^(c + d*Sqrt[x])] + (20*I)*b*d^3*x^(3/2)*PolyLog[2, I*E^(c + d*Sqrt[x])]) + (60*I)*b*d^2*x*PolyLog[3, (-I)*E^(c + d*Sqrt[x])] - (60*I)*b*d^2*x*PolyLog[3, I*E^(c + d*Sqrt[x])] - (120*I)*b*d*Sqrt[x]*PolyLog[4, (-I)*E^(c + d*Sqrt[x])] + (120*I)*b*d*Sqrt[x]*PolyLog[4, I*E^(c + d*Sqrt[x])] + (120*I)*b*PolyLog[5, (-I)*E^(c + d*Sqrt[x])] - (120*I)*b*PolyLog[5, I*E^(c + d*Sqrt[x])])/(5*d^5)`

### 3.52.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

$$\downarrow \text{2010}$$

$$\int (ax^{3/2} + bx^{3/2} \operatorname{sech}(c + d\sqrt{x})) dx$$

$$\downarrow \text{2009}$$

$$\frac{2}{5}ax^{5/2} + \frac{4bx^2 \arctan(e^{c+d\sqrt{x}})}{d} + \frac{48ib \operatorname{PolyLog}(5, -ie^{c+d\sqrt{x}})}{d^5} - \frac{48ib \operatorname{PolyLog}(5, ie^{c+d\sqrt{x}})}{d^5} - \frac{48ib\sqrt{x} \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} + \frac{48ib\sqrt{x} \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} + \frac{24ibx \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{24ibx \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} - \frac{8ibx^{3/2} \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{8ibx^{3/2} \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2}$$

input `Int[x^(3/2)*(a + b*Sech[c + d*Sqrt[x]]),x]`

output `(2*a*x^(5/2))/5 + (4*b*x^2*ArcTan[E^(c + d*Sqrt[x])])/d - ((8*I)*b*x^(3/2)*PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((8*I)*b*x^(3/2)*PolyLog[2, I*E^(c + d*Sqrt[x])])/d^2 + ((24*I)*b*x*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((24*I)*b*x*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 - ((48*I)*b*Sqrt[x]*PolyLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((48*I)*b*Sqrt[x]*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 + ((48*I)*b*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 - ((48*I)*b*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5`

### 3.52.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

### 3.52.4 Maple [F]

$$\int x^{\frac{3}{2}}(a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

input `int(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x)`

output `int(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x)`

**3.52.5 Fricas [F]**

$$\int x^{3/2}(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)x^{\frac{3}{2}} dx$$

input `integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(b*x^(3/2)*sech(d*sqrt(x) + c) + a*x^(3/2), x)`

**3.52.6 Sympy [F]**

$$\int x^{3/2}(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int x^{\frac{3}{2}}(a + b\operatorname{sech}(c + d\sqrt{x})) dx$$

input `integrate(x**(3/2)*(a+b*sech(c+d*x**(1/2))),x)`

output `Integral(x**(3/2)*(a + b*sech(c + d*sqrt(x))), x)`

**3.52.7 Maxima [F]**

$$\int x^{3/2}(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)x^{\frac{3}{2}} dx$$

input `integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

output `2/5*a*x^(5/2) + 2*b*integrate(x^(3/2)*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2*c) + 1), x)`

**3.52.8 Giac [F]**

$$\int x^{3/2}(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)x^{\frac{3}{2}} dx$$

input `integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)*x^(3/2), x)`

**3.52.9 Mupad [F(-1)]**

Timed out.

$$\int x^{3/2}(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int x^{3/2} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right) dx$$

input `int(x^(3/2)*(a + b/cosh(c + d*x^(1/2))),x)`

output `int(x^(3/2)*(a + b/cosh(c + d*x^(1/2))), x)`



### 3.53 $\int \sqrt{x}(a + b\operatorname{sech}(c + d\sqrt{x})) dx$

3.53.1	Optimal result	320
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3.53.4	Maple [F]	322
3.53.5	Fricas [F]	322
3.53.6	Sympy [F]	322
3.53.7	Maxima [F]	323
3.53.8	Giac [F]	323
3.53.9	Mupad [F(-1)]	323

#### 3.53.1 Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \sqrt{x}(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \frac{2}{3}ax^{3/2} + \frac{4bx \arctan(e^{c+d\sqrt{x}})}{d} - \frac{4ib\sqrt{x} \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{4ib\sqrt{x} \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} + \frac{4ib \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{4ib \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3}$$

output  $\frac{2}{3}a*x^{(3/2)}+4*b*x*\arctan(\exp(c+d*x^{(1/2)}))/d+4*I*b*\operatorname{polylog}(3,-I*\exp(c+d*x^{(1/2)}))/d^3-4*I*b*\operatorname{polylog}(3,I*\exp(c+d*x^{(1/2)}))/d^3-4*I*b*\operatorname{polylog}(2,-I*\exp(c+d*x^{(1/2)}))*x^{(1/2)}/d^2+4*I*b*\operatorname{polylog}(2,I*\exp(c+d*x^{(1/2)}))*x^{(1/2)}/d^2$

#### 3.53.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

$$\int \sqrt{x}(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \frac{2(ad^3x^{3/2} + 3ibd^2x \log(1 - ie^{c+d\sqrt{x}}) - 3ibd^2x \log(1 + ie^{c+d\sqrt{x}}) - 6ibd\sqrt{x} \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}}) + 6ibd\sqrt{x} \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}}))}{3d^3}$$

input `Integrate[Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]]),x]`

output  $(2*(a*d^3*x^{3/2}) + (3*I)*b*d^2*x*\text{Log}[1 - I*E^{(c + d*\text{Sqrt}[x])}] - (3*I)*b*d^2*x*\text{Log}[1 + I*E^{(c + d*\text{Sqrt}[x])}] - (6*I)*b*d*\text{Sqrt}[x]*\text{PolyLog}[2, (-I)*E^{(c + d*\text{Sqrt}[x])}] + (6*I)*b*d*\text{Sqrt}[x]*\text{PolyLog}[2, I*E^{(c + d*\text{Sqrt}[x])}] + (6*I)*b*\text{PolyLog}[3, (-I)*E^{(c + d*\text{Sqrt}[x])}] - (6*I)*b*\text{PolyLog}[3, I*E^{(c + d*\text{Sqrt}[x])}]))/(3*d^3)$

### 3.53.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a + b\text{sech}(c + d\sqrt{x})) dx$$

↓ 2010

$$\int (a\sqrt{x} + b\sqrt{x}\text{sech}(c + d\sqrt{x})) dx$$

↓ 2009

$$\frac{2}{3}ax^{3/2} + \frac{4bx \arctan(e^{c+d\sqrt{x}})}{d} + \frac{4ib \text{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{4ib \text{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} - \frac{4ib\sqrt{x} \text{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{4ib\sqrt{x} \text{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2}$$

input `Int[Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]]),x]`

output  $(2*a*x^{3/2})/3 + (4*b*x*\text{ArcTan}[E^{(c + d*\text{Sqrt}[x])}])/d - ((4*I)*b*\text{Sqrt}[x]*\text{PolyLog}[2, (-I)*E^{(c + d*\text{Sqrt}[x])}])/d^2 + ((4*I)*b*\text{Sqrt}[x]*\text{PolyLog}[2, I*E^{(c + d*\text{Sqrt}[x])}])/d^2 + ((4*I)*b*\text{PolyLog}[3, (-I)*E^{(c + d*\text{Sqrt}[x])}])/d^3 - ((4*I)*b*\text{PolyLog}[3, I*E^{(c + d*\text{Sqrt}[x])}])/d^3$

## 3.53.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

## 3.53.4 Maple [F]

$$\int (a + b \operatorname{sech}(c + d\sqrt{x})) \sqrt{x} dx$$

input `int((a+b*sech(c+d*x^(1/2)))*x^(1/2),x)`

output `int((a+b*sech(c+d*x^(1/2)))*x^(1/2),x)`

## 3.53.5 Fracas [F]

$$\int \sqrt{x}(a + b \operatorname{sech}(c + d\sqrt{x})) dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a) \sqrt{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))*x^(1/2),x, algorithm="fricas")`

output `integral(b*sqrt(x)*sech(d*sqrt(x) + c) + a*sqrt(x), x)`

## 3.53.6 Sympy [F]

$$\int \sqrt{x}(a + b \operatorname{sech}(c + d\sqrt{x})) dx = \int \sqrt{x}(a + b \operatorname{sech}(c + d\sqrt{x})) dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))*x**(1/2),x)`

output `Integral(sqrt(x)*(a + b*sech(c + d*sqrt(x))), x)`

**3.53.7 Maxima [F]**

$$\int \sqrt{x}(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)\sqrt{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))*x^(1/2),x, algorithm="maxima")`

output `2/3*a*x^(3/2) + 2*b*integrate(sqrt(x)*e^(d*sqrt(x) + c)/(e^(2*d*sqrt(x) + 2*c) + 1), x)`

**3.53.8 Giac [F]**

$$\int \sqrt{x}(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int (b\operatorname{sech}(d\sqrt{x} + c) + a)\sqrt{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))*x^(1/2),x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)*sqrt(x), x)`

**3.53.9 Mupad [F(-1)]**

Timed out.

$$\int \sqrt{x}(a + b\operatorname{sech}(c + d\sqrt{x})) dx = \int \sqrt{x} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right) dx$$

input `int(x^(1/2)*(a + b/cosh(c + d*x^(1/2))),x)`

output `int(x^(1/2)*(a + b/cosh(c + d*x^(1/2))), x)`

### 3.54 $\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{\sqrt{x}} dx$

3.54.1	Optimal result	324
3.54.2	Mathematica [A] (verified)	324
3.54.3	Rubi [A] (verified)	325
3.54.4	Maple [A] (verified)	326
3.54.5	Fricas [A] (verification not implemented)	326
3.54.6	Sympy [A] (verification not implemented)	326
3.54.7	Maxima [A] (verification not implemented)	327
3.54.8	Giac [A] (verification not implemented)	327
3.54.9	Mupad [B] (verification not implemented)	327

#### 3.54.1 Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} + \frac{2b \arctan(\sinh(c + d\sqrt{x}))}{d}$$

output `2*b*arctan(sinh(c+d*x^(1/2)))/d+2*a*x^(1/2)`

#### 3.54.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} + \frac{2b \arctan(\sinh(c + d\sqrt{x}))}{d}$$

input `Integrate[(a + b*Sech[c + d*Sqrt[x]])/Sqrt[x], x]`

output `2*a*Sqrt[x] + (2*b*ArcTan[Sinh[c + d*Sqrt[x]]])/d`

### 3.54.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{\sqrt{x}} dx$$

↓ 2010

$$\int \left( \frac{a}{\sqrt{x}} + \frac{b \operatorname{sech}(c + d\sqrt{x})}{\sqrt{x}} \right) dx$$

↓ 2009

$$2a\sqrt{x} + \frac{2b \arctan(\sinh(c + d\sqrt{x}))}{d}$$

input `Int[(a + b*Sech[c + d*Sqrt[x]])/Sqrt[x], x]`

output `2*a*Sqrt[x] + (2*b*ArcTan[Sinh[c + d*Sqrt[x]]])/d`

#### 3.54.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_.))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

**3.54.4 Maple [A] (verified)**

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{2b \arctan(\sinh(c+d\sqrt{x}))}{d} + 2a\sqrt{x}$	23
default	$\frac{2b \arctan(\sinh(c+d\sqrt{x}))}{d} + 2a\sqrt{x}$	23
parts	$\frac{2b \arctan(\sinh(c+d\sqrt{x}))}{d} + 2a\sqrt{x}$	23

input `int((a+b*sech(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)`output `2*b*arctan(sinh(c+d*x^(1/2)))/d+2*a*x^(1/2)`**3.54.5 Fracas [A] (verification not implemented)**

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{\sqrt{x}} dx = \frac{2(ad\sqrt{x} + 2b \arctan(\cosh(d\sqrt{x} + c) + \sinh(d\sqrt{x} + c)))}{d}$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="fracas")`output `2*(a*d*sqrt(x) + 2*b*arctan(cosh(d*sqrt(x) + c) + sinh(d*sqrt(x) + c)))/d`**3.54.6 Sympy [A] (verification not implemented)**

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} + 2b \begin{cases} \sqrt{x} \operatorname{sech}(c) & \text{for } d = 0 \\ \frac{2 \operatorname{atan}(\tanh(\frac{c}{2} + \frac{d\sqrt{x}}{2}))}{d} & \text{otherwise} \end{cases}$$

input `integrate((a+b*sech(c+d*x**(1/2)))/x**(1/2),x)`output `2*a*sqrt(x) + 2*b*Piecewise((sqrt(x)*sech(c), Eq(d, 0)), (2*atan(tanh(c/2 + d*sqrt(x)/2))/d, True))`

---

3.54.  $\int \frac{a+b \operatorname{sech}(c+d\sqrt{x})}{\sqrt{x}} dx$

**3.54.7 Maxima [A] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} + \frac{2b \arctan(\sinh(d\sqrt{x} + c))}{d}$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")`output `2*a*sqrt(x) + 2*b*arctan(sinh(d*sqrt(x) + c))/d`**3.54.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{\sqrt{x}} dx = \frac{2(d\sqrt{x} + c)a}{d} + \frac{4b \arctan(e^{(d\sqrt{x}+c)})}{d}$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")`output `2*(d*sqrt(x) + c)*a/d + 4*b*arctan(e^(d*sqrt(x) + c))/d`**3.54.9 Mupad [B] (verification not implemented)**

Time = 1.98 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{\sqrt{x}} dx = 2a\sqrt{x} + \frac{4 \operatorname{atan}\left(\frac{be^{d\sqrt{x}} e^c \sqrt{d^2}}{d\sqrt{b^2}}\right) \sqrt{b^2}}{\sqrt{d^2}}$$

input `int((a + b/cosh(c + d*x^(1/2)))/x^(1/2),x)`output `2*a*x^(1/2) + (4*atan((b*exp(d*x^(1/2))*exp(c)*(d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2)`



### 3.55 $\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^{3/2}} dx$

3.55.1	Optimal result	328
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3.55.4	Maple [N/A] (verified)	330
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3.55.6	Sympy [N/A]	330
3.55.7	Maxima [N/A]	331
3.55.8	Giac [N/A]	331
3.55.9	Mupad [N/A]	331

#### 3.55.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx = -\frac{2a}{\sqrt{x}} + b\operatorname{Int}\left(\frac{\operatorname{sech}(c + d\sqrt{x})}{x^{3/2}}, x\right)$$

output `-2*a/x^(1/2)+b*Unintegrable(sech(c+d*x^(1/2))/x^(3/2),x)`

#### 3.55.2 Mathematica [N/A]

Not integrable

Time = 13.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx$$

input `Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^(3/2),x]`

output `Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^(3/2), x]`

### 3.55.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx$$

↓ 2010

$$\int \left( \frac{a}{x^{3/2}} + \frac{b \operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} \right) dx$$

↓ 2009

$$b \int \frac{\operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx - \frac{2a}{\sqrt{x}}$$

input `Int[(a + b*Sech[c + d*Sqrt[x]])/x^(3/2),x]`

output `$Aborted`

#### 3.55.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

**3.55.4 Maple [N/A] (verified)**

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{\frac{3}{2}}} dx$$

input `int((a+b*sech(c+d*x^(1/2)))/x^(3/2),x)`output `int((a+b*sech(c+d*x^(1/2)))/x^(3/2),x)`**3.55.5 Fricas [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^{\frac{3}{2}}} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^(3/2),x, algorithm="fricas")`output `integral((b*sqrt(x)*sech(d*sqrt(x) + c) + a*sqrt(x))/x^2, x)`**3.55.6 Sympy [N/A]**

Not integrable

Time = 1.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{\frac{3}{2}}} dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))/x**(3/2),x)`output `Integral((a + b*sech(c + d*sqrt(x)))/x**(3/2), x)`

**3.55.7 Maxima [N/A]**

Not integrable

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^{3/2}} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^(3/2),x, algorithm="maxima")`

output `2*b*integrate(e^(d*sqrt(x) + c)/(x^(3/2))*e^(2*d*sqrt(x) + 2*c) + x^(3/2), x) - 2*a/sqrt(x)`

**3.55.8 Giac [N/A]**

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^{3/2}} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^(3/2),x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)/x^(3/2), x)`

**3.55.9 Mupad [N/A]**

Not integrable

Time = 2.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{3/2}} dx = \int \frac{a + \frac{b}{\cosh(c+d\sqrt{x})}}{x^{3/2}} dx$$

input `int((a + b/cosh(c + d*x^(1/2)))/x^(3/2),x)`

output `int((a + b/cosh(c + d*x^(1/2)))/x^(3/2), x)`

---

3.55.  $\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^{3/2}} dx$

### 3.56 $\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^{5/2}} dx$

3.56.1	Optimal result	332
3.56.2	Mathematica [N/A]	332
3.56.3	Rubi [N/A]	333
3.56.4	Maple [N/A] (verified)	334
3.56.5	Fricas [N/A]	334
3.56.6	Sympy [N/A]	334
3.56.7	Maxima [N/A]	335
3.56.8	Giac [N/A]	335
3.56.9	Mupad [N/A]	335

#### 3.56.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx = -\frac{2a}{3x^{3/2}} + b\operatorname{Int}\left(\frac{\operatorname{sech}(c + d\sqrt{x})}{x^{5/2}}, x\right)$$

output `-2/3*a/x^(3/2)+b*Unintegrable(sech(c+d*x^(1/2))/x^(5/2), x)`

#### 3.56.2 Mathematica [N/A]

Not integrable

Time = 15.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{a + b\operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx$$

input `Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^(5/2), x]`

output `Integrate[(a + b*Sech[c + d*Sqrt[x]])/x^(5/2), x]`

**3.56.3 Rubi [N/A]**

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx$$

↓ 2010

$$\int \left( \frac{a}{x^{5/2}} + \frac{b \operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} \right) dx$$

↓ 2009

$$b \int \frac{\operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx - \frac{2a}{3x^{3/2}}$$

input `Int[(a + b*Sech[c + d*Sqrt[x]])/x^(5/2),x]`

output `$Aborted`

**3.56.3.1 Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

**3.56.4 Maple [N/A] (verified)**

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{\frac{5}{2}}} dx$$

input `int((a+b*sech(c+d*x^(1/2)))/x^(5/2), x)`output `int((a+b*sech(c+d*x^(1/2)))/x^(5/2), x)`**3.56.5 Fricas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^{\frac{5}{2}}} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^(5/2), x, algorithm="fricas")`output `integral((b*sqrt(x)*sech(d*sqrt(x) + c) + a*sqrt(x))/x^3, x)`**3.56.6 Sympy [N/A]**

Not integrable

Time = 4.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{\frac{5}{2}}} dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))/x**(5/2), x)`output `Integral((a + b*sech(c + d*sqrt(x)))/x**(5/2), x)`

**3.56.7 Maxima [N/A]**

Not integrable

Time = 0.45 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^{5/2}} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^(5/2),x, algorithm="maxima")`

output `2*b*integrate(e^(d*sqrt(x) + c)/(x^(5/2))*e^(2*d*sqrt(x) + 2*c) + x^(5/2), x) - 2/3*a/x^(3/2)`

**3.56.8 Giac [N/A]**

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{b \operatorname{sech}(d\sqrt{x} + c) + a}{x^{5/2}} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))/x^(5/2),x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)/x^(5/2), x)`

**3.56.9 Mupad [N/A]**

Not integrable

Time = 2.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b \operatorname{sech}(c + d\sqrt{x})}{x^{5/2}} dx = \int \frac{a + \frac{b}{\cosh(c+d\sqrt{x})}}{x^{5/2}} dx$$

input `int((a + b/cosh(c + d*x^(1/2)))/x^(5/2),x)`

output `int((a + b/cosh(c + d*x^(1/2)))/x^(5/2), x)`

---

3.56.  $\int \frac{a+b\operatorname{sech}(c+d\sqrt{x})}{x^{5/2}} dx$



### 3.57 $\int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$

3.57.1	Optimal result	336
3.57.2	Mathematica [A] (verified)	337
3.57.3	Rubi [A] (verified)	338
3.57.4	Maple [F]	339
3.57.5	Fricas [F]	339
3.57.6	Sympy [F]	340
3.57.7	Maxima [F]	340
3.57.8	Giac [F]	340
3.57.9	Mupad [F(-1)]	341

#### 3.57.1 Optimal result

Integrand size = 22, antiderivative size = 407

$$\begin{aligned} \int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx &= \frac{2b^2 x^2}{d} + \frac{2}{5} a^2 x^{5/2} \\ &+ \frac{8abx^2 \arctan(e^{c+d\sqrt{x}})}{d} - \frac{8b^2 x^{3/2} \log(1 + e^{2(c+d\sqrt{x})})}{d^2} \\ &- \frac{16iabx^{3/2} \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{16iabx^{3/2} \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} \\ &- \frac{12b^2 x \operatorname{PolyLog}(2, -e^{2(c+d\sqrt{x})})}{d^3} + \frac{48iabx \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} \\ &- \frac{48iabx \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} + \frac{12b^2 \sqrt{x} \operatorname{PolyLog}(3, -e^{2(c+d\sqrt{x})})}{d^4} \\ &- \frac{96iab\sqrt{x} \operatorname{PolyLog}(4, -ie^{c+d\sqrt{x}})}{d^4} + \frac{96iab\sqrt{x} \operatorname{PolyLog}(4, ie^{c+d\sqrt{x}})}{d^4} \\ &- \frac{6b^2 \operatorname{PolyLog}(4, -e^{2(c+d\sqrt{x})})}{d^5} + \frac{96iab \operatorname{PolyLog}(5, -ie^{c+d\sqrt{x}})}{d^5} \\ &- \frac{96iab \operatorname{PolyLog}(5, ie^{c+d\sqrt{x}})}{d^5} + \frac{2b^2 x^2 \tanh(c + d\sqrt{x})}{d} \end{aligned}$$

```
output 2*b^2*x^2/d+2/5*a^2*x^(5/2)+8*a*b*x^2*arctan(exp(c+d*x^(1/2)))/d-8*b^2*x^(
3/2)*ln(1+exp(2*c+2*d*x^(1/2)))/d^2-96*I*a*b*polylog(5,I*exp(c+d*x^(1/2)))
/d^5+16*I*a*b*x^(3/2)*polylog(2,I*exp(c+d*x^(1/2)))/d^2-12*b^2*x*polylog(2
,-exp(2*c+2*d*x^(1/2)))/d^3+96*I*a*b*polylog(5,-I*exp(c+d*x^(1/2)))/d^5-48
*I*a*b*x*polylog(3,I*exp(c+d*x^(1/2)))/d^3-6*b^2*polylog(4,-exp(2*c+2*d*x^(
1/2)))/d^5-16*I*a*b*x^(3/2)*polylog(2,-I*exp(c+d*x^(1/2)))/d^2+48*I*a*b*x
*polylog(3,-I*exp(c+d*x^(1/2)))/d^3+12*b^2*polylog(3,-exp(2*c+2*d*x^(1/2))
)*x^(1/2)/d^4+96*I*a*b*polylog(4,I*exp(c+d*x^(1/2)))*x^(1/2)/d^4-96*I*a*b*
polylog(4,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^4+2*b^2*x^2*tanh(c+d*x^(1/2))/d
```

### 3.57.2 Mathematica [A] (verified)

Time = 5.89 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.22

$$\int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \frac{2 \cosh(c + d\sqrt{x}) (a + b \operatorname{sech}(c + d\sqrt{x}))^2 \left( \frac{10b^2 e^{2c} x^2 \cosh(c + d\sqrt{x})}{d(1 + e^{2c})} + a^2 x^{5/2} \cosh(c + d\sqrt{x}) \right)}{d}$$

```
input Integrate[x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

```
output (2*Cosh[c + d*Sqrt[x]]*(a + b*Sech[c + d*Sqrt[x]])^2*((10*b^2*E^(2*c)*x^2*
Cosh[c + d*Sqrt[x]]/(d*(1 + E^(2*c))) + a^2*x^(5/2)*Cosh[c + d*Sqrt[x]] +
((5*I)*b*Cosh[c + d*Sqrt[x]]*(2*a*d^4*x^2*Log[1 - I*E^(c + d*Sqrt[x]]) -
2*a*d^4*x^2*Log[1 + I*E^(c + d*Sqrt[x]]) + (4*I)*b*d^3*x^(3/2)*Log[1 + E(
2*(c + d*Sqrt[x]))] - 8*a*d^3*x^(3/2)*PolyLog[2, (-I)*E^(c + d*Sqrt[x])] +
8*a*d^3*x^(3/2)*PolyLog[2, I*E^(c + d*Sqrt[x])] + (6*I)*b*d^2*x*PolyLog[2
,-E^(2*(c + d*Sqrt[x]))] + 24*a*d^2*x*PolyLog[3, (-I)*E^(c + d*Sqrt[x])]
- 24*a*d^2*x*PolyLog[3, I*E^(c + d*Sqrt[x])] - (6*I)*b*d*Sqrt[x]*PolyLog[3
,-E^(2*(c + d*Sqrt[x]))] - 48*a*d*Sqrt[x]*PolyLog[4, (-I)*E^(c + d*Sqrt[x
])] + 48*a*d*Sqrt[x]*PolyLog[4, I*E^(c + d*Sqrt[x])] + (3*I)*b*PolyLog[4,
-E^(2*(c + d*Sqrt[x]))] + 48*a*PolyLog[5, (-I)*E^(c + d*Sqrt[x])] - 48*a*P
olyLog[5, I*E^(c + d*Sqrt[x])])/d^5 + (5*b^2*x^2*Sech[c]*Sinh[d*Sqrt[x]]
/d))/(5*(b + a*Cosh[c + d*Sqrt[x]])^2)
```

### 3.57.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {5959, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx \\
 & \quad \downarrow \text{5959} \\
 & 2 \int x^2 (a + b \operatorname{sech}(c + d\sqrt{x}))^2 d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int x^2 \left( a + b \operatorname{csc} \left( ic + id\sqrt{x} + \frac{\pi}{2} \right) \right)^2 d\sqrt{x} \\
 & \quad \downarrow \text{4678} \\
 & 2 \int (a^2 x^2 + b^2 \operatorname{sech}^2(c + d\sqrt{x}) x^2 + 2ab \operatorname{sech}(c + d\sqrt{x}) x^2) d\sqrt{x} \\
 & \quad \downarrow \text{2009} \\
 & 2 \left( \frac{1}{5} a^2 x^{5/2} + \frac{4abx^2 \arctan(e^{c+d\sqrt{x}})}{d} + \frac{48iab \operatorname{PolyLog}(5, -ie^{c+d\sqrt{x}})}{d^5} - \frac{48iab \operatorname{PolyLog}(5, ie^{c+d\sqrt{x}})}{d^5} - \frac{48iab\sqrt{x}}{d^5} \right)
 \end{aligned}$$

input `Int[x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output `2*((b^2*x^2)/d + (a^2*x^(5/2))/5 + (4*a*b*x^2*ArcTan[E^(c + d*Sqrt[x])])/d - (4*b^2*x^(3/2)*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((8*I)*a*b*x^(3/2)*PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((8*I)*a*b*x^(3/2)*PolyLog[2, I*E^(c + d*Sqrt[x])])/d^2 - (6*b^2*x*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d^3 + ((24*I)*a*b*x*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((24*I)*a*b*x*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (6*b^2*Sqrt[x]*PolyLog[3, -E^(2*(c + d*Sqrt[x]))])/d^4 - ((48*I)*a*b*Sqrt[x]*PolyLog[4, (-I)*E^(c + d*Sqrt[x])])/d^4 + ((48*I)*a*b*Sqrt[x]*PolyLog[4, I*E^(c + d*Sqrt[x])])/d^4 - (3*b^2*PolyLog[4, -E^(2*(c + d*Sqrt[x]))])/d^5 + ((48*I)*a*b*PolyLog[5, (-I)*E^(c + d*Sqrt[x])])/d^5 - ((48*I)*a*b*PolyLog[5, I*E^(c + d*Sqrt[x])])/d^5 + (b^2*x^2*Tanh[c + d*Sqrt[x]])/d`

---

3.57.  $\int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$

## 3.57.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

## 3.57.4 Maple [F]

$$\int x^{\frac{3}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

input `int(x^(3/2)*(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(x^(3/2)*(a+b*sech(c+d*x^(1/2)))^2,x)`

## 3.57.5 Fracas [F]

$$\int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^{\frac{3}{2}} dx$$

input `integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(b^2*x^(3/2)*sech(d*sqrt(x) + c)^2 + 2*a*b*x^(3/2)*sech(d*sqrt(x) + c) + a^2*x^(3/2), x)`

**3.57.6 Sympy [F]**

$$\int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

input `integrate(x**(3/2)*(a+b*sech(c+d*x**(1/2)))**2,x)`

output `Integral(x**(3/2)*(a + b*sech(c + d*sqrt(x)))**2, x)`

**3.57.7 Maxima [F]**

$$\int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^{3/2} dx$$

input `integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `2/5*(a^2*d*x^(5/2)*e^(2*d*sqrt(x) + 2*c) + a^2*d*x^(5/2) - 10*b^2*x^2)/(d*e^(2*d*sqrt(x) + 2*c) + d) + integrate(4*(a*b*d*x^(5/2)*e^(d*sqrt(x) + c) + 2*b^2*x^2)/(d*x*e^(2*d*sqrt(x) + 2*c) + d*x), x)`

**3.57.8 Giac [F]**

$$\int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^{3/2} dx$$

input `integrate(x^(3/2)*(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)^2*x^(3/2), x)`

**3.57.9 Mupad [F(-1)]**

Timed out.

$$\int x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int x^{3/2} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2 dx$$

input `int(x^(3/2)*(a + b/cosh(c + d*x^(1/2)))^2,x)`output `int(x^(3/2)*(a + b/cosh(c + d*x^(1/2)))^2, x)`

### 3.58 $\int \sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$

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#### 3.58.1 Optimal result

Integrand size = 22, antiderivative size = 229

$$\int \sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \frac{2b^2x}{d} + \frac{2}{3}a^2x^{3/2} + \frac{8abx \arctan(e^{c+d\sqrt{x}})}{d} - \frac{4b^2\sqrt{x} \log(1 + e^{2(c+d\sqrt{x})})}{d^2} - \frac{8iab\sqrt{x} \operatorname{PolyLog}(2, -ie^{c+d\sqrt{x}})}{d^2} + \frac{8iab\sqrt{x} \operatorname{PolyLog}(2, ie^{c+d\sqrt{x}})}{d^2} - \frac{2b^2 \operatorname{PolyLog}(2, -e^{2(c+d\sqrt{x})})}{d^3} + \frac{8iab \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{8iab \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} + \frac{2b^2x \tanh(c + d\sqrt{x})}{d}$$

output

```
2*b^2*x/d+2/3*a^2*x^(3/2)+8*a*b*x*arctan(exp(c+d*x^(1/2)))/d-2*b^2*polylog(2,-exp(2*c+2*d*x^(1/2)))/d^3+8*I*a*b*polylog(3,-I*exp(c+d*x^(1/2)))/d^3-8*I*a*b*polylog(3,I*exp(c+d*x^(1/2)))/d^3-4*b^2*ln(1+exp(2*c+2*d*x^(1/2)))*x^(1/2)/d^2-8*I*a*b*polylog(2,-I*exp(c+d*x^(1/2)))*x^(1/2)/d^2+8*I*a*b*polylog(2,I*exp(c+d*x^(1/2)))*x^(1/2)/d^2+2*b^2*x*tanh(c+d*x^(1/2))/d
```

### 3.58.2 Mathematica [A] (verified)

Time = 3.35 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.50

$$\int \sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

$$= \frac{2 \cosh(c + d\sqrt{x}) (a + b \operatorname{sech}(c + d\sqrt{x}))^2 \left( a^2 x^{3/2} \cosh(c + d\sqrt{x}) + \frac{3b \cosh(c + d\sqrt{x}) \left( 2be^{2c}x - 2b(1+e^{2c})x + \frac{b(1+e^{2c})}{x} \right)}{2be^{2c}x - 2b(1+e^{2c})x + \frac{b(1+e^{2c})}{x}} \right)}{2be^{2c}x - 2b(1+e^{2c})x + \frac{b(1+e^{2c})}{x}}$$

input `Integrate[Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output `(2*Cosh[c + d*Sqrt[x]]*(a + b*Sech[c + d*Sqrt[x]])^2*(a^2*x^(3/2)*Cosh[c + d*Sqrt[x]] + (3*b*Cosh[c + d*Sqrt[x]]*(2*b*E^(2*c)*x - 2*b*(1 + E^(2*c))*x + (b*(1 + E^(2*c))*(2*d^2*x - 2*d*Sqrt[x]*Log[1 + E^(2*(c + d*Sqrt[x]))] - PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d^2 + ((2*I)*a*(1 + E^(2*c))*(d^2*x*Log[1 - I*E^(c + d*Sqrt[x]]) - d^2*x*Log[1 + I*E^(c + d*Sqrt[x]]) - 2*d*Sqrt[x]*PolyLog[2, (-I)*E^(c + d*Sqrt[x]]) + 2*d*Sqrt[x]*PolyLog[2, I*E^(c + d*Sqrt[x]]) + 2*PolyLog[3, (-I)*E^(c + d*Sqrt[x]]) - 2*PolyLog[3, I*E^(c + d*Sqrt[x]])])/d^2))/(d*(1 + E^(2*c))) + (3*b^2*x*Sech[c]*Sinh[d*Sqrt[x]])/d)/(3*(b + a*Cosh[c + d*Sqrt[x]])^2)`

### 3.58.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {5959, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

$$\downarrow \text{5959}$$

$$2 \int x (a + b \operatorname{sech}(c + d\sqrt{x}))^2 d\sqrt{x}$$

$$\downarrow \text{3042}$$

---

3.58.  $\int \sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$



$$\begin{aligned}
& 2 \int x \left( a + b \operatorname{csc} \left( ic + id\sqrt{x} + \frac{\pi}{2} \right) \right)^2 d\sqrt{x} \\
& \quad \downarrow \text{4678} \\
& 2 \int (xa^2 + 2bx \operatorname{sech}(c + d\sqrt{x}) a + b^2 x \operatorname{sech}^2(c + d\sqrt{x})) d\sqrt{x} \\
& \quad \downarrow \text{2009} \\
& 2 \left( \frac{1}{3} a^2 x^{3/2} + \frac{4abx \arctan(e^{c+d\sqrt{x}})}{d} + \frac{4iab \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} - \frac{4iab \operatorname{PolyLog}(3, ie^{c+d\sqrt{x}})}{d^3} - \frac{4iab\sqrt{x} \operatorname{PolyLog}(3, -ie^{c+d\sqrt{x}})}{d^3} \right)
\end{aligned}$$

input `Int[Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output `2*((b^2*x)/d + (a^2*x^(3/2))/3 + (4*a*b*x*ArcTan[E^(c + d*Sqrt[x])])/d - (2*b^2*Sqrt[x]*Log[1 + E^(2*(c + d*Sqrt[x]))])/d^2 - ((4*I)*a*b*Sqrt[x]*PolyLog[2, (-I)*E^(c + d*Sqrt[x])])/d^2 + ((4*I)*a*b*Sqrt[x]*PolyLog[2, I*E^(c + d*Sqrt[x])])/d^2 - (b^2*PolyLog[2, -E^(2*(c + d*Sqrt[x]))])/d^3 + ((4*I)*a*b*PolyLog[3, (-I)*E^(c + d*Sqrt[x])])/d^3 - ((4*I)*a*b*PolyLog[3, I*E^(c + d*Sqrt[x])])/d^3 + (b^2*x*Tanh[c + d*Sqrt[x]])/d)`

### 3.58.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

**3.58.4 Maple [F]**

$$\int (a + b \operatorname{sech}(c + d\sqrt{x}))^2 \sqrt{x} dx$$

input `int((a+b*sech(c+d*x^(1/2)))^2*x^(1/2),x)`

output `int((a+b*sech(c+d*x^(1/2)))^2*x^(1/2),x)`

**3.58.5 Fricas [F]**

$$\int \sqrt{x}(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 \sqrt{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="fricas")`

output `integral(b^2*sqrt(x)*sech(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sech(d*sqrt(x) + c) + a^2*sqrt(x), x)`

**3.58.6 Sympy [F]**

$$\int \sqrt{x}(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int \sqrt{x}(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))**2*x**(1/2),x)`

output `Integral(sqrt(x)*(a + b*sech(c + d*sqrt(x)))**2, x)`

**3.58.7 Maxima [F]**

$$\int \sqrt{x}(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 \sqrt{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="maxima")`

output `2/3*(a^2*d*x^(3/2)*e^(2*d*sqrt(x) + 2*c) + a^2*d*x^(3/2) - 6*b^2*x)/(d*e^(2*d*sqrt(x) + 2*c) + d) + integrate(4*(a*b*d*x^(3/2)*e^(d*sqrt(x) + c) + b^2*x)/(d*x*e^(2*d*sqrt(x) + 2*c) + d*x), x)`

**3.58.8 Giac [F]**

$$\int \sqrt{x}(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int (b \operatorname{sech}(d\sqrt{x} + c) + a)^2 \sqrt{x} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)^2*sqrt(x), x)`

**3.58.9 Mupad [F(-1)]**

Timed out.

$$\int \sqrt{x}(a + b \operatorname{sech}(c + d\sqrt{x}))^2 dx = \int \sqrt{x} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)^2 dx$$

input `int(x^(1/2)*(a + b/cosh(c + d*x^(1/2)))^2,x)`

output `int(x^(1/2)*(a + b/cosh(c + d*x^(1/2)))^2, x)`

**3.59** 
$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{\sqrt{x}} dx$$

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 3.59.2 Mathematica [A] (verified) . . . . . 347  
 3.59.3 Rubi [A] (verified) . . . . . 348  
 3.59.4 Maple [A] (verified) . . . . . 350  
 3.59.5 Fricas [B] (verification not implemented) . . . . . 350  
 3.59.6 Sympy [F] . . . . . 351  
 3.59.7 Maxima [A] (verification not implemented) . . . . . 351  
 3.59.8 Giac [A] (verification not implemented) . . . . . 351  
 3.59.9 Mupad [B] (verification not implemented) . . . . . 352

**3.59.1 Optimal result**

Integrand size = 22, antiderivative size = 47

$$\int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{\sqrt{x}} dx = 2a^2\sqrt{x} + \frac{4ab \arctan(\sinh(c + d\sqrt{x}))}{d} + \frac{2b^2 \tanh(c + d\sqrt{x})}{d}$$

output `4*a*b*arctan(sinh(c+d*x^(1/2)))/d+2*a^2*x^(1/2)+2*b^2*tanh(c+d*x^(1/2))/d`

**3.59.2 Mathematica [A] (verified)**

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{\sqrt{x}} dx = 2a^2\sqrt{x} + \frac{4ab \arctan(\sinh(c + d\sqrt{x}))}{d} + \frac{2b^2 \tanh(c + d\sqrt{x})}{d}$$

input `Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/Sqrt[x],x]`

output `2*a^2*Sqrt[x] + (4*a*b*ArcTan[Sinh[c + d*Sqrt[x]]])/d + (2*b^2*Tanh[c + d*Sqrt[x]])/d`

---

3.59. 
$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{\sqrt{x}} dx$$

### 3.59.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$ , Rules used = {5959, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{\sqrt{x}} dx \\
 & \quad \downarrow \text{5959} \\
 & 2 \int (a + b \operatorname{sech}(c + d\sqrt{x}))^2 d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int \left( a + b \csc \left( ic + id\sqrt{x} + \frac{\pi}{2} \right) \right)^2 d\sqrt{x} \\
 & \quad \downarrow \text{4260} \\
 & 2 \left( 2ab \int \operatorname{sech}(c + d\sqrt{x}) d\sqrt{x} + b^2 \int \operatorname{sech}^2(c + d\sqrt{x}) d\sqrt{x} + a^2 \sqrt{x} \right) \\
 & \quad \downarrow \text{3042} \\
 & 2 \left( 2ab \int \csc \left( ic + id\sqrt{x} + \frac{\pi}{2} \right) d\sqrt{x} + b^2 \int \csc \left( ic + id\sqrt{x} + \frac{\pi}{2} \right)^2 d\sqrt{x} + a^2 \sqrt{x} \right) \\
 & \quad \downarrow \text{4254} \\
 & 2 \left( 2ab \int \csc \left( ic + id\sqrt{x} + \frac{\pi}{2} \right) d\sqrt{x} + \frac{ib^2 \int 1d(-i \tanh(c + d\sqrt{x}))}{d} + a^2 \sqrt{x} \right) \\
 & \quad \downarrow \text{24} \\
 & 2 \left( 2ab \int \csc \left( ic + id\sqrt{x} + \frac{\pi}{2} \right) d\sqrt{x} + a^2 \sqrt{x} + \frac{b^2 \tanh(c + d\sqrt{x})}{d} \right) \\
 & \quad \downarrow \text{4257} \\
 & 2 \left( a^2 \sqrt{x} + \frac{2ab \arctan(\sinh(c + d\sqrt{x}))}{d} + \frac{b^2 \tanh(c + d\sqrt{x})}{d} \right)
 \end{aligned}$$

input `Int[(a + b*Sech[c + d*Sqrt[x]])^2/Sqrt[x],x]`

---

3.59.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{\sqrt{x}} dx$

output  $2*(a^2*\text{Sqrt}[x] + (2*a*b*\text{ArcTan}[\text{Sinh}[c + d*\text{Sqrt}[x]]])/d + (b^2*\text{Tanh}[c + d*\text{Sqrt}[x]])/d)$

### 3.59.3.1 Defintions of rubi rules used

rule 24  $\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

rule 3042  $\text{Int}[u_, x\_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] /; \text{FunctionOfTrigOfLinearQ}[u, x]$

rule 4254  $\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[-d^{(-1)} \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \ \&\& \ \text{IGtQ}[n/2, 0]$

rule 4257  $\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[-\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

rule 4260  $\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^{2}, x\_Symbol] \rightarrow \text{Simp}[a^2*x, x] + (\text{Simp}[2*a*b \text{Int}[\text{Csc}[c + d*x], x], x] + \text{Simp}[b^2 \text{Int}[\text{Csc}[c + d*x]^2, x], x]) /; \text{FreeQ}[\{a, b, c, d\}, x]$

rule 5959  $\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*\text{Sech}[(c_.) + (d_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[1/n \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*\text{Sech}[c + d*x])^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p\}, x] \ \&\& \ \text{IGtQ}[\text{Simplify}[(m + 1)/n], 0] \ \&\& \ \text{IntegerQ}[p]$

---

3.59.  $\int \frac{(a+b\text{sech}(c+d\sqrt{x}))^2}{\sqrt{x}} dx$

**3.59.4 Maple [A] (verified)**

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

method	result	size
parts	$\frac{4ab \arctan(\sinh(c+d\sqrt{x}))}{d} + 2a^2\sqrt{x} + \frac{2b^2 \tanh(c+d\sqrt{x})}{d}$	42
derivativedivides	$\frac{2a^2(c+d\sqrt{x})+8ab \arctan(e^{c+d\sqrt{x}})+2b^2 \tanh(c+d\sqrt{x})}{d}$	43
default	$\frac{2a^2(c+d\sqrt{x})+8ab \arctan(e^{c+d\sqrt{x}})+2b^2 \tanh(c+d\sqrt{x})}{d}$	43

```
input int((a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
output 4*a*b*arctan(sinh(c+d*x^(1/2)))/d+2*a^2*x^(1/2)+2*b^2*tanh(c+d*x^(1/2))/d
```

**3.59.5 Fracas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(41) = 82.

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.13

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{\sqrt{x}} dx$$

$$= \frac{2 \left( a^2 d \sqrt{x} \cosh(d\sqrt{x} + c)^2 + 2 a^2 d \sqrt{x} \cosh(d\sqrt{x} + c) \sinh(d\sqrt{x} + c) + a^2 d \sqrt{x} \sinh(d\sqrt{x} + c)^2 + a^2 d \sqrt{x} \right)}{d \cosh(d\sqrt{x} + c)^2 + 2 d \cosh(d\sqrt{x} + c) \sinh(d\sqrt{x} + c) + d \sinh(d\sqrt{x} + c)^2 + d}$$

```
input integrate((a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="fracas")
```

```
output 2*(a^2*d*sqrt(x)*cosh(d*sqrt(x) + c)^2 + 2*a^2*d*sqrt(x)*cosh(d*sqrt(x) + c)*sinh(d*sqrt(x) + c) + a^2*d*sqrt(x)*sinh(d*sqrt(x) + c)^2 + a^2*d*sqrt(x) - 2*b^2 + 4*(a*b*cosh(d*sqrt(x) + c)^2 + 2*a*b*cosh(d*sqrt(x) + c)*sinh(d*sqrt(x) + c) + a*b*sinh(d*sqrt(x) + c)^2 + a*b)*arctan(cosh(d*sqrt(x) + c) + sinh(d*sqrt(x) + c)))/(d*cosh(d*sqrt(x) + c)^2 + 2*d*cosh(d*sqrt(x) + c)*sinh(d*sqrt(x) + c) + d*sinh(d*sqrt(x) + c)^2 + d)
```

---

3.59.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{\sqrt{x}} dx$

**3.59.6 Sympy [F]**

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{\sqrt{x}} dx = \int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{\sqrt{x}} dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))**2/x**(1/2),x)`

output `Integral((a + b*sech(c + d*sqrt(x)))**2/sqrt(x), x)`

**3.59.7 Maxima [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{\sqrt{x}} dx = 2a^2\sqrt{x} + \frac{4ab \arctan(\sinh(d\sqrt{x} + c))}{d} + \frac{4b^2}{d(e^{(-2d\sqrt{x}-2c)} + 1)}$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")`

output `2*a^2*sqrt(x) + 4*a*b*arctan(sinh(d*sqrt(x) + c))/d + 4*b^2/(d*(e^(-2*d*sqrt(x) - 2*c) + 1))`

**3.59.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{\sqrt{x}} dx = \frac{2(d\sqrt{x} + c)a^2}{d} + \frac{8ab \arctan(e^{(d\sqrt{x}+c)})}{d} - \frac{4b^2}{d(e^{(2d\sqrt{x}+2c)} + 1)}$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")`

output `2*(d*sqrt(x) + c)*a^2/d + 8*a*b*arctan(e^(d*sqrt(x) + c))/d - 4*b^2/(d*(e^(2*d*sqrt(x) + 2*c) + 1))`

---

3.59.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{\sqrt{x}} dx$



**3.59.9 Mupad [B] (verification not implemented)**

Time = 2.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{\sqrt{x}} dx = 2a^2 \sqrt{x} + \frac{8 \operatorname{atan}\left(\frac{ab e^{d\sqrt{x}} e^c \sqrt{d^2}}{d \sqrt{a^2 b^2}}\right) \sqrt{a^2 b^2}}{\sqrt{d^2}} - \frac{4b^2}{d(e^{2c+2d\sqrt{x}} + 1)}$$

input `int((a + b/cosh(c + d*x^(1/2)))^2/x^(1/2),x)`

output `2*a^2*x^(1/2) + (8*atan((a*b*exp(d*x^(1/2))*exp(c)*(d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d^2)^(1/2) - (4*b^2)/(d*(exp(2*c + 2*d*x^(1/2)) + 1))`

**3.60** 
$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{3/2}} dx$$

3.60.1	Optimal result	353
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3.60.7	Maxima [N/A]	355
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3.60.9	Mupad [N/A]	356

**3.60.1 Optimal result**

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}} dx = \operatorname{Int}\left(\frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}}, x\right)$$

output `Unintegrable((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x)`

**3.60.2 Mathematica [N/A]**

Not integrable

Time = 36.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}} dx$$

input `Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^(3/2),x]`

output `Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^(3/2), x]`

---

3.60. 
$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{3/2}} dx$$

### 3.60.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}} dx$$

↓ 5961

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}} dx$$

input `Int[(a + b*Sech[c + d*Sqrt[x]])^2/x^(3/2),x]`

output `$Aborted`

#### 3.60.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.60.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{\frac{3}{2}}} dx$$

input `int((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x)`

output `int((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x)`

---

3.60.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{3/2}} dx$

**3.60.5 Fracas [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2}{x^{3/2}} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="fricas")`

output `integral((b^2*sqrt(x)*sech(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sech(d*sqrt(x) + c) + a^2*sqrt(x))/x^2, x)`

**3.60.6 Sympy [N/A]**

Not integrable

Time = 2.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}} dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))**2/x**(3/2),x)`

output `Integral((a + b*sech(c + d*sqrt(x)))**2/x**(3/2), x)`

**3.60.7 Maxima [N/A]**

Not integrable

Time = 0.55 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.82

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2}{x^{3/2}} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="maxima")`

output `-2*(a^2*d*sqrt(x)*e^(2*d*sqrt(x) + 2*c) + a^2*d*sqrt(x) + 2*b^2)/(d*x*e^(2*d*sqrt(x) + 2*c) + d*x) + integrate(4*(a*b*d*x*e^(d*sqrt(x) + c) - b^2*sqrt(x))/(d*x^(5/2)*e^(2*d*sqrt(x) + 2*c) + d*x^(5/2)), x)`

---

3.60.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{3/2}} dx$

**3.60.8 Giac [N/A]**

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2}{x^{3/2}} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)^2/x^(3/2), x)`

**3.60.9 Mupad [N/A]**

Not integrable

Time = 2.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{3/2}} dx = \int \frac{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2}{x^{3/2}} dx$$

input `int((a + b/cosh(c + d*x^(1/2)))^2/x^(3/2),x)`

output `int((a + b/cosh(c + d*x^(1/2)))^2/x^(3/2), x)`

**3.61** 
$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}} dx$$

3.61.1	Optimal result	357
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3.61.7	Maxima [F(-1)]	359
3.61.8	Giac [N/A]	360
3.61.9	Mupad [N/A]	360

**3.61.1 Optimal result**

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx = \operatorname{Int}\left(\frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}}, x\right)$$

output `Unintegrable((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x)`

**3.61.2 Mathematica [N/A]**

Not integrable

Time = 34.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{(a + b\operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx$$

input `Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^(5/2),x]`

output `Integrate[(a + b*Sech[c + d*Sqrt[x]])^2/x^(5/2), x]`

---

3.61. 
$$\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}} dx$$

### 3.61.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx$$

↓ 5961

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx$$

input `Int[(a + b*Sech[c + d*Sqrt[x]])^2/x^(5/2),x]`

output `$Aborted`

#### 3.61.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.61.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx$$

input `int((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x)`

output `int((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x)`

---

3.61.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}} dx$

**3.61.5 Fricas [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2}{x^{5/2}} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="fricas")`

output `integral((b^2*sqrt(x)*sech(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*sech(d*sqrt(x) + c) + a^2*sqrt(x))/x^3, x)`

**3.61.6 Sympy [N/A]**

Not integrable

Time = 5.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx$$

input `integrate((a+b*sech(c+d*x**(1/2)))**2/x**(5/2),x)`

output `Integral((a + b*sech(c + d*sqrt(x)))**2/x**(5/2), x)`

**3.61.7 Maxima [F(-1)]**

Timed out.

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx = \text{Timed out}$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="maxima")`

output `Timed out`

---

3.61.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}} dx$



**3.61.8 Giac [N/A]**

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2}{x^{5/2}} dx$$

input `integrate((a+b*sech(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="giac")`

output `integrate((b*sech(d*sqrt(x) + c) + a)^2/x^(5/2), x)`

**3.61.9 Mupad [N/A]**

Not integrable

Time = 2.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \operatorname{sech}(c + d\sqrt{x}))^2}{x^{5/2}} dx = \int \frac{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2}{x^{5/2}} dx$$

input `int((a + b/cosh(c + d*x^(1/2)))^2/x^(5/2),x)`

output `int((a + b/cosh(c + d*x^(1/2)))^2/x^(5/2), x)`

---

3.61.  $\int \frac{(a+b\operatorname{sech}(c+d\sqrt{x}))^2}{x^{5/2}} dx$

### 3.62 $\int \frac{x^{3/2}}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$

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3.62.5	Fricas [F]	365
3.62.6	Sympy [F]	365
3.62.7	Maxima [F(-2)]	366
3.62.8	Giac [F]	366
3.62.9	Mupad [F(-1)]	366

#### 3.62.1 Optimal result

Integrand size = 22, antiderivative size = 601

$$\begin{aligned} \int \frac{x^{3/2}}{a+b\operatorname{sech}(c+d\sqrt{x})} dx = & \frac{2x^{5/2}}{5a} - \frac{2bx^2 \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\ & + \frac{2bx^2 \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\ & + \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{24bx \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\ & - \frac{24bx \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\ & + \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\ & + \frac{48b \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{48b \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \end{aligned}$$

output  $\frac{2}{5}x^{5/2}/a-2bx^2\ln(1+a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2}))/a/d/(-a^2+b^2)^{1/2}+2bx^2\ln(1+a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2}))/a/d/(-a^2+b^2)^{1/2}-8b^2x^{3/2}\operatorname{polylog}(2,-a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2}))/a/d^2/(-a^2+b^2)^{1/2}+8b^2x^{3/2}\operatorname{polylog}(2,-a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2}))/a/d^2/(-a^2+b^2)^{1/2}+24b^2x\operatorname{polylog}(3,-a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2}))/a/d^3/(-a^2+b^2)^{1/2}-24b^2x\operatorname{polylog}(3,-a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2}))/a/d^3/(-a^2+b^2)^{1/2}+48b^2\operatorname{polylog}(5,-a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2}))/a/d^5/(-a^2+b^2)^{1/2}-48b^2\operatorname{polylog}(5,-a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2}))/a/d^5/(-a^2+b^2)^{1/2}-48b^2\operatorname{polylog}(4,-a\exp(c+d\sqrt{x})/(b-\sqrt{-a^2+b^2}))/a/d^4/(-a^2+b^2)^{1/2}+48b^2\operatorname{polylog}(4,-a\exp(c+d\sqrt{x})/(b+\sqrt{-a^2+b^2}))/a/d^4/(-a^2+b^2)^{1/2}$

### 3.62.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.77

$$\int \frac{x^{3/2}}{a + b\operatorname{sech}(c + d\sqrt{x})} dx = \frac{2\left(\sqrt{-a^2 + b^2}d^5x^{5/2} - 5bd^4x^2 \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right) + 5bd^4x^2 \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)\right)}{5a\sqrt{-a^2+b^2}}$$

input `Integrate[x^(3/2)/(a + b*Sech[c + d*Sqrt[x]]),x]`

output  $(2*(\operatorname{Sqrt}[-a^2 + b^2]*d^5x^{5/2} - 5*b*d^4*x^2*\operatorname{Log}[1 + (a*E^{(c + d*\operatorname{Sqrt}[x])})/(b - \operatorname{Sqrt}[-a^2 + b^2])]) + 5*b*d^4*x^2*\operatorname{Log}[1 + (a*E^{(c + d*\operatorname{Sqrt}[x])})/(b + \operatorname{Sqrt}[-a^2 + b^2])]) - 20*b*d^3*x^{3/2}*PolyLog[2, (a*E^{(c + d*\operatorname{Sqrt}[x])})/(-b + \operatorname{Sqrt}[-a^2 + b^2])]) + 20*b*d^3*x^{3/2}*PolyLog[2, -((a*E^{(c + d*\operatorname{Sqrt}[x])})/(b + \operatorname{Sqrt}[-a^2 + b^2]))]) + 60*b*d^2*x*PolyLog[3, (a*E^{(c + d*\operatorname{Sqrt}[x])})/(-b + \operatorname{Sqrt}[-a^2 + b^2])]) - 60*b*d^2*x*PolyLog[3, -((a*E^{(c + d*\operatorname{Sqrt}[x])})/(b + \operatorname{Sqrt}[-a^2 + b^2]))]) - 120*b*d*\operatorname{Sqrt}[x]*PolyLog[4, (a*E^{(c + d*\operatorname{Sqrt}[x])})/(-b + \operatorname{Sqrt}[-a^2 + b^2])]) + 120*b*d*\operatorname{Sqrt}[x]*PolyLog[4, -((a*E^{(c + d*\operatorname{Sqrt}[x])})/(b + \operatorname{Sqrt}[-a^2 + b^2]))]) + 120*b*PolyLog[5, (a*E^{(c + d*\operatorname{Sqrt}[x])})/(-b + \operatorname{Sqrt}[-a^2 + b^2])]) - 120*b*PolyLog[5, -((a*E^{(c + d*\operatorname{Sqrt}[x])})/(b + \operatorname{Sqrt}[-a^2 + b^2]))])]/(5*a*\operatorname{Sqrt}[-a^2 + b^2]*d^5)$

### 3.62.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{x^{3/2}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx \\
 & \quad \downarrow \text{5959} \\
 & 2 \int \frac{x^2}{a + b \operatorname{sech}(c + d\sqrt{x})} d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int \frac{x^2}{a + b \csc\left(ic + id\sqrt{x} + \frac{\pi}{2}\right)} d\sqrt{x} \\
 & \quad \downarrow \text{4679} \\
 & 2 \int \left( \frac{x^2}{a} - \frac{bx^2}{a(b + a \cosh(c + d\sqrt{x}))} \right) d\sqrt{x} \\
 & \quad \downarrow \text{2009} \\
 & 2 \left( \frac{24b \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{24b \operatorname{PolyLog}\left(5, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} - \frac{24b\sqrt{x} \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{24b\sqrt{x} \operatorname{PolyLog}\left(4, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} \right)
 \end{aligned}$$

input `Int[x^(3/2)/(a + b*Sech[c + d*Sqrt[x]]),x]`

```
output 2*(x^(5/2)/(5*a) - (b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (b*x^2*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (4*b*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (4*b*x^(3/2)*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (12*b*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (12*b*x*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (24*b*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (24*b*Sqrt[x]*PolyLog[4, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^4) + (24*b*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5) - (24*b*PolyLog[5, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^5))
```

### 3.62.3.1 Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

```
rule 4679 Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]
```

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

**3.62.4 Maple [F]**

$$\int \frac{x^{\frac{3}{2}}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

input `int(x^(3/2)/(a+b*sech(c+d*x^(1/2))),x)`

output `int(x^(3/2)/(a+b*sech(c+d*x^(1/2))),x)`

**3.62.5 Fricas [F]**

$$\int \frac{x^{3/2}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^{\frac{3}{2}}}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

input `integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(x^(3/2)/(b*sech(d*sqrt(x) + c) + a), x)`

**3.62.6 Sympy [F]**

$$\int \frac{x^{3/2}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^{\frac{3}{2}}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

input `integrate(x**(3/2)/(a+b*sech(c+d*x**(1/2))),x)`

output `Integral(x**(3/2)/(a + b*sech(c + d*sqrt(x))), x)`

**3.62.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{x^{3/2}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \text{Exception raised: ValueError}$$

input `integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

**3.62.8 Giac [F]**

$$\int \frac{x^{3/2}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^{3/2}}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

input `integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate(x^(3/2)/(b*sech(d*sqrt(x) + c) + a), x)`

**3.62.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x^{3/2}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{x^{3/2}}{a + \frac{b}{\cosh(c + d\sqrt{x})}} dx$$

input `int(x^(3/2)/(a + b/cosh(c + d*x^(1/2))),x)`

output `int(x^(3/2)/(a + b/cosh(c + d*x^(1/2))), x)`

### 3.63 $\int \frac{\sqrt{x}}{a+b\operatorname{sech}(c+d\sqrt{x})} dx$

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#### 3.63.1 Optimal result

Integrand size = 22, antiderivative size = 361

$$\int \frac{\sqrt{x}}{a+b\operatorname{sech}(c+d\sqrt{x})} dx = \frac{2x^{3/2}}{3a} - \frac{2bx \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{2bx \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$

$$- \frac{4b\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$+ \frac{4b\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$+ \frac{4b \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{4b \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

output  $2/3*x^{(3/2)}/a-2*b*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a/d/(-a^2+b^2)^{(1/2)}+2*b*x*\ln(1+a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a/d/(-a^2+b^2)^{(1/2)}+4*b*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))/a/d^3/(-a^2+b^2)^{(1/2)}-4*b*polylog(3,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))/a/d^3/(-a^2+b^2)^{(1/2)}-4*b*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b-(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a/d^2/(-a^2+b^2)^{(1/2)}+4*b*polylog(2,-a*\exp(c+d*x^{(1/2)})/(b+(-a^2+b^2)^{(1/2)}))*x^{(1/2)}/a/d^2/(-a^2+b^2)^{(1/2)}$



### 3.63.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

$$= \frac{2\left(\sqrt{-a^2 + b^2} d^3 x^{3/2} - 3bd^2 x \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b - \sqrt{-a^2 + b^2}}\right) + 3bd^2 x \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}}\right) - 6bd\sqrt{x} \operatorname{PolyLog}\left(2, \frac{ae^{c+d\sqrt{x}}}{-b + \sqrt{-a^2 + b^2}}\right) + 6bd\sqrt{x} \operatorname{PolyLog}\left(2, \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}}\right) - 6bd \operatorname{PolyLog}\left(3, \frac{ae^{c+d\sqrt{x}}}{-b + \sqrt{-a^2 + b^2}}\right) + 6bd \operatorname{PolyLog}\left(3, \frac{ae^{c+d\sqrt{x}}}{b + \sqrt{-a^2 + b^2}}\right)\right)}{3a\sqrt{-a^2 + b^2}}$$

input `Integrate[Sqrt[x]/(a + b*Sech[c + d*Sqrt[x]]),x]`

output `(2*(Sqrt[-a^2 + b^2]*d^3*x^(3/2) - 3*b*d^2*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])] + 3*b*d^2*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])] - 6*b*d*Sqrt[x]*PolyLog[2, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[-a^2 + b^2])] + 6*b*d*Sqrt[x]*PolyLog[2, -(a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])] + 6*b*PolyLog[3, (a*E^(c + d*Sqrt[x]))/(-b + Sqrt[-a^2 + b^2])] - 6*b*PolyLog[3, -(a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(3*a*Sqrt[-a^2 + b^2]*d^3)`

### 3.63.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

$$\downarrow \text{5959}$$

$$2 \int \frac{x}{a + b \operatorname{sech}(c + d\sqrt{x})} d\sqrt{x}$$

$$\downarrow \text{3042}$$

$$2 \int \frac{x}{a + b \csc\left(ic + id\sqrt{x} + \frac{\pi}{2}\right)} d\sqrt{x}$$

$$\downarrow \text{4679}$$

---

3.63.  $\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$

$$2 \int \left( \frac{x}{a} - \frac{bx}{a(b + a \cosh(c + d\sqrt{x}))} \right) d\sqrt{x}$$

↓ 2009

$$2 \left( \frac{2b \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{2b \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{2b\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{2b\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} \right)$$

input `Int[Sqrt[x]/(a + b*Sech[c + d*Sqrt[x]]), x]`

output `2*(x^(3/2)/(3*a) - (b*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (b*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - (2*b*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (2*b*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (2*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (2*b*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3))`

### 3.63.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

**3.63.4 Maple [F]**

$$\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

input `int(x^(1/2)/(a+b*sech(c+d*x^(1/2))),x)`

output `int(x^(1/2)/(a+b*sech(c+d*x^(1/2))),x)`

**3.63.5 Fricas [F]**

$$\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{\sqrt{x}}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

input `integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")`

output `integral(sqrt(x)/(b*sech(d*sqrt(x) + c) + a), x)`

**3.63.6 Sympy [F]**

$$\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx$$

input `integrate(x**(1/2)/(a+b*sech(c+d*x**(1/2))),x)`

output `Integral(sqrt(x)/(a + b*sech(c + d*sqrt(x))), x)`

**3.63.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \text{Exception raised: ValueError}$$

input `integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

**3.63.8 Giac [F]**

$$\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{\sqrt{x}}{b \operatorname{sech}(d\sqrt{x} + c) + a} dx$$

input `integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`

output `integrate(sqrt(x)/(b*sech(d*sqrt(x) + c) + a), x)`

**3.63.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{\sqrt{x}}{a + b \operatorname{sech}(c + d\sqrt{x})} dx = \int \frac{\sqrt{x}}{a + \frac{b}{\cosh(c + d\sqrt{x})}} dx$$

input `int(x^(1/2)/(a + b/cosh(c + d*x^(1/2))),x)`

output `int(x^(1/2)/(a + b/cosh(c + d*x^(1/2))), x)`

### 3.64 $\int \frac{1}{\sqrt{x}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$

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#### 3.64.1 Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{1}{\sqrt{x}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx = \frac{2\sqrt{x}}{a} - \frac{4b \arctan\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+bd}}$$

```
output -4*b*arctan((a-b)^(1/2)*tanh(1/2*c+1/2*d*x^(1/2))/(a+b)^(1/2))/a/d/(a-b)^(1/2)/(a+b)^(1/2)+2*x^(1/2)/a
```

#### 3.64.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt{x}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx = \frac{2\left(\frac{c}{d} + \sqrt{x} + \frac{2b \arctan\left(\frac{(-a+b) \tanh\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}\right)}{a}$$

```
input Integrate[1/(Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])),x]
```

```
output (2*(c/d + Sqrt[x] + (2*b*ArcTan[((-a + b)*Tanh[(c + d*Sqrt[x])/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*d)))/a
```

### 3.64.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {5959, 3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx \\
 & \quad \downarrow \text{5959} \\
 & 2 \int \frac{1}{a + b \operatorname{sech}(c + d\sqrt{x})} d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int \frac{1}{a + b \csc\left(ic + id\sqrt{x} + \frac{\pi}{2}\right)} d\sqrt{x} \\
 & \quad \downarrow \text{4270} \\
 & 2 \left( \frac{\sqrt{x}}{a} - \frac{\int \frac{1}{a \cosh\left(\frac{c+d\sqrt{x}}{b} + 1\right)} d\sqrt{x}}{a} \right) \\
 & \quad \downarrow \text{3042} \\
 & 2 \left( \frac{\sqrt{x}}{a} - \frac{\int \frac{1}{a \sin\left(\frac{ic+id\sqrt{x}+\frac{\pi}{2}}{b} + 1\right)} d\sqrt{x}}{a} \right) \\
 & \quad \downarrow \text{3138} \\
 & 2 \left( \frac{\sqrt{x}}{a} + \frac{2i \int \frac{1}{\frac{a+b}{b} + (1-\frac{a}{b})x} d\left(i \tanh\left(\frac{1}{2}(c + d\sqrt{x})\right)\right)}{ad} \right) \\
 & \quad \downarrow \text{221} \\
 & 2 \left( \frac{\sqrt{x}}{a} - \frac{2b \arctan\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}} \right)
 \end{aligned}$$

input `Int[1/(Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])),x]`

output  $2*(\text{Sqrt}[x]/a - (2*b*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tanh}[(c + d*\text{Sqrt}[x])/2])]/\text{Sqrt}[a + b]))/(a*\text{Sqrt}[a - b]*\text{Sqrt}[a + b]*d)$

### 3.64.3.1 Defintions of rubi rules used

rule 221  $\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-a/b, 2]/a)*\text{ArcTanh}[x/\text{Rt}[-a/b, 2]], x] \text{ ; FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b]$

rule 3042  $\text{Int}[u_, x\_Symbol] \rightarrow \text{Int}[\text{DeactivateTrig}[u, x], x] \text{ ; FunctionOfTrigOfLinearQ}[u, x]$

rule 3138  $\text{Int}[(a_ + (b_)*\sin[\text{Pi}/2 + (c_.) + (d_)*(x_)])^{-1}, x\_Symbol] \rightarrow \text{With}\{e = \text{FreeFactors}[\text{Tan}[(c + d*x)/2], x]\}, \text{Simp}[2*(e/d) \ \text{Subst}[\text{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \text{Tan}[(c + d*x)/2]/e], x] \text{ ; FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

rule 4270  $\text{Int}[(\text{csc}[(c_.) + (d_)*(x_)])*(b_.) + (a_))^{-1}, x\_Symbol] \rightarrow \text{Simp}[x/a, x] - \text{Simp}[1/a \ \text{Int}[1/(1 + (a/b)*\text{Sin}[c + d*x]), x], x] \text{ ; FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

rule 5959  $\text{Int}[(x_)^{(m_)}*((a_.) + (b_)*\text{Sech}[(c_.) + (d_)*(x_)^{(n_)}])^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[1/n \ \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1)}*(a + b*\text{Sech}[c + d*x])^p, x], x, x^n], x] \text{ ; FreeQ}\{a, b, c, d, m, n, p, x\} \ \&\& \ \text{IGtQ}[\text{Simplify}[(m + 1)/n], 0] \ \&\& \ \text{IntegerQ}[p]$

### 3.64.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{-\frac{2 \ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right) - 1\right)}{a} + \frac{2 \ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right) + 1\right)}{a} - \frac{4b \arctan\left(\frac{(a-b) \tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	90
default	$\frac{-\frac{2 \ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right) - 1\right)}{a} + \frac{2 \ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right) + 1\right)}{a} - \frac{4b \arctan\left(\frac{(a-b) \tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	90

input `int(1/(a+b*sech(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)`

output `2/d*(-1/a*ln(tanh(1/2*c+1/2*d*x^(1/2))-1)+1/a*ln(tanh(1/2*c+1/2*d*x^(1/2))+1)-2*b/a/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*c+1/2*d*x^(1/2))/((a+b)*(a-b))^(1/2)))`

### 3.64.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(55) = 110.

Time = 0.30 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.74

$$\int \frac{1}{\sqrt{x}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

$$= \left[ \frac{2 \left( (a^2 - b^2)d\sqrt{x} - \sqrt{-a^2 + b^2}b \log \left( \frac{ab + (b^2 + \sqrt{-a^2 + b^2}b) \cosh(d\sqrt{x} + c) + (a^2 - b^2 - \sqrt{-a^2 + b^2}b) \sinh(d\sqrt{x} + c) + \sqrt{-a^2 + b^2}a}{a \cosh(d\sqrt{x} + c) + b} \right)}{(a^3 - ab^2)d} \right]$$

input `integrate(1/(a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="fracas")`

output `[2*((a^2 - b^2)*d*sqrt(x) - sqrt(-a^2 + b^2)*b*log((a*b + (b^2 + sqrt(-a^2 + b^2)*b)*cosh(d*sqrt(x) + c) + (a^2 - b^2 - sqrt(-a^2 + b^2)*b)*sinh(d*sqrt(x) + c) + sqrt(-a^2 + b^2)*a)/(a*cosh(d*sqrt(x) + c) + b)))/((a^3 - a*b^2)*d), 2*((a^2 - b^2)*d*sqrt(x) + 2*sqrt(a^2 - b^2)*b*arctan(-(sqrt(a^2 - b^2)*a*cosh(d*sqrt(x) + c) + sqrt(a^2 - b^2)*a*sinh(d*sqrt(x) + c) + sqrt(a^2 - b^2)*b)/(a^2 - b^2)))/((a^3 - a*b^2)*d)]`

---

3.64.  $\int \frac{1}{\sqrt{x}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$



**3.64.6 Sympy [F]**

$$\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

input `integrate(1/(a+b*sech(c+d*x**(1/2)))/x**(1/2),x)`

output `Integral(1/(sqrt(x)*(a + b*sech(c + d*sqrt(x)))), x)`

**3.64.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \text{Exception raised: ValueError}$$

input `integrate(1/(a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f or more de`

**3.64.8 Giac [A] (verification not implemented)**

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = -\frac{4 b \arctan\left(\frac{ae^{(d\sqrt{x}+c)}+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}ad} + \frac{2(d\sqrt{x}+c)}{ad}$$

input `integrate(1/(a+b*sech(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")`

output `-4*b*arctan((a*e^(d*sqrt(x) + c) + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a*d) + 2*(d*sqrt(x) + c)/(a*d)`

---

3.64.  $\int \frac{1}{\sqrt{x}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$

**3.64.9 Mupad [B] (verification not implemented)**

Time = 2.56 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.28

$$\int \frac{1}{\sqrt{x}(a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \frac{2\sqrt{x}}{a} + \frac{2b \ln\left(\frac{2be^{d\sqrt{x}}e^c}{a^2\sqrt{x}} - \frac{2b(a+be^{d\sqrt{x}}e^c)}{a^2\sqrt{x}\sqrt{a+b}\sqrt{b-a}}\right)}{ad\sqrt{a+b}\sqrt{b-a}} - \frac{2b \ln\left(\frac{2be^{d\sqrt{x}}e^c}{a^2\sqrt{x}} + \frac{2b(a+be^{d\sqrt{x}}e^c)}{a^2\sqrt{x}\sqrt{a+b}\sqrt{b-a}}\right)}{ad\sqrt{a+b}\sqrt{b-a}}$$

input `int(1/(x^(1/2)*(a + b/cosh(c + d*x^(1/2)))) ,x)`

output `(2*x^(1/2))/a + (2*b*log((2*b*exp(d*x^(1/2))*exp(c))/(a^2*x^(1/2)) - (2*b*(a + b*exp(d*x^(1/2))*exp(c)))/(a^2*x^(1/2)*(a + b)^(1/2)*(b - a)^(1/2))))/(a*d*(a + b)^(1/2)*(b - a)^(1/2)) - (2*b*log((2*b*exp(d*x^(1/2))*exp(c))/(a^2*x^(1/2)) + (2*b*(a + b*exp(d*x^(1/2))*exp(c)))/(a^2*x^(1/2)*(a + b)^(1/2)*(b - a)^(1/2))))/(a*d*(a + b)^(1/2)*(b - a)^(1/2))`

**3.65**  $\int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$

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**3.65.1 Optimal result**

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx = \operatorname{Int}\left(\frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))}, x\right)$$

output `Unintegrable(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x)`

**3.65.2 Mathematica [N/A]**

Not integrable

Time = 10.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx = \int \frac{1}{x^{3/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

input `Integrate[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])),x]`

output `Integrate[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])), x]`

### 3.65.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

↓ 5961

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

input `Int[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])),x]`

output `$Aborted`

#### 3.65.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.65.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

input `int(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x)`

output `int(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x)`

---

3.65.  $\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$

**3.65.5 Fracas [N/A]**

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a) x^{\frac{3}{2}}} dx$$

```
input integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
```

```
output integral(sqrt(x)/(b*x^2*sech(d*sqrt(x) + c) + a*x^2), x)
```

**3.65.6 Sympy [N/A]**

Not integrable

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{x^{\frac{3}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

```
input integrate(1/x**(3/2)/(a+b*sech(c+d*x**(1/2))),x)
```

```
output Integral(1/(x**(3/2)*(a + b*sech(c + d*sqrt(x)))), x)
```

**3.65.7 Maxima [N/A]**

Not integrable

Time = 0.49 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a) x^{\frac{3}{2}}} dx$$

```
input integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
```

```
output -2*b*integrate(e^(d*sqrt(x) + c)/(a^2*x^(3/2)*e^(2*d*sqrt(x) + 2*c) + 2*a*
b*x^(3/2)*e^(d*sqrt(x) + c) + a^2*x^(3/2)), x) - 2/(a*sqrt(x))
```

---

3.65.  $\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$

**3.65.8 Giac [N/A]**

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a) x^{3/2}} dx$$

input `integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`output `integrate(1/((b*sech(d*sqrt(x) + c) + a)*x^(3/2)), x)`**3.65.9 Mupad [N/A]**

Not integrable

Time = 2.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{x^{3/2} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)} dx$$

input `int(1/(x^(3/2)*(a + b/cosh(c + d*x^(1/2))))),x)`output `int(1/(x^(3/2)*(a + b/cosh(c + d*x^(1/2))))), x)`

**3.66**  $\int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$

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**3.66.1 Optimal result**

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx = \operatorname{Int}\left(\frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))}, x\right)$$

output `Unintegrable(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x)`

**3.66.2 Mathematica [N/A]**

Not integrable

Time = 10.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx = \int \frac{1}{x^{5/2}(a+b\operatorname{sech}(c+d\sqrt{x}))} dx$$

input `Integrate[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])),x]`

output `Integrate[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])), x]`

**3.66.3 Rubi [N/A]**

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

↓ 5961

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

input `Int[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])),x]`

output `$Aborted`

**3.66.3.1 Defintions of rubi rules used**

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

**3.66.4 Maple [N/A] (verified)**

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

input `int(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x)`

output `int(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x)`

---

3.66.  $\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$



**3.66.5 Fracas [N/A]**

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a) x^{\frac{5}{2}}} dx$$

```
input integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="fricas")
```

```
output integral(sqrt(x)/(b*x^3*sech(d*sqrt(x) + c) + a*x^3), x)
```

**3.66.6 Sympy [N/A]**

Not integrable

Time = 6.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{x^{\frac{5}{2}} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$$

```
input integrate(1/x**(5/2)/(a+b*sech(c+d*x**(1/2))),x)
```

```
output Integral(1/(x**(5/2)*(a + b*sech(c + d*sqrt(x)))), x)
```

**3.66.7 Maxima [N/A]**

Not integrable

Time = 0.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a) x^{\frac{5}{2}}} dx$$

```
input integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="maxima")
```

```
output -2*b*integrate(e^(d*sqrt(x) + c)/(a^2*x^(5/2)*e^(2*d*sqrt(x) + 2*c) + 2*a*
b*x^(5/2)*e^(d*sqrt(x) + c) + a^2*x^(5/2)), x) - 2/3/(a*x^(3/2))
```

---

3.66.  $\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx$

**3.66.8 Giac [N/A]**

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a) x^{5/2}} dx$$

input `integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2))),x, algorithm="giac")`output `integrate(1/((b*sech(d*sqrt(x) + c) + a)*x^(5/2)), x)`**3.66.9 Mupad [N/A]**

Not integrable

Time = 2.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))} dx = \int \frac{1}{x^{5/2} \left( a + \frac{b}{\cosh(c + d\sqrt{x})} \right)} dx$$

input `int(1/(x^(5/2)*(a + b/cosh(c + d*x^(1/2))))),x)`output `int(1/(x^(5/2)*(a + b/cosh(c + d*x^(1/2))))), x)`

**3.67** 
$$\int \frac{x^{3/2}}{\left(a+b\operatorname{sech}(c+d\sqrt{x})\right)^2} dx$$

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**3.67.1 Optimal result**

Integrand size = 22, antiderivative size = 1755

$$\int \frac{x^{3/2}}{\left(a+b\operatorname{sech}(c+d\sqrt{x})\right)^2} dx = \text{Too large to display}$$

```
output 2*b^2*x^2*sinh(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cosh(c+d*x^(1/2)))+2/5*x^(5
/2)/a^2-4*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+
b^2)^(1/2)+4*b*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a
^2+b^2)^(1/2)-16*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/
2)))/a^2/d^2/(-a^2+b^2)^(1/2)+16*b*x^(3/2)*polylog(2,-a*exp(c+d*x^(1/2)))/(
b+(-a^2+b^2)^(1/2))/a^2/d^2/(-a^2+b^2)^(1/2)+48*b*x*polylog(3,-a*exp(c+d*x
^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-48*b*x*polylog(3,-
a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)+48*b^2*p
olylog(3,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d
^4+48*b^2*polylog(3,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/
(a^2-b^2)/d^4+48*b^3*polylog(4,-a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x
^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^4-48*b^3*polylog(4,-a*exp(c+d*x^(1/2))/(b+(-
a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^4-96*b*polylog(4,-a*exp(c+
d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^4/(-a^2+b^2)^(1/2)+96*b*pol
ylog(4,-a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^4/(-a^2+b^2
)^(1/2)-8*b^2*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(a
^2-b^2)/d^2+2*b^3*x^2*ln(1+a*exp(c+d*x^(1/2))/(b-(-a^2+b^2)^(1/2)))/a^2/(-
a^2+b^2)^(3/2)/d-8*b^2*x^(3/2)*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2
)))/a^2/(a^2-b^2)/d^2-2*b^3*x^2*ln(1+a*exp(c+d*x^(1/2))/(b+(-a^2+b^2)^(1/2
)))/a^2/(-a^2+b^2)^(3/2)/d-24*b^2*x*polylog(2,-a*exp(c+d*x^(1/2))/(b-(-a...
```

3.67. 
$$\int \frac{x^{3/2}}{\left(a+b\operatorname{sech}(c+d\sqrt{x})\right)^2} dx$$

### 3.67.2 Mathematica [A] (verified)

Time = 8.27 (sec) , antiderivative size = 1769, normalized size of antiderivative = 1.01

$$\int \frac{x^{3/2}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

input `Integrate[x^(3/2)/(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output

```
(2*(b + a*Cosh[c + d*Sqrt[x]])*Sech[c + d*Sqrt[x]]^2*(x^(5/2)*(b + a*Cosh[
c + d*Sqrt[x]]) + (5*b*E^c*(b + a*Cosh[c + d*Sqrt[x]])*(2*b*E^c*x^2 - ((1
+ E^(2*c))*(4*b*d^3*Sqrt[(-a^2 + b^2)*E^(2*c)])*x^(3/2)*Log[1 + (a*E^(2*c +
d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] + 2*a^2*d^4*E^c*x^2*Log
[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])] - b^2*d
^4*E^c*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2
*c)])] + 4*b*d^3*Sqrt[(-a^2 + b^2)*E^(2*c)]*x^(3/2)*Log[1 + (a*E^(2*c + d*
Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] - 2*a^2*d^4*E^c*x^2*Log[1
+ (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])] + b^2*d^4*
E^c*x^2*Log[1 + (a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)
])] + 4*d^2*(3*b*Sqrt[(-a^2 + b^2)*E^(2*c)] + 2*a^2*d*E^c*Sqrt[x] - b^2*d*
E^c*Sqrt[x])*x*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 +
b^2)*E^(2*c)])]) + 4*d^2*(3*b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqr
t[x] + b^2*d*E^c*Sqrt[x])*x*PolyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c +
Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 24*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*
PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])
)] - 24*a^2*d^2*E^c*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-
a^2 + b^2)*E^(2*c)])]) + 12*b^2*d^2*E^c*x*PolyLog[3, -((a*E^(2*c + d*Sqrt[
x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])]) - 24*b*d*Sqrt[(-a^2 + b^2)*E^(
2*c)]*Sqrt[x]*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 ...
```

### 3.67.3 Rubi [A] (verified)

Time = 3.03 (sec) , antiderivative size = 1754, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

---

3.67.  $\int \frac{x^{3/2}}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$

$$\begin{aligned}
& \int \frac{x^{3/2}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx \\
& \quad \downarrow \text{5959} \\
& 2 \int \frac{x^2}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} d\sqrt{x} \\
& \quad \downarrow \text{3042} \\
& 2 \int \frac{x^2}{(a + b \csc(ic + id\sqrt{x} + \frac{\pi}{2}))^2} d\sqrt{x} \\
& \quad \downarrow \text{4679} \\
& 2 \int \left( -\frac{2bx^2}{a^2(b + a \cosh(c + d\sqrt{x}))} + \frac{x^2}{a^2} + \frac{b^2x^2}{a^2(b + a \cosh(c + d\sqrt{x}))^2} \right) d\sqrt{x} \\
& \quad \downarrow \text{2009} \\
& 2 \left( \frac{x^2 \log\left(\frac{e^{c+d\sqrt{x}}a}{b-\sqrt{b^2-a^2}} + 1\right) b^3}{a^2(b^2 - a^2)^{3/2} d} - \frac{x^2 \log\left(\frac{e^{c+d\sqrt{x}}a}{b+\sqrt{b^2-a^2}} + 1\right) b^3}{a^2(b^2 - a^2)^{3/2} d} + \frac{4x^{3/2} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{b^2-a^2}}\right) b^3}{a^2(b^2 - a^2)^{3/2} d^2} - \frac{4x^{3/2} \operatorname{PolyLog}\left(2, \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{b^2-a^2}}\right) b^3}{a^2(b^2 - a^2)^{3/2} d^2} \right)
\end{aligned}$$

input `Int[x^(3/2)/(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output  $2*((b^2*x^2)/(a^2*(a^2 - b^2)*d) + x^{(5/2)}/(5*a^2) - (4*b^2*x^{(3/2)}*Log[1 + (a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])]/(a^2*(a^2 - b^2)*d^2) + (b^3*x^2*Log[1 + (a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])]/(a^2*(-a^2 + b^2)^{(3/2)}*d) - (2*b*x^2*Log[1 + (a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])]/(a^2*Sqrt[-a^2 + b^2]*d) - (4*b^2*x^{(3/2)}*Log[1 + (a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])]/(a^2*(a^2 - b^2)*d^2) - (b^3*x^2*Log[1 + (a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])]/(a^2*(-a^2 + b^2)^{(3/2)}*d) + (2*b*x^2*Log[1 + (a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])]/(a^2*Sqrt[-a^2 + b^2]*d) - (12*b^2*x*PolyLog[2, -((a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])])]/(a^2*(a^2 - b^2)*d^3) + (4*b^3*x^{(3/2)}*PolyLog[2, -((a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])])]/(a^2*(-a^2 + b^2)^{(3/2)}*d^2) - (8*b*x^{(3/2)}*PolyLog[2, -((a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])])]/(a^2*Sqrt[-a^2 + b^2]*d^2) - (12*b^2*x*PolyLog[2, -((a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])])]/(a^2*(a^2 - b^2)*d^3) - (4*b^3*x^{(3/2)}*PolyLog[2, -((a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])])]/(a^2*(-a^2 + b^2)^{(3/2)}*d^2) + (8*b*x^{(3/2)}*PolyLog[2, -((a*E^{(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])])]/(a^2*Sqrt[-a^2 + b^2]*d^2) + (24*b^2*Sqrt[x]*PolyLog[3, -((a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])])]/(a^2*(a^2 - b^2)*d^4) - (12*b^3*x*PolyLog[3, -((a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])])]/(a^2*(-a^2 + b^2)^{(3/2)}*d^3) + (24*b*x*PolyLog[3, -((a*E^{(c + d*Sqrt[x]))/(b - Sqrt[-a...$

### 3.67.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

$$3.67. \int \frac{x^{3/2}}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

**3.67.4 Maple [F]**

$$\int \frac{x^{\frac{3}{2}}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `int(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

**3.67.5 Fricas [F]**

$$\int \frac{x^{3/2}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^{\frac{3}{2}}}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(x^(3/2)/(b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2), x)`

**3.67.6 Sympy [F]**

$$\int \frac{x^{3/2}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^{\frac{3}{2}}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `integrate(x**(3/2)/(a+b*sech(c+d*x**(1/2)))**2,x)`

output `Integral(x**(3/2)/(a + b*sech(c + d*sqrt(x)))**2, x)`

**3.67.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{x^{3/2}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

**3.67.8 Giac [F]**

$$\int \frac{x^{3/2}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^{3/2}}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(x^(3/2)/(b*sech(d*sqrt(x) + c) + a)^2, x)`

**3.67.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{x^{3/2}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{x^{3/2}}{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2} dx$$

input `int(x^(3/2)/(a + b/cosh(c + d*x^(1/2)))^2,x)`

output `int(x^(3/2)/(a + b/cosh(c + d*x^(1/2)))^2, x)`



$$3.68 \quad \int \frac{\sqrt{x}}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

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## 3.68.1 Optimal result

Integrand size = 22, antiderivative size = 1027

$$\begin{aligned}
\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx &= \frac{2b^2 x}{a^2 (a^2 - b^2) d} + \frac{2x^{3/2}}{3a^2} - \frac{4b^2 \sqrt{x} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 (a^2 - b^2) d^2} \\
&+ \frac{2b^3 x \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d} - \frac{4bx \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d} \\
&- \frac{4b^2 \sqrt{x} \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 (a^2 - b^2) d^2} - \frac{2b^3 x \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d} \\
&+ \frac{4bx \log\left(1 + \frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d} - \frac{4b^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 (a^2 - b^2) d^3} \\
&+ \frac{4b^3 \sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^2} \\
&- \frac{8b\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^2} \\
&- \frac{4b^2 \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 (a^2 - b^2) d^3} \\
&- \frac{4b^3 \sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^2} \\
&+ \frac{8b\sqrt{x} \operatorname{PolyLog}\left(2, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^2} \\
&- \frac{4b^3 \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^3} \\
&+ \frac{8b \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b-\sqrt{-a^2+b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^3} \\
&+ \frac{4b^3 \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 (-a^2 + b^2)^{3/2} d^3} \\
&- \frac{8b \operatorname{PolyLog}\left(3, -\frac{ae^{c+d\sqrt{x}}}{b+\sqrt{-a^2+b^2}}\right)}{a^2 \sqrt{-a^2 + b^2} d^3} \\
&+ \frac{2b^2 x \sinh(c + d\sqrt{x})}{a (a^2 - b^2) d (b + a \cosh(c + d\sqrt{x}))}
\end{aligned}$$

---

3.68.  $\int \frac{\sqrt{x}}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$

```
output 2*b^2*x/a^2/(a^2-b^2)/d+2/3*x^(3/2)/a^2+2*b^3*x*ln(1+a*exp(c+d*x^(1/2)))/(b
-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d-2*b^3*x*ln(1+a*exp(c+d*x^(1/2))
/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d-4*b^2*polylog(2,-a*exp(c+d*x
^(1/2)))/(b-(-a^2+b^2)^(1/2))/a^2/(a^2-b^2)/d^3-4*b^2*polylog(2,-a*exp(c+d
*x^(1/2)))/(b+(-a^2+b^2)^(1/2))/a^2/(a^2-b^2)/d^3-4*b^3*polylog(3,-a*exp(c
+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^3+4*b^3*polylog(3
,-a*exp(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2))/a^2/(-a^2+b^2)^(3/2)/d^3+2*b^2*
x*sinh(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*cosh(c+d*x^(1/2)))-4*b*x*ln(1+a*exp
(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))/a^2/d/(-a^2+b^2)^(1/2)+4*b*x*ln(1+a*ex
p(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2))/a^2/d/(-a^2+b^2)^(1/2)+8*b*polylog(3,
-a*exp(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))/a^2/d^3/(-a^2+b^2)^(1/2)-8*b*pol
ylog(3,-a*exp(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2))/a^2/d^3/(-a^2+b^2)^(1/2)-
4*b^2*ln(1+a*exp(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))*x^(1/2)/a^2/(a^2-b^2)/
d^2-4*b^2*ln(1+a*exp(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2))*x^(1/2)/a^2/(a^2-b
^2)/d^2+4*b^3*polylog(2,-a*exp(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2))*x^(1/2)/
a^2/(-a^2+b^2)^(3/2)/d^2-4*b^3*polylog(2,-a*exp(c+d*x^(1/2)))/(b+(-a^2+b^2)
^(1/2))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^2-8*b*polylog(2,-a*exp(c+d*x^(1/2)
))/(b-(-a^2+b^2)^(1/2))*x^(1/2)/a^2/d^2/(-a^2+b^2)^(1/2)+8*b*polylog(2,-a*
exp(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2))*x^(1/2)/a^2/d^2/(-a^2+b^2)^(1/2)
```

### 3.68.2 Mathematica [A] (verified)

Time = 5.15 (sec) , antiderivative size = 986, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

$$= \frac{2(b + a \cosh(c + d\sqrt{x})) \operatorname{sech}^2(c + d\sqrt{x})}{x^{3/2}(b + a \cosh(c + d\sqrt{x}))} + \frac{3be^c(b + a \cosh(c + d\sqrt{x}))}{2be^c x - \frac{e^{-c}(1 + e^{2c + 2d\sqrt{x}})}{2be^c}}$$

```
input Integrate[Sqrt[x]/(a + b*Sech[c + d*Sqrt[x]])^2,x]
```

output

```
(2*(b + a*Cosh[c + d*Sqrt[x]])*Sech[c + d*Sqrt[x]]^2*(x^(3/2)*(b + a*Cosh[
c + d*Sqrt[x])) + (3*b*E^c*(b + a*Cosh[c + d*Sqrt[x]])*(2*b*E^c*x - ((1 +
E^(2*c))*(2*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*Log[1 + (a*E^(2*c + d*S
qrt[x]))]/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])) + 2*a^2*d^2*E^c*x*Log[1 + (
a*E^(2*c + d*Sqrt[x]))]/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])) - b^2*d^2*E^c
*x*Log[1 + (a*E^(2*c + d*Sqrt[x]))]/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)])) +
2*b*d*Sqrt[(-a^2 + b^2)*E^(2*c)]*Sqrt[x]*Log[1 + (a*E^(2*c + d*Sqrt[x]))]/
(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])) - 2*a^2*d^2*E^c*x*Log[1 + (a*E^(2*c
+ d*Sqrt[x]))]/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])) + b^2*d^2*E^c*x*Log[1
+ (a*E^(2*c + d*Sqrt[x]))]/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])) + 2*(b*Sqr
t[(-a^2 + b^2)*E^(2*c)] + 2*a^2*d*E^c*Sqrt[x] - b^2*d*E^c*Sqrt[x])*PolyLog
[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2)*E^(2*c)]))] + 2*(
b*Sqrt[(-a^2 + b^2)*E^(2*c)] - 2*a^2*d*E^c*Sqrt[x] + b^2*d*E^c*Sqrt[x])*Po
lyLog[2, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))]
- 4*a^2*E^c*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sqrt[(-a^2 + b^2
)*E^(2*c)]))] + 2*b^2*E^c*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]))/(b*E^c - Sq
rt[(-a^2 + b^2)*E^(2*c)]))] + 4*a^2*E^c*PolyLog[3, -((a*E^(2*c + d*Sqrt[x]
))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)]))] - 2*b^2*E^c*PolyLog[3, -((a*E^(2
*c + d*Sqrt[x]))/(b*E^c + Sqrt[(-a^2 + b^2)*E^(2*c)])))]/(d^2*E^c*Sqrt[(-
a^2 + b^2)*E^(2*c)])))/((a^2 - b^2)*d*(1 + E^(2*c))) + (3*b^2*x*Sech[c]...
```

### 3.68.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 1026, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

$$\downarrow \text{5959}$$

$$2 \int \frac{x}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} d\sqrt{x}$$

$$\downarrow \text{3042}$$

$$2 \int \frac{x}{(a + b \operatorname{csc}(ic + id\sqrt{x} + \frac{\pi}{2}))^2} d\sqrt{x}$$

$$\downarrow \text{4679}$$

---

3.68.  $\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$

$$2 \int \left( \frac{xb^2}{a^2 (b + a \cosh(c + d\sqrt{x}))^2} - \frac{2xb}{a^2 (b + a \cosh(c + d\sqrt{x}))} + \frac{x}{a^2} \right) d\sqrt{x}$$

↓ 2009

$$2 \left( \frac{x \log \left( \frac{e^{c+d\sqrt{x}} a}{b - \sqrt{b^2 - a^2}} + 1 \right) b^3}{a^2 (b^2 - a^2)^{3/2} d} - \frac{x \log \left( \frac{e^{c+d\sqrt{x}} a}{b + \sqrt{b^2 - a^2}} + 1 \right) b^3}{a^2 (b^2 - a^2)^{3/2} d} + \frac{2\sqrt{x} \text{PolyLog} \left( 2, -\frac{ae^{c+d\sqrt{x}}}{b - \sqrt{b^2 - a^2}} \right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} - \frac{2\sqrt{x} \text{PolyLog} \left( 2, -\frac{ae^{c+d\sqrt{x}}}{b + \sqrt{b^2 - a^2}} \right) b^3}{a^2 (b^2 - a^2)^{3/2} d^2} \right)$$

input `Int[Sqrt[x]/(a + b*Sech[c + d*Sqrt[x]])^2,x]`

output

```

2*((b^2*x)/(a^2*(a^2 - b^2)*d) + x^(3/2)/(3*a^2) - (2*b^2*Sqrt[x]*Log[1 +
(a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) + (b^
3*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^
2)^(3/2)*d) - (2*b*x*Log[1 + (a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2])])
)/(a^2*Sqrt[-a^2 + b^2]*d) - (2*b^2*Sqrt[x]*Log[1 + (a*E^(c + d*Sqrt[x]))/
(b + Sqrt[-a^2 + b^2])])/(a^2*(a^2 - b^2)*d^2) - (b^3*x*Log[1 + (a*E^(c +
d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + (2*b*x*L
og[1 + (a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^
2]*d) - (2*b^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))]
)/(a^2*(a^2 - b^2)*d^3) + (2*b^3*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))
/(b - Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (4*b*Sqrt[x]*Pol
yLog[2, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 +
b^2]*d^2) - (2*b^2*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^
2]))])/(a^2*(a^2 - b^2)*d^3) - (2*b^3*Sqrt[x]*PolyLog[2, -((a*E^(c + d*Sqr
t[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (4*b*Sqrt[
x]*PolyLog[2, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[
-a^2 + b^2]*d^2) - (2*b^3*PolyLog[3, -((a*E^(c + d*Sqrt[x]))/(b - Sqrt[-a^
2 + b^2]))])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + (4*b*PolyLog[3, -((a*E^(c + d*
Sqrt[x]))/(b - Sqrt[-a^2 + b^2]))])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (2*b^3*Po
lyLog[3, -((a*E^(c + d*Sqrt[x]))/(b + Sqrt[-a^2 + b^2]))])/(a^2*(-a^2 + ...

```

### 3.68.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

---

3.68.  $\int \frac{\sqrt{x}}{(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :=> Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

### 3.68.4 Maple [F]

$$\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `int(x^(1/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(x^(1/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

### 3.68.5 Fricas [F]

$$\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{\sqrt{x}}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`

output `integral(sqrt(x)/(b^2*sech(d*sqrt(x) + c)^2 + 2*a*b*sech(d*sqrt(x) + c) + a^2), x)`

**3.68.6 Sympy [F]**

$$\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `integrate(x**(1/2)/(a+b*sech(c+d*x**(1/2)))**2,x)`

output `Integral(sqrt(x)/(a + b*sech(c + d*sqrt(x)))**2, x)`

**3.68.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

input `integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

output `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more details)Is`

**3.68.8 Giac [F]**

$$\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{\sqrt{x}}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2} dx$$

input `integrate(x^(1/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(sqrt(x)/(b*sech(d*sqrt(x) + c) + a)^2, x)`

**3.68.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{\sqrt{x}}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{\sqrt{x}}{\left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2} dx$$

input `int(x^(1/2)/(a + b/cosh(c + d*x^(1/2)))^2,x)`output `int(x^(1/2)/(a + b/cosh(c + d*x^(1/2)))^2, x)`



**3.69** 
$$\int \frac{1}{\sqrt{x} \left( a + b \operatorname{sech}(c + d\sqrt{x}) \right)^2} dx$$

3.69.1	Optimal result	400
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**3.69.1 Optimal result**

Integrand size = 22, antiderivative size = 127

$$\int \frac{1}{\sqrt{x} \left( a + b \operatorname{sech}(c + d\sqrt{x}) \right)^2} dx = \frac{2\sqrt{x}}{a^2} - \frac{4b(2a^2 - b^2) \arctan\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{2b^2 \tanh(c + d\sqrt{x})}{a(a^2 - b^2)d(a + b \operatorname{sech}(c + d\sqrt{x}))}$$

output

```
-4*b*(2*a^2-b^2)*arctan((a-b)^(1/2)*tanh(1/2*c+1/2*d*x^(1/2))/(a+b)^(1/2))
/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+2*x^(1/2)/a^2+2*b^2*tanh(c+d*x^(1/2))/a/(a^
2-b^2)/d/(a+b*sech(c+d*x^(1/2)))
```

**3.69.2 Mathematica [A] (verified)**

Time = 0.56 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.83

$$\int \frac{1}{\sqrt{x} \left( a + b \operatorname{sech}(c + d\sqrt{x}) \right)^2} dx = \frac{2 \left( a \left( (a^2 - b^2)^{3/2} (c + d\sqrt{x}) + (4a^2b - 2b^3) \arctan\left(\frac{(-a+b) \tanh\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a^2-b^2}}\right) \right) \cosh(c + d\sqrt{x}) + b \left( (a^2 - b^2)^{3/2} \right) \right)}{a^2(a-b)(a+b)\sqrt{a^2-b^2}d(b + \dots)}$$

input

```
Integrate[1/(Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])^2),x]
```

---

3.69. 
$$\int \frac{1}{\sqrt{x} \left( a + b \operatorname{sech}(c + d\sqrt{x}) \right)^2} dx$$

```
output (2*(a*((a^2 - b^2)^(3/2)*(c + d*Sqrt[x]) + (4*a^2*b - 2*b^3)*ArcTan[((-a +
b)*Tanh[(c + d*Sqrt[x])/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*Sqrt[x]] + b*((a
^2 - b^2)^(3/2)*(c + d*Sqrt[x]) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[
(c + d*Sqrt[x])/2])/Sqrt[a^2 - b^2]] + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*Sqrt
[x]])))/(a^2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d*(b + a*Cosh[c + d*Sqrt[x]]
)
```

### 3.69.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {5959, 3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx \\
 & \quad \downarrow \text{5959} \\
 & 2 \int \frac{1}{(a + b \operatorname{sech}(c + d\sqrt{x}))^2} d\sqrt{x} \\
 & \quad \downarrow \text{3042} \\
 & 2 \int \frac{1}{(a + b \csc(ic + id\sqrt{x} + \frac{\pi}{2}))^2} d\sqrt{x} \\
 & \quad \downarrow \text{4272} \\
 & 2 \left( \frac{b^2 \tanh(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \operatorname{sech}(c + d\sqrt{x}))} - \frac{\int -\frac{a^2 - b \operatorname{sech}(c + d\sqrt{x}) a - b^2}{a + b \operatorname{sech}(c + d\sqrt{x})} d\sqrt{x}}{a(a^2 - b^2)} \right) \\
 & \quad \downarrow \text{25} \\
 & 2 \left( \frac{\int \frac{a^2 - b \operatorname{sech}(c + d\sqrt{x}) a - b^2}{a + b \operatorname{sech}(c + d\sqrt{x})} d\sqrt{x}}{a(a^2 - b^2)} + \frac{b^2 \tanh(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \operatorname{sech}(c + d\sqrt{x}))} \right) \\
 & \quad \downarrow \text{3042} \\
 & 2 \left( \frac{b^2 \tanh(c + d\sqrt{x})}{ad(a^2 - b^2)(a + b \operatorname{sech}(c + d\sqrt{x}))} + \frac{\int \frac{a^2 - b \csc(ic + id\sqrt{x} + \frac{\pi}{2}) a - b^2}{a + b \csc(ic + id\sqrt{x} + \frac{\pi}{2})} d\sqrt{x}}{a(a^2 - b^2)} \right)
 \end{aligned}$$

---

3.69.  $\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$

$$\begin{aligned}
& \downarrow 4407 \\
& 2 \left( \frac{\frac{\sqrt{x}(a^2-b^2)}{a} - \frac{b(2a^2-b^2) \int \frac{\operatorname{sech}(c+d\sqrt{x})}{a+b\operatorname{sech}(c+d\sqrt{x})} d\sqrt{x}}{a}}{a(a^2-b^2)} + \frac{b^2 \tanh(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\operatorname{sech}(c+d\sqrt{x}))} \right) \\
& \downarrow 3042 \\
& 2 \left( \frac{b^2 \tanh(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\operatorname{sech}(c+d\sqrt{x}))} + \frac{\frac{\sqrt{x}(a^2-b^2)}{a} - \frac{b(2a^2-b^2) \int \frac{\csc(ic+id\sqrt{x}+\frac{\pi}{2})}{a+b\csc(ic+id\sqrt{x}+\frac{\pi}{2})} d\sqrt{x}}{a}}{a(a^2-b^2)} \right) \\
& \downarrow 4318 \\
& 2 \left( \frac{\frac{\sqrt{x}(a^2-b^2)}{a} - \frac{(2a^2-b^2) \int \frac{1}{a \cosh(\frac{c+d\sqrt{x}}{b}) + 1} d\sqrt{x}}{a}}{a(a^2-b^2)} + \frac{b^2 \tanh(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\operatorname{sech}(c+d\sqrt{x}))} \right) \\
& \downarrow 3042 \\
& 2 \left( \frac{b^2 \tanh(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\operatorname{sech}(c+d\sqrt{x}))} + \frac{\frac{\sqrt{x}(a^2-b^2)}{a} - \frac{(2a^2-b^2) \int \frac{1}{a \sin(\frac{ic+id\sqrt{x}+\frac{\pi}{2}}{b}) + 1} d\sqrt{x}}{a}}{a(a^2-b^2)} \right) \\
& \downarrow 3138 \\
& 2 \left( \frac{b^2 \tanh(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\operatorname{sech}(c+d\sqrt{x}))} + \frac{\frac{\sqrt{x}(a^2-b^2)}{a} + \frac{2i(2a^2-b^2) \int \frac{1}{\frac{a+b}{b} + (1-\frac{a}{b})x} d(i \tanh(\frac{1}{2}(c+d\sqrt{x})))}{ad}}{a(a^2-b^2)} \right) \\
& \downarrow 221 \\
& 2 \left( \frac{\frac{\sqrt{x}(a^2-b^2)}{a} - \frac{2b(2a^2-b^2) \arctan\left(\frac{\sqrt{a-b} \tanh(\frac{1}{2}(c+d\sqrt{x}))}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)} + \frac{b^2 \tanh(c+d\sqrt{x})}{ad(a^2-b^2)(a+b\operatorname{sech}(c+d\sqrt{x}))} \right)
\end{aligned}$$

input `Int[1/(Sqrt[x]*(a + b*Sech[c + d*Sqrt[x]])^2),x]`

```
output 2*(((a^2 - b^2)*Sqrt[x])/a - (2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[
(c + d*Sqrt[x])/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(a*(a^2 -
b^2)) + (b^2*Tanh[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(a + b*Sech[c + d*Sqrt
[x]])))
```

### 3.69.3.1 Defintions of rubi rules used

```
rule 25 Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]
```

```
rule 221 Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

```
rule 3138 Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

```
rule 4272 Int[(csc[(c_) + (d_)*(x_)])*(b_) + (a_)^(n_), x_Symbol] := Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Sim
p[1/(a*(n + 1)*(a^2 - b^2)) Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 -
b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x
], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Integ
erQ[2*n]
```

```
rule 4318 Int[csc[(e_) + (f_)*(x_)]/(csc[(e_) + (f_)*(x_)])*(b_) + (a_), x_Symbo
l] := Simp[1/b Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

```
rule 4407 Int[(csc[(e_) + (f_)*(x_)])*(d_) + (c_)/(csc[(e_) + (f_)*(x_)])*(b_) +
(a_), x_Symbol] := Simp[c*(x/a), x] - Simp[(b*c - a*d)/a Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

---


$$3.69. \int \frac{1}{\sqrt{x}(a+b\operatorname{sech}(c+d\sqrt{x}))^2} dx$$

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
:= Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

### 3.69.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{-\frac{2 \ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right) - 1\right)}{a^2}}{d} - \frac{4b \left( \frac{ab \tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{(a^2 - b^2) \left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)^2 a - \tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)^2 b + a + b} + \frac{(2a^2 - b^2) \arctan\left(\frac{(a-b) \tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}} \right)}{a^2}$
default	$\frac{-\frac{2 \ln\left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right) - 1\right)}{a^2}}{d} - \frac{4b \left( \frac{ab \tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{(a^2 - b^2) \left(\tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)\right)^2 a - \tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)^2 b + a + b} + \frac{(2a^2 - b^2) \arctan\left(\frac{(a-b) \tanh\left(\frac{c}{2} + \frac{d\sqrt{x}}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}} \right)}{a^2}$

```
input int(1/(a+b*sech(c+d*x^(1/2)))^2/x^(1/2), x, method=_RETURNVERBOSE)
```

```
output 2/d*(-1/a^2*ln(tanh(1/2*c+1/2*d*x^(1/2))-1)-2/a^2*b*(-a*b/(a^2-b^2)*tanh(1/2*c+1/2*d*x^(1/2))/(tanh(1/2*c+1/2*d*x^(1/2))^2*a-tanh(1/2*c+1/2*d*x^(1/2))^2*b+a+b)+(2*a^2-b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*c+1/2*d*x^(1/2))/((a+b)*(a-b))^(1/2)))+1/a^2*ln(tanh(1/2*c+1/2*d*x^(1/2))+1))
```

### 3.69.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(110) = 220.

Time = 0.30 (sec) , antiderivative size = 1387, normalized size of antiderivative = 10.92

$$\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Too large to display}$$

```
input integrate(1/(a+b*sech(c+d*x^(1/2)))^2/x^(1/2), x, algorithm="fracas")
```

---

3.69.  $\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$

output

```

[-2*(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*cosh(d*sqrt(x) + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*sinh(d*sqrt(x) + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x) + 2*(a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*sqrt(x))*cosh(d*sqrt(x) + c) + ((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2))*cosh(d*sqrt(x) + c)^2 + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*sinh(d*sqrt(x) + c)^2 + 2*(2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*cosh(d*sqrt(x) + c) + 2*((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2))*cosh(d*sqrt(x) + c) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*sinh(d*sqrt(x) + c) + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2))*log((a*b + (b^2 + sqrt(-a^2 + b^2))*b)*cosh(d*sqrt(x) + c) + (a^2 - b^2 - sqrt(-a^2 + b^2))*b)*sinh(d*sqrt(x) + c) + sqrt(-a^2 + b^2)*a)/(a*cosh(d*sqrt(x) + c) + b)) + 2*(a^2*b^3 - b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x))*cosh(d*sqrt(x) + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*sqrt(x))*sinh(d*sqrt(x) + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*sqrt(x) + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^4)*d*sinh(d*sqrt(x) + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cosh(d*sqrt(x) + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*sqrt(x) + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*sqrt(x) + c)), -2*(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x))*cosh(d*sqrt(x) + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x))*sinh(d*sqrt(x) + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x) - 2*((2*a^3*b - a*b^3)*sqrt(a^2 - b^2))*cosh(d*sqrt(x) + c)^2 + (2*a^3*b - a*b^3)*sqrt(a^2 - ...

```

### 3.69.6 Sympy [F]

$$\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `integrate(1/(a+b*sech(c+d*x**(1/2)))*2/x**(1/2),x)`

output `Integral(1/(sqrt(x)*(a + b*sech(c + d*sqrt(x)))*2), x)`

**3.69.7 Maxima [F(-2)]**

Exception generated.

$$\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \text{Exception raised: ValueError}$$

```
input integrate(1/(a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")
```

```
output Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

**3.69.8 Giac [A] (verification not implemented)**

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = -\frac{4(2a^2b - b^3) \arctan\left(\frac{ae^{(d\sqrt{x}+c)}+b}{\sqrt{a^2-b^2}}\right)}{(a^4d - a^2b^2d)\sqrt{a^2 - b^2}} - \frac{4(b^3e^{(d\sqrt{x}+c)} + ab^2)}{(a^4d - a^2b^2d)(ae^{(2d\sqrt{x}+2c)} + 2be^{(d\sqrt{x}+c)} + a)} + \frac{2(d\sqrt{x} + c)}{a^2d}$$

```
input integrate(1/(a+b*sech(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")
```

```
output -4*(2*a^2*b - b^3)*arctan((a*e^(d*sqrt(x) + c) + b)/sqrt(a^2 - b^2))/((a^4
*d - a^2*b^2*d)*sqrt(a^2 - b^2)) - 4*(b^3*e^(d*sqrt(x) + c) + a*b^2)/((a^4
*d - a^2*b^2*d)*(a*e^(2*d*sqrt(x) + 2*c) + 2*b*e^(d*sqrt(x) + c) + a)) + 2
*(d*sqrt(x) + c)/(a^2*d)
```

**3.69.9 Mupad [B] (verification not implemented)**

Time = 2.50 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.71

$$\int \frac{1}{\sqrt{x} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

$$= \frac{2\sqrt{x}}{a^2} - \frac{\frac{4b^2\sqrt{x}}{d(a^3\sqrt{x}-ab^2\sqrt{x})} + \frac{4b^3\sqrt{x}e^{c+d\sqrt{x}}}{ad(a^3\sqrt{x}-ab^2\sqrt{x})}}{a + 2be^{c+d\sqrt{x}} + ae^{2c+2d\sqrt{x}}}$$

$$+ \frac{\ln\left(\frac{2e^{c+d\sqrt{x}}(2a^2b-b^3)}{a^3\sqrt{x}(a^2-b^2)} - \frac{(4a^2b-2b^3)(a+be^{c+d\sqrt{x}})}{a^3\sqrt{x}(a+b)^{3/2}(b-a)^{3/2}}\right) (4a^2b - 2b^3)}{a^2 d (a+b)^{3/2} (b-a)^{3/2}}$$

$$- \frac{2b \ln\left(\frac{2e^{c+d\sqrt{x}}(2a^2b-b^3)}{a^3\sqrt{x}(a^2-b^2)} + \frac{2b(a+be^{c+d\sqrt{x}})(2a^2-b^2)}{a^3\sqrt{x}(a+b)^{3/2}(b-a)^{3/2}}\right) (2a^2 - b^2)}{a^2 d (a+b)^{3/2} (b-a)^{3/2}}$$

input `int(1/(x^(1/2)*(a + b/cosh(c + d*x^(1/2)))^2),x)`

output

```
(2*x^(1/2))/a^2 - ((4*b^2*x^(1/2))/(d*(a^3*x^(1/2) - a*b^2*x^(1/2))) + (4*b^3*x^(1/2)*exp(c + d*x^(1/2)))/(a*d*(a^3*x^(1/2) - a*b^2*x^(1/2))))/(a + 2*b*exp(c + d*x^(1/2)) + a*exp(2*c + 2*d*x^(1/2))) + (log((2*exp(c + d*x^(1/2)))*(2*a^2*b - b^3))/(a^3*x^(1/2)*(a^2 - b^2)) - ((4*a^2*b - 2*b^3)*(a + b*exp(c + d*x^(1/2)))/(a^3*x^(1/2)*(a + b)^(3/2)*(b - a)^(3/2))))*(4*a^2*b - 2*b^3)/(a^2*d*(a + b)^(3/2)*(b - a)^(3/2)) - (2*b*log((2*exp(c + d*x^(1/2)))*(2*a^2*b - b^3))/(a^3*x^(1/2)*(a^2 - b^2)) + (2*b*(a + b*exp(c + d*x^(1/2)))*(2*a^2 - b^2))/(a^3*x^(1/2)*(a + b)^(3/2)*(b - a)^(3/2)))*(2*a^2 - b^2)/(a^2*d*(a + b)^(3/2)*(b - a)^(3/2))
```



**3.70** 
$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

3.70.1	Optimal result	408
3.70.2	Mathematica [N/A]	408
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3.70.4	Maple [N/A] (verified)	409
3.70.5	Fricas [N/A]	410
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3.70.7	Maxima [N/A]	410
3.70.8	Giac [N/A]	411
3.70.9	Mupad [N/A]	411

**3.70.1 Optimal result**

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \operatorname{Int}\left(\frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2}, x\right)$$

output `Unintegrable(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

**3.70.2 Mathematica [N/A]**

Not integrable

Time = 81.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `Integrate[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2),x]`

output `Integrate[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2), x]`

### 3.70.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

↓ 5961

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `Int[1/(x^(3/2)*(a + b*Sech[c + d*Sqrt[x]])^2),x]`

output `$Aborted`

#### 3.70.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.70.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `int(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

---

3.70.  $\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$

**3.70.5 Fricas [N/A]**

Not integrable

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^{3/2}} dx$$

input `integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`output `integral(sqrt(x)/(b^2*x^2*sech(d*sqrt(x) + c)^2 + 2*a*b*x^2*sech(d*sqrt(x) + c) + a^2*x^2), x)`**3.70.6 Sympy [N/A]**

Not integrable

Time = 5.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `integrate(1/x**(3/2)/(a+b*sech(c+d*x**(1/2)))**2,x)`output `Integral(1/(x**(3/2)*(a + b*sech(c + d*sqrt(x)))**2), x)`**3.70.7 Maxima [N/A]**

Not integrable

Time = 1.02 (sec) , antiderivative size = 317, normalized size of antiderivative = 14.41

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^{3/2}} dx$$

input `integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

---

3.70.  $\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$

output 
$$\begin{aligned} & -2*(2*a*b^2 + (a^3*d*e^{(2*c)} - a*b^2*d*e^{(2*c)})*\text{sqrt}(x)*e^{(2*d*\text{sqrt}(x))} + \\ & 2*(b^3*e^c + (a^2*b*d*e^c - b^3*d*e^c)*\text{sqrt}(x))*e^{(d*\text{sqrt}(x))} + (a^3*d - a \\ & *b^2*d)*\text{sqrt}(x))/((a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*x*e^{(2*d*\text{sqrt}(x))} + \\ & 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x*e^{(d*\text{sqrt}(x))} + (a^5*d - a^3*b^2*d)*x) - \\ & \text{integrate}(2*(2*a*b^2*\text{sqrt}(x) + (2*b^3*\text{sqrt}(x))*e^c + (2*a^2*b*d*e^c - b^3* \\ & d*e^c)*x)*e^{(d*\text{sqrt}(x))})/((a^5*d*e^{(2*c)} - a^3*b^2*d*e^{(2*c)})*x^{(5/2)}*e^{(2 \\ & *d*\text{sqrt}(x))} + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^{(5/2)}*e^{(d*\text{sqrt}(x))} + (a^5 \\ & *d - a^3*b^2*d)*x^{(5/2)}), x) \end{aligned}$$

### 3.70.8 Giac [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^{3/2}} dx$$

input `integrate(1/x^(3/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")`

output `integrate(1/((b*sech(d*sqrt(x) + c) + a)^2*x^(3/2)), x)`

### 3.70.9 Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{3/2} \left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2} dx$$

input `int(1/(x^(3/2)*(a + b/cosh(c + d*x^(1/2)))^2),x)`

output `int(1/(x^(3/2)*(a + b/cosh(c + d*x^(1/2)))^2), x)`

**3.71** 
$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

3.71.1	Optimal result	412
3.71.2	Mathematica [N/A]	412
3.71.3	Rubi [N/A]	413
3.71.4	Maple [N/A] (verified)	413
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3.71.7	Maxima [N/A]	414
3.71.8	Giac [N/A]	415
3.71.9	Mupad [N/A]	415

**3.71.1 Optimal result**

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \operatorname{Int}\left(\frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2}, x\right)$$

output `Unintegrable(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

**3.71.2 Mathematica [N/A]**

Not integrable

Time = 81.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `Integrate[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])^2),x]`

output `Integrate[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])^2), x]`

### 3.71.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

↓ 5961

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `Int[1/(x^(5/2)*(a + b*Sech[c + d*Sqrt[x]])^2),x]`

output `$Aborted`

#### 3.71.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

### 3.71.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `int(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

output `int(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x)`

---

3.71.  $\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$

**3.71.5 Fricas [N/A]**

Not integrable

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^{5/2}} dx$$

input `integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="fricas")`output `integral(sqrt(x)/(b^2*x^3*sech(d*sqrt(x) + c)^2 + 2*a*b*x^3*sech(d*sqrt(x) + c) + a^2*x^3), x)`**3.71.6 Sympy [N/A]**

Not integrable

Time = 50.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$$

input `integrate(1/x**(5/2)/(a+b*sech(c+d*x**(1/2)))**2,x)`output `Integral(1/(x**(5/2)*(a + b*sech(c + d*sqrt(x)))**2), x)`**3.71.7 Maxima [N/A]**

Not integrable

Time = 1.31 (sec) , antiderivative size = 324, normalized size of antiderivative = 14.73

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^{5/2}} dx$$

input `integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="maxima")`

---

3.71.  $\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$

```
output -2/3*(6*a*b^2 + (a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*sqrt(x)*e^(2*d*sqrt(x))
+ 2*(3*b^3*e^c + (a^2*b*d*e^c - b^3*d*e^c)*sqrt(x))*e^(d*sqrt(x)) + (a^3*d
- a*b^2*d)*sqrt(x))/((a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*x^2*e^(2*d*sqrt(
x)) + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^2*e^(d*sqrt(x)) + (a^5*d - a^3*b^2
*d)*x^2) - integrate(2*(4*a*b^2*sqrt(x) + (4*b^3*sqrt(x)*e^c + (2*a^2*b*d*
e^c - b^3*d*e^c)*x)*e^(d*sqrt(x)))/((a^5*d*e^(2*c) - a^3*b^2*d*e^(2*c))*x^
(7/2)*e^(2*d*sqrt(x)) + 2*(a^4*b*d*e^c - a^2*b^3*d*e^c)*x^(7/2)*e^(d*sqrt(
x)) + (a^5*d - a^3*b^2*d)*x^(7/2)), x)
```

### 3.71.8 Giac [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{(b \operatorname{sech}(d\sqrt{x} + c) + a)^2 x^{5/2}} dx$$

```
input integrate(1/x^(5/2)/(a+b*sech(c+d*x^(1/2)))^2,x, algorithm="giac")
```

```
output sage0*x
```

### 3.71.9 Mupad [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx = \int \frac{1}{x^{5/2} \left(a + \frac{b}{\cosh(c + d\sqrt{x})}\right)^2} dx$$

```
input int(1/(x^(5/2)*(a + b/cosh(c + d*x^(1/2)))^2),x)
```

```
output int(1/(x^(5/2)*(a + b/cosh(c + d*x^(1/2)))^2), x)
```

---

3.71.  $\int \frac{1}{x^{5/2} (a + b \operatorname{sech}(c + d\sqrt{x}))^2} dx$



## 3.72 $\int (ex)^m (a + b\operatorname{sech}(c + dx^n))^p dx$

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### 3.72.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a + b\operatorname{sech}(c + dx^n))^p dx = x^{-m} (ex)^m \operatorname{Int}(x^m (a + b\operatorname{sech}(c + dx^n))^p, x)$$

output `(e*x)^m*Unintegrable(x^m*(a+b*sech(c+d*x^n))^p,x)/(x^m)`

### 3.72.2 Mathematica [N/A]

Not integrable

Time = 20.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\operatorname{sech}(c + dx^n))^p dx = \int (ex)^m (a + b\operatorname{sech}(c + dx^n))^p dx$$

input `Integrate[(e*x)^m*(a + b*Sech[c + d*x^n])^p,x]`

output `Integrate[(e*x)^m*(a + b*Sech[c + d*x^n])^p, x]`

### 3.72.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {5963, 5961}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx$$

$$\downarrow \text{5963}$$

$$x^{-m} (ex)^m \int x^m (a + b \operatorname{sech}(dx^n + c))^p dx$$

$$\downarrow \text{5961}$$

$$x^{-m} (ex)^m \int x^m (a + b \operatorname{sech}(dx^n + c))^p dx$$

input `Int[(e*x)^m*(a + b*Sech[c + d*x^n])^p,x]`

output `$Aborted`

#### 3.72.3.1 Defintions of rubi rules used

rule 5961 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Unintegrable[x^m*(a + b*Sech[c + d*x^n])^p, x] /; FreeQ[{a, b, c, d, m, n, p}, x]`

rule 5963 `Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]`

**3.72.4 Maple [N/A] (verified)**

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx$$

input `int((e*x)^m*(a+b*sech(c+d*x^n))^p,x)`output `int((e*x)^m*(a+b*sech(c+d*x^n))^p,x)`**3.72.5 Fricas [N/A]**

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx = \int (ex)^m (b \operatorname{sech}(dx^n + c) + a)^p dx$$

input `integrate((e*x)^m*(a+b*sech(c+d*x^n))^p,x, algorithm="fricas")`output `integral((e*x)^m*(b*sech(d*x^n + c) + a)^p, x)`**3.72.6 Sympy [N/A]**

Not integrable

Time = 44.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx = \int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx$$

input `integrate((e*x)**m*(a+b*sech(c+d*x**n))**p,x)`output `Integral((e*x)**m*(a + b*sech(c + d*x**n))**p, x)`

**3.72.7 Maxima [N/A]**

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx = \int (ex)^m (b \operatorname{sech}(dx^n + c) + a)^p dx$$

input `integrate((e*x)^m*(a+b*sech(c+d*x^n))^p,x, algorithm="maxima")`output `integrate((e*x)^m*(b*sech(d*x^n + c) + a)^p, x)`**3.72.8 Giac [N/A]**

Not integrable

Time = 0.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx = \int (ex)^m (b \operatorname{sech}(dx^n + c) + a)^p dx$$

input `integrate((e*x)^m*(a+b*sech(c+d*x^n))^p,x, algorithm="giac")`output `integrate((e*x)^m*(b*sech(d*x^n + c) + a)^p, x)`**3.72.9 Mupad [N/A]**

Not integrable

Time = 2.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m (a + b \operatorname{sech}(c + dx^n))^p dx = \int \left( a + \frac{b}{\cosh(c + dx^n)} \right)^p (ex)^m dx$$

input `int((a + b/cosh(c + d*x^n))^p*(e*x)^m,x)`output `int((a + b/cosh(c + d*x^n))^p*(e*x)^m, x)`

### 3.73 $\int (ex)^{-1+n} (a + b\operatorname{sech}(c + dx^n)) dx$

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#### 3.73.1 Optimal result

Integrand size = 20, antiderivative size = 44

$$\int (ex)^{-1+n} (a + b\operatorname{sech}(c + dx^n)) dx = \frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \arctan(\sinh(c + dx^n))}{den}$$

output `a*(e*x)^n/e/n+b*(e*x)^n*arctan(sinh(c+d*x^n))/d/e/n/(x^n)`

#### 3.73.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (ex)^{-1+n} (a + b\operatorname{sech}(c + dx^n)) dx = \frac{x^{-n}(ex)^n (a(c + dx^n) + b \arctan(\sinh(c + dx^n)))}{den}$$

input `Integrate[(e*x)^(-1 + n)*(a + b*Sech[c + d*x^n]),x]`

output `((e*x)^n*(a*(c + d*x^n) + b*ArcTan[Sinh[c + d*x^n]]))/(d*e*n*x^n)`

### 3.73.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{n-1} (a + b\operatorname{sech}(c + dx^n)) dx$$

↓ 2010

$$\int (a(ex)^{n-1} + b(ex)^{n-1}\operatorname{sech}(c + dx^n)) dx$$

↓ 2009

$$\frac{a(ex)^n}{en} + \frac{bx^{-n}(ex)^n \arctan(\sinh(c + dx^n))}{den}$$

input `Int[(e*x)^(-1 + n)*(a + b*Sech[c + d*x^n]),x]`

output `(a*(e*x)^n)/(e*n) + (b*(e*x)^n*ArcTan[Sinh[c + d*x^n]])/(d*e*n*x^n)`

#### 3.73.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

### 3.73.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.52

method	result
risch	$\frac{ax e^{\frac{(-1+n)(-i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie) \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie) \pi - i \operatorname{csgn}(ie) \operatorname{csgn}(ie) \pi + 2 \ln(e) + 2 \ln(x))}{2}}}{n} + \frac{2 \arctan(e^{c+dx^n})}{n}$

input `int((e*x)^(-1+n)*(a+b*sech(c+d*x^n)),x,method=_RETURNVERBOSE)`

output `a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*ln(x))+2*arctan(exp(c+d*x^n))/d/e*e^n/n*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))`

### 3.73.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs.  $2(44) = 88$ .

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.77

$$\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n)) dx$$

$$= \frac{ad \cosh((n-1) \log(e)) \cosh(n \log(x)) + ad \cosh(n \log(x)) \sinh((n-1) \log(e)) + 2(b \cosh((n-1) \log(e)) \sinh(n \log(x)) + b \cosh(n \log(x)) \sinh((n-1) \log(e))) \arctan(\cosh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c) + \sinh(d \cosh(n \log(x)) + d \sinh(n \log(x)) + c)) + (a*d*\cosh((n-1)*\log(e)) + a*d*\sinh((n-1)*\log(e)))*\sinh(n*\log(x))}{(d*n)}$$

input `integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n)),x, algorithm="fracas")`

output `(a*d*cosh((n-1)*log(e))*cosh(n*log(x)) + a*d*cosh(n*log(x))*sinh((n-1)*log(e)) + 2*(b*cosh((n-1)*log(e)) + b*sinh((n-1)*log(e)))*arctan(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) + (a*d*cosh((n-1)*log(e)) + a*d*sinh((n-1)*log(e)))*sinh(n*log(x)))/(d*n)`

**3.73.6 Sympy [F]**

$$\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n)) dx = \int (ex)^{n-1} (a + b \operatorname{sech}(c + dx^n)) dx$$

input `integrate((e*x)**(-1+n)*(a+b*sech(c+d*x**n)),x)`

output `Integral((e*x)**(n - 1)*(a + b*sech(c + d*x**n)), x)`

**3.73.7 Maxima [F]**

$$\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n)) dx = \int (b \operatorname{sech}(dx^n + c) + a)(ex)^{n-1} dx$$

input `integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n)),x, algorithm="maxima")`

output `2*b*integrate((e*x)^(n - 1)/(e^(d*x^n + c) + e^(-d*x^n - c)), x) + (e*x)^n *a/(e*n)`

**3.73.8 Giac [F]**

$$\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n)) dx = \int (b \operatorname{sech}(dx^n + c) + a)(ex)^{n-1} dx$$

input `integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n)),x, algorithm="giac")`

output `integrate((b*sech(d*x^n + c) + a)*(e*x)^(n - 1), x)`



**3.73.9 Mupad [B] (verification not implemented)**

Time = 2.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.50

$$\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n)) dx = \frac{2 \operatorname{atan}\left(\frac{bx e^{dx^n} e^c (ex)^{n-1} \sqrt{d^2 n^2 x^{2n}}}{dn x^n \sqrt{b^2 x^2 (ex)^{2n-2}}}\right) \sqrt{b^2 x^2 (ex)^{2n-2}}}{\sqrt{d^2 n^2 x^{2n}}} + \frac{ax (ex)^{n-1}}{n}$$

input `int((a + b/cosh(c + d*x^n))*(e*x)^(n - 1),x)`output `(2*atan((b*x*exp(d*x^n)*exp(c)*(e*x)^(n - 1)*(d^2*n^2*x^(2*n))^(1/2))/(d*n*x^n*(b^2*x^2*(e*x)^(2*n - 2))^(1/2)))*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(d^2*n^2*x^(2*n))^(1/2) + (a*x*(e*x)^(n - 1))/n`

### 3.74 $\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n)) dx$

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#### 3.74.1 Optimal result

Integrand size = 22, antiderivative size = 135

$$\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n)) dx = \frac{a(ex)^{2n}}{2en} + \frac{2bx^{-n}(ex)^{2n} \arctan(e^{c+dx^n})}{den} - \frac{ibx^{-2n}(ex)^{2n} \operatorname{PolyLog}(2, -ie^{c+dx^n})}{d^2en} + \frac{ibx^{-2n}(ex)^{2n} \operatorname{PolyLog}(2, ie^{c+dx^n})}{d^2en}$$

```
output 1/2*a*(e*x)^(2*n)/e/n+2*b*(e*x)^(2*n)*arctan(exp(c+d*x^n))/d/e/n/(x^n)-I*b
*(e*x)^(2*n)*polylog(2,-I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))+I*b*(e*x)^(2*n)*
polylog(2,I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))
```

#### 3.74.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.93

$$\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n)) dx = \frac{x^{-2n}(ex)^{2n} (ad^2x^{2n} + 2ibc \log(1 - ie^{c+dx^n}) - b\pi \log(1 - ie^{c+dx^n}) + 2ibdx^n \log(1 - ie^{c+dx^n}) - 2ibc \log(1 - ie^{c+dx^n}))}{den}$$

```
input Integrate[(e*x)^(-1 + 2*n)*(a + b*Sech[c + d*x^n]),x]
```

output  $((e*x)^{(2*n)}*(a*d^{2*x^{(2*n)}} + (2*I)*b*c*\text{Log}[1 - I*E^{(c + d*x^n)}] - b*Pi*\text{Log}[1 - I*E^{(c + d*x^n)}] + (2*I)*b*d*x^n*\text{Log}[1 - I*E^{(c + d*x^n)}] - (2*I)*b*c*\text{Log}[1 + I*E^{(c + d*x^n)}] + b*Pi*\text{Log}[1 + I*E^{(c + d*x^n)}] - (2*I)*b*d*x^n*\text{Log}[1 + I*E^{(c + d*x^n)}] - (2*I)*b*c*\text{Log}[\text{Cot}[\frac{((2*I)*c + Pi + (2*I)*d*x^n)}{4}]] + b*Pi*\text{Log}[\text{Cot}[\frac{((2*I)*c + Pi + (2*I)*d*x^n)}{4}]] - (2*I)*b*\text{PolyLog}[2, (-I)*E^{(c + d*x^n)}] + (2*I)*b*\text{PolyLog}[2, I*E^{(c + d*x^n)}]))/(2*d^{2*e*n*x^{(2*n)}})$

### 3.74.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1} (a + b\text{sech}(c + dx^n)) dx$$

↓ 2010

$$\int (a(ex)^{2n-1} + b(ex)^{2n-1}\text{sech}(c + dx^n)) dx$$

↓ 2009

$$\frac{a(ex)^{2n}}{2en} + \frac{2bx^{-n}(ex)^{2n} \arctan(e^{c+dx^n})}{\frac{den}{ibx^{-2n}(ex)^{2n} \text{PolyLog}(2, ie^{dx^n+c})}} - \frac{ibx^{-2n}(ex)^{2n} \text{PolyLog}(2, -ie^{dx^n+c})}{d^2en} +$$

input  $\text{Int}[(e*x)^{(-1 + 2*n)}*(a + b*\text{Sech}[c + d*x^n]), x]$

output  $(a*(e*x)^{(2*n)})/(2*e*n) + (2*b*(e*x)^{(2*n)}*\text{ArcTan}[E^{(c + d*x^n)}])/(d*e*n*x^n) - (I*b*(e*x)^{(2*n)}*\text{PolyLog}[2, (-I)*E^{(c + d*x^n)}])/(d^2*e*n*x^{(2*n)}) + (I*b*(e*x)^{(2*n)}*\text{PolyLog}[2, I*E^{(c + d*x^n)}])/(d^2*e*n*x^{(2*n)})$

### 3.74.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

### 3.74.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.70 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.73

method	result
risch	$\frac{ax e^{\frac{(2n-1)(-i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie x) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie x)^2 \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie x)^2 \pi - i \operatorname{csgn}(ie x)^3 \pi + 2 \ln(e) + 2 \ln(x))}{2n}}}{2n} + \frac{2b e^{-i\pi n \operatorname{csgn}(ie)}}{2n}$

input `int((e*x)^(2*n-1)*(a+b*sech(c+d*x^n)), x, method=_RETURNVERBOSE)`

output `1/2*a/n*x*exp(1/2*(2*n-1)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*ln(x))+2*b*exp(-I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(I*Pi*n*csgn(I*e)*csgn(I*e*x)^2)*exp(I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-I*Pi*n*csgn(I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(-1/2*I*Pi*csgn(I*e)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*exp(1/2*I*Pi*csgn(I*e*x)^3)*(e^n)^2/e*exp(c)/n/d^2*(-1/2*(-exp(2*c))^(1/2)*x^n*d*(ln(1+exp(d*x^n)*(-exp(2*c))^(1/2))-ln(1-exp(d*x^n)*(-exp(2*c))^(1/2)))*exp(-2*c)-1/2*(-exp(2*c))^(1/2)*(dilog(1+exp(d*x^n)*(-exp(2*c))^(1/2))-dilog(1-exp(d*x^n)*(-exp(2*c))^(1/2)))*exp(-2*c))`

---

3.74.  $\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n)) dx$

### 3.74.5 Fracas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 664 vs.  $2(124) = 248$ .

Time = 0.28 (sec) , antiderivative size = 664, normalized size of antiderivative = 4.92

$$\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n)) dx = \text{Too large to display}$$

input `integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n)),x, algorithm="fracas")`

output `1/2*(a*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + a*d^2*cosh(n*log(x))^2*sinh((2*n - 1)*log(e)) + (a*d^2*cosh((2*n - 1)*log(e)) + a*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 - 2*(-I*b*cosh((2*n - 1)*log(e)) - I*b*sinh((2*n - 1)*log(e)))*dilog(I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 2*(I*b*cosh((2*n - 1)*log(e)) + I*b*sinh((2*n - 1)*log(e)))*dilog(-I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 2*(I*b*c*cosh((2*n - 1)*log(e)) + I*b*c*sinh((2*n - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I) - 2*(-I*b*c*cosh((2*n - 1)*log(e)) - I*b*c*sinh((2*n - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I) - 2*(I*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + I*b*c*cosh((2*n - 1)*log(e)) + (I*b*d*cosh(n*log(x)) + I*b*c)*sinh((2*n - 1)*log(e)) + (I*b*d*cosh((2*n - 1)*log(e)) + I*b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*log(I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 1) - 2*(-I*b*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) - I*b*c*cosh((2*n - 1)*log(e)) + (-I*b*d*cosh(n*log(x)) - I*b*c)*sinh((2*n - 1)*log(e)) + (-I*b*d*cosh((2*n - 1)*log(e)) - I*b*d*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*log(-I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I*sinh(d*cosh(n*log(x)) + ...`

### 3.74.6 SymPy [F]

$$\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n)) dx = \int (ex)^{2n-1} (a + b \operatorname{sech}(c + dx^n)) dx$$

input `integrate((e*x)**(-1+2*n)*(a+b*sech(c+d*x**n)),x)`

output `Integral((e*x)**(2*n - 1)*(a + b*sech(c + d*x**n)), x)`

---

3.74.  $\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n)) dx$

**3.74.7 Maxima [F]**

$$\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n)) dx = \int (b\operatorname{sech}(dx^n + c) + a)(ex)^{2n-1} dx$$

input `integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n)),x, algorithm="maxima")`

output `2*b*integrate((e*x)^(2*n - 1)/(e^(d*x^n + c) + e^(-d*x^n - c)), x) + 1/2*(e*x)^(2*n)*a/(e*n)`

**3.74.8 Giac [F]**

$$\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n)) dx = \int (b\operatorname{sech}(dx^n + c) + a)(ex)^{2n-1} dx$$

input `integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n)),x, algorithm="giac")`

output `integrate((b*sech(d*x^n + c) + a)*(e*x)^(2*n - 1), x)`

**3.74.9 Mupad [F(-1)]**

Timed out.

$$\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n)) dx = \int \left( a + \frac{b}{\cosh(c + dx^n)} \right) (ex)^{2n-1} dx$$

input `int((a + b/cosh(c + d*x^n))*(e*x)^(2*n - 1),x)`

output `int((a + b/cosh(c + d*x^n))*(e*x)^(2*n - 1), x)`

### 3.75 $\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n)) dx$

3.75.1	Optimal result	430
3.75.2	Mathematica [F]	431
3.75.3	Rubi [A] (verified)	431
3.75.4	Maple [F]	432
3.75.5	Fricas [B] (verification not implemented)	432
3.75.6	Sympy [F]	433
3.75.7	Maxima [F]	434
3.75.8	Giac [F]	434
3.75.9	Mupad [F(-1)]	434

#### 3.75.1 Optimal result

Integrand size = 22, antiderivative size = 217

$$\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n)) dx = \frac{a(ex)^{3n}}{3en} + \frac{2bx^{-n}(ex)^{3n} \arctan(e^{c+dx^n})}{den} - \frac{2ibx^{-2n}(ex)^{3n} \operatorname{PolyLog}(2, -ie^{c+dx^n})}{d^2en} + \frac{2ibx^{-2n}(ex)^{3n} \operatorname{PolyLog}(2, ie^{c+dx^n})}{d^2en} + \frac{2ibx^{-3n}(ex)^{3n} \operatorname{PolyLog}(3, -ie^{c+dx^n})}{d^3en} - \frac{2ibx^{-3n}(ex)^{3n} \operatorname{PolyLog}(3, ie^{c+dx^n})}{d^3en}$$

```
output 1/3*a*(e*x)^(3*n)/e/n+2*b*(e*x)^(3*n)*arctan(exp(c+d*x^n))/d/e/n/(x^n)-2*I*b*(e*x)^(3*n)*polylog(2,-I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))+2*I*b*(e*x)^(3*n)*polylog(2,I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))+2*I*b*(e*x)^(3*n)*polylog(3,-I*exp(c+d*x^n))/d^3/e/n/(x^(3*n))-2*I*b*(e*x)^(3*n)*polylog(3,I*exp(c+d*x^n))/d^3/e/n/(x^(3*n))
```

### 3.75.2 Mathematica [F]

$$\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n)) dx = \int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n)) dx$$

input `Integrate[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n]),x]`

output `Integrate[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n]), x]`

### 3.75.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (ex)^{3n-1} (a + b\operatorname{sech}(c + dx^n)) dx \\ & \quad \downarrow \text{2010} \\ & \int (a(ex)^{3n-1} + b(ex)^{3n-1}\operatorname{sech}(c + dx^n)) dx \\ & \quad \downarrow \text{2009} \\ & \frac{a(ex)^{3n}}{3en} + \frac{2bx^{-n}(ex)^{3n} \arctan(e^{c+dx^n})}{2ibx^{-3n}(ex)^{3n} \operatorname{PolyLog}(3, ie^{dx^n+c})} + \frac{2ibx^{-3n}(ex)^{3n} \operatorname{PolyLog}(3, -ie^{dx^n+c})}{2ibx^{-2n}(ex)^{3n} \operatorname{PolyLog}(2, -ie^{dx^n+c})} - \\ & \quad \frac{d^3en}{2ibx^{-2n}(ex)^{3n} \operatorname{PolyLog}(2, ie^{dx^n+c})} + \frac{d^2en}{d^2en} \end{aligned}$$

input `Int[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n]),x]`

output `(a*(e*x)^(3*n))/(3*e*n) + (2*b*(e*x)^(3*n)*ArcTan[E^(c + d*x^n)])/(d*e*n*x^n) - ((2*I)*b*(e*x)^(3*n)*PolyLog[2, (-I)*E^(c + d*x^n)]/(d^2*e*n*x^(2*n))) + ((2*I)*b*(e*x)^(3*n)*PolyLog[2, I*E^(c + d*x^n)]/(d^2*e*n*x^(2*n))) + ((2*I)*b*(e*x)^(3*n)*PolyLog[3, (-I)*E^(c + d*x^n)]/(d^3*e*n*x^(3*n))) - ((2*I)*b*(e*x)^(3*n)*PolyLog[3, I*E^(c + d*x^n)]/(d^3*e*n*x^(3*n)))`

---

3.75.  $\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n)) dx$



### 3.75.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2010 `Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)+(b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

### 3.75.4 Maple [F]

$$\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + dx^n)) dx$$

input `int((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n)),x)`

output `int((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n)),x)`

### 3.75.5 Fracas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1082 vs.  $2(200) = 400$ .

Time = 0.30 (sec) , antiderivative size = 1082, normalized size of antiderivative = 4.99

$$\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + dx^n)) dx = \text{Too large to display}$$

input `integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n)),x, algorithm="fracas")`

```

output 1/3*(a*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^3 + a*d^3*cosh(n*log(x))^
3*sinh((3*n - 1)*log(e)) + (a*d^3*cosh((3*n - 1)*log(e)) + a*d^3*sinh((3*n
- 1)*log(e)))*sinh(n*log(x))^3 + 3*(a*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))
+ a*d^3*cosh(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 - 6
*(-I*b*d*cosh((3*n - 1)*log(e))*cosh(n*log(x)) - I*b*d*cosh(n*log(x))*sinh
((3*n - 1)*log(e)) + (-I*b*d*cosh((3*n - 1)*log(e)) - I*b*d*sinh((3*n - 1)
*log(e)))*sinh(n*log(x)))*dilog(I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x))
+ c) + I*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 6*(I*b*d*cosh((
3*n - 1)*log(e))*cosh(n*log(x)) + I*b*d*cosh(n*log(x))*sinh((3*n - 1)*log(
e)) + (I*b*d*cosh((3*n - 1)*log(e)) + I*b*d*sinh((3*n - 1)*log(e)))*sinh(n
*log(x)))*dilog(-I*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - I*sinh(
d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)) - 3*(-I*b*c^2*cosh((3*n - 1)*log
(e)) - I*b*c^2*sinh((3*n - 1)*log(e)))*log(cosh(d*cosh(n*log(x)) + d*sinh(
n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + I) - 3*(I
*b*c^2*cosh((3*n - 1)*log(e)) + I*b*c^2*sinh((3*n - 1)*log(e)))*log(cosh(d
*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sinh(d*cosh(n*log(x)) + d*sinh(n
*log(x)) + c) - I) - 3*(I*b*d^2*cosh((3*n - 1)*log(e))*cosh(n*log(x))^2 -
I*b*c^2*cosh((3*n - 1)*log(e)) + (I*b*d^2*cosh((3*n - 1)*log(e)) + I*b*d^2
*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + (I*b*d^2*cosh(n*log(x))^2 - I*
b*c^2)*sinh((3*n - 1)*log(e)) + 2*(I*b*d^2*cosh((3*n - 1)*log(e))*cosh(...

```

### 3.75.6 Sympy [F]

$$\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + dx^n)) dx = \int (ex)^{3n-1} (a + b \operatorname{sech}(c + dx^n)) dx$$

```
input integrate((e*x)**(-1+3*n)*(a+b*sech(c+d*x**n)),x)
```

```
output Integral((e*x)**(3*n - 1)*(a + b*sech(c + d*x**n)), x)
```

**3.75.7 Maxima [F]**

$$\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + dx^n)) dx = \int (b \operatorname{sech}(dx^n + c) + a)(ex)^{3n-1} dx$$

input `integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n)),x, algorithm="maxima")`

output `2*b*integrate((e*x)^(3*n - 1)/(e^(d*x^n + c) + e^(-d*x^n - c)), x) + 1/3*(e*x)^(3*n)*a/(e*n)`

**3.75.8 Giac [F]**

$$\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + dx^n)) dx = \int (b \operatorname{sech}(dx^n + c) + a)(ex)^{3n-1} dx$$

input `integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n)),x, algorithm="giac")`

output `integrate((b*sech(d*x^n + c) + a)*(e*x)^(3*n - 1), x)`

**3.75.9 Mupad [F(-1)]**

Timed out.

$$\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + dx^n)) dx = \int \left( a + \frac{b}{\cosh(c + dx^n)} \right) (ex)^{3n-1} dx$$

input `int((a + b/cosh(c + d*x^n))*(e*x)^(3*n - 1),x)`

output `int((a + b/cosh(c + d*x^n))*(e*x)^(3*n - 1), x)`

### 3.76 $\int (ex)^{-1+n} (a + b\operatorname{sech}(c + dx^n))^2 dx$

3.76.1	Optimal result . . . . .	435
3.76.2	Mathematica [A] (verified) . . . . .	435
3.76.3	Rubi [A] (verified) . . . . .	436
3.76.4	Maple [C] (warning: unable to verify) . . . . .	438
3.76.5	Fricas [B] (verification not implemented) . . . . .	438
3.76.6	Sympy [F] . . . . .	439
3.76.7	Maxima [F] . . . . .	440
3.76.8	Giac [F] . . . . .	440
3.76.9	Mupad [B] (verification not implemented) . . . . .	440

#### 3.76.1 Optimal result

Integrand size = 22, antiderivative size = 79

$$\int (ex)^{-1+n} (a + b\operatorname{sech}(c + dx^n))^2 dx = \frac{a^2(ex)^n}{en} + \frac{2abx^{-n}(ex)^n \arctan(\sinh(c + dx^n))}{den} + \frac{b^2x^{-n}(ex)^n \tanh(c + dx^n)}{den}$$

output `a^2*(e*x)^n/e/n+2*a*b*(e*x)^n*arctan(sinh(c+d*x^n))/d/e/n/(x^n)+b^2*(e*x)^n*tanh(c+d*x^n)/d/e/n/(x^n)`

#### 3.76.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int (ex)^{-1+n} (a + b\operatorname{sech}(c + dx^n))^2 dx = \frac{x^{-n}(ex)^n (a(a(c + dx^n) + 2b \arctan(\sinh(c + dx^n))) + b^2 \tanh(c + dx^n))}{den}$$

input `Integrate[(e*x)^(-1 + n)*(a + b*Sech[c + d*x^n])^2,x]`

output `((e*x)^n*(a*(a*(c + d*x^n) + 2*b*ArcTan[Sinh[c + d*x^n]]) + b^2*Tanh[c + d*x^n]))/(d*e*n*x^n)`

**3.76.3 Rubi [A] (verified)**

Time = 0.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71, number of steps used = 9, number of rules used = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {5963, 5959, 3042, 4260, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int (ex)^{n-1} (a + b\operatorname{sech}(c + dx^n))^2 dx \\
 \downarrow \text{5963} \\
 \frac{x^{-n}(ex)^n \int x^{n-1} (a + b\operatorname{sech}(dx^n + c))^2 dx}{e} \\
 \downarrow \text{5959} \\
 \frac{x^{-n}(ex)^n \int (a + b\operatorname{sech}(dx^n + c))^2 dx^n}{en} \\
 \downarrow \text{3042} \\
 \frac{x^{-n}(ex)^n \int (a + b \csc (idx^n + ic + \frac{\pi}{2}))^2 dx^n}{en} \\
 \downarrow \text{4260} \\
 \frac{x^{-n}(ex)^n (2ab \int \operatorname{sech}(dx^n + c) dx^n + b^2 \int \operatorname{sech}^2(dx^n + c) dx^n + a^2 x^n)}{en} \\
 \downarrow \text{3042} \\
 \frac{x^{-n}(ex)^n (2ab \int \csc (idx^n + ic + \frac{\pi}{2}) dx^n + b^2 \int \csc (idx^n + ic + \frac{\pi}{2})^2 dx^n + a^2 x^n)}{en} \\
 \downarrow \text{4254} \\
 \frac{x^{-n}(ex)^n (2ab \int \csc (idx^n + ic + \frac{\pi}{2}) dx^n + \frac{ib^2 \int 1d(-i \tanh(dx^n + c))}{d} + a^2 x^n)}{en} \\
 \downarrow \text{24} \\
 \frac{x^{-n}(ex)^n (2ab \int \csc (idx^n + ic + \frac{\pi}{2}) dx^n + a^2 x^n + \frac{b^2 \tanh(c + dx^n)}{d})}{en} \\
 \downarrow \text{4257} \\
 \frac{x^{-n}(ex)^n (a^2 x^n + \frac{2ab \arctan(\sinh(c + dx^n))}{d} + \frac{b^2 \tanh(c + dx^n)}{d})}{en}
 \end{array}$$

input `Int[(e*x)^(-1 + n)*(a + b*Sech[c + d*x^n])^2,x]`

output `((e*x)^n*(a^2*x^n + (2*a*b*ArcTan[Sinh[c + d*x^n]])/d + (b^2*Tanh[c + d*x^n])/d))/(e*n*x^n)`

### 3.76.3.1 Defintions of rubi rules used

rule 24 `Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4254 `Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[-d^(-1) Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

rule 4257 `Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

rule 4260 `Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^2, x_Symbol] := Simp[a^2*x, x] + (Simp[2*a*b Int[Csc[c + d*x], x], x] + Simp[b^2 Int[Csc[c + d*x]^2, x], x]) /; FreeQ[{a, b, c, d}, x]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

rule 5963 `Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]`

### 3.76.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 17.01 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.43

method	result
risch	$\frac{a^2 x e^{(-1+n) \left( -i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie x) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie x)^2 \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie x)^2 \pi - i \operatorname{csgn}(ie x)^3 \pi + 2 \ln(e) + 2 \ln(x) \right)}}{n} - \frac{2 x x^{-n} b^2 e^{(-1+n) \left( -i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie x) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie x)^2 \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie x)^2 \pi - i \operatorname{csgn}(ie x)^3 \pi + 2 \ln(e) + 2 \ln(x) \right)}}{n}$

input `int((e*x)^(-1+n)*(a+b*sech(c+d*x^n))^2,x,method=_RETURNVERBOSE)`

output `a^2/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*ln(x))-2/d/n*x/(x^n)*b^2*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*ln(x)))/(1+exp(2*c+2*d*x^n))+4*arctan(exp(c+d*x^n))/d/e*e^n/n*a*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))`

### 3.76.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(79) = 158.

Time = 0.27 (sec) , antiderivative size = 646, normalized size of antiderivative = 8.18

$$\int (ex)^{-1+n} (a + b\operatorname{sech}(c + dx^n))^2 dx = \text{Too large to display}$$

input `integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")`

output  $(a^2 d \cosh((n-1)\log(e)) \cosh(n\log(x)) + a^2 d \cosh((n-1)\log(e)) \cosh(n\log(x)) + a^2 d \cosh(n\log(x)) \sinh((n-1)\log(e)) + (a^2 d \cosh((n-1)\log(e)) + a^2 d \sinh((n-1)\log(e))) \sinh(n\log(x))) \cosh(d \cosh(n\log(x)) + d \sinh(n\log(x)) + c)^2 - 2b^2 \cosh((n-1)\log(e)) + 2(a^2 d \cosh((n-1)\log(e)) \cosh(n\log(x)) + a^2 d \cosh(n\log(x)) \sinh((n-1)\log(e)) + (a^2 d \cosh((n-1)\log(e)) + a^2 d \sinh((n-1)\log(e))) \sinh(n\log(x))) \cosh(d \cosh(n\log(x)) + d \sinh(n\log(x)) + c) \sinh(d \cosh(n\log(x)) + d \sinh(n\log(x)) + c) + (a^2 d \cosh((n-1)\log(e)) \cosh(n\log(x)) + a^2 d \cosh(n\log(x)) \sinh((n-1)\log(e)) + (a^2 d \cosh((n-1)\log(e)) + a^2 d \sinh((n-1)\log(e))) \sinh(n\log(x))) \sinh(d \cosh(n\log(x)) + d \sinh(n\log(x)) + c)^2 + 4((a b \cosh((n-1)\log(e)) + a b \sinh((n-1)\log(e))) \cosh(d \cosh(n\log(x)) + d \sinh(n\log(x)) + c)^2 + a b \cosh((n-1)\log(e)) + 2(a b \cosh((n-1)\log(e)) + a b \sinh((n-1)\log(e))) \cosh(d \cosh(n\log(x)) + d \sinh(n\log(x)) + c) \sinh(d \cosh(n\log(x)) + d \sinh(n\log(x)) + c) + (a b \cosh((n-1)\log(e)) + a b \sinh((n-1)\log(e))) \sinh(d \cosh(n\log(x)) + d \sinh(n\log(x)) + c)^2 + a b \sinh((n-1)\log(e)) \arctan(\cosh(d \cosh(n\log(x)) + d \sinh(n\log(x)) + c) + \sinh(d \cosh(n\log(x)) + d \sinh(n\log(x)) + c)) + (a^2 d \cosh(n\log(x)) - 2b^2) \sinh((n-1)\log(e)) + (a^2 d \cosh((n-1)\log(e)) + a^2 d \sinh((n-1)\log(e))) \sinh(n\log(x)) / (d n \cosh(d \cosh(n\log(x)) + d \sinh(n\log(x)) + c)^2 + 2 d n \cosh(d \cosh...$

### 3.76.6 Sympy [F]

$$\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n))^2 dx = \int (ex)^{n-1} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

input `integrate((e*x)**(-1+n)*(a+b*sech(c+d*x**n))**2,x)`

output `Integral((e*x)**(n-1)*(a+b*sech(c+d*x**n))**2,x)`



**3.76.7 Maxima [F]**

$$\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n))^2 dx = \int (b \operatorname{sech}(dx^n + c) + a)^2 (ex)^{n-1} dx$$

input `integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")`

output `4*a*b*e^n*integrate(e^(d*x^n + n*log(x) + c)/(e*x*e^(2*d*x^n + 2*c) + e*x), x) - 2*b^2*e^n/(d*e*n*e^(2*d*x^n + 2*c) + d*e*n) + (e*x)^n*a^2/(e*n)`

**3.76.8 Giac [F]**

$$\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n))^2 dx = \int (b \operatorname{sech}(dx^n + c) + a)^2 (ex)^{n-1} dx$$

input `integrate((e*x)^(-1+n)*(a+b*sech(c+d*x^n))^2,x, algorithm="giac")`

output `integrate((b*sech(d*x^n + c) + a)^2*(e*x)^(n - 1), x)`

**3.76.9 Mupad [B] (verification not implemented)**

Time = 2.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.00

$$\int (ex)^{-1+n} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

$$= \frac{4 \operatorname{atan}\left(\frac{a b x e^{d x^n} e^c (e x)^{n-1} \sqrt{d^2 n^2 x^{2n}}}{d n x^n \sqrt{a^2 b^2 x^2 (e x)^{2n-2}}}\right) \sqrt{a^2 b^2 x^2 (e x)^{2n-2}}}{\frac{\sqrt{d^2 n^2 x^{2n}}}{n} + \frac{a^2 x (e x)^{n-1}}{d n x^n (e^{2c+2 d x^n} + 1)} - \frac{2 b^2 x (e x)^{n-1}}{d n x^n (e^{2c+2 d x^n} + 1)}}$$

input `int((a + b/cosh(c + d*x^n))^2*(e*x)^(n - 1),x)`

output `(4*atan((a*b*x*exp(d*x^n)*exp(c)*(e*x)^(n - 1)*(d^2*n^2*x^(2*n))^(1/2))/(d*n*x^n*(a^2*b^2*x^2*(e*x)^(2*n - 2))^(1/2)))*(a^2*b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(d^2*n^2*x^(2*n))^(1/2) + (a^2*x*(e*x)^(n - 1))/n - (2*b^2*x*(e*x)^(n - 1))/(d*n*x^n*(exp(2*c + 2*d*x^n) + 1))`

### 3.77 $\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n))^2 dx$

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#### 3.77.1 Optimal result

Integrand size = 24, antiderivative size = 208

$$\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n))^2 dx = \frac{a^2(ex)^{2n}}{2en} + \frac{4abx^{-n}(ex)^{2n} \arctan(e^{c+dx^n})}{den} - \frac{b^2x^{-2n}(ex)^{2n} \log(\cosh(c + dx^n))}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n} \operatorname{PolyLog}(2, -ie^{c+dx^n})}{d^2en} + \frac{2iabx^{-2n}(ex)^{2n} \operatorname{PolyLog}(2, ie^{c+dx^n})}{d^2en} + \frac{b^2x^{-n}(ex)^{2n} \tanh(c + dx^n)}{den}$$

```
output 1/2*a^2*(e*x)^(2*n)/e/n+4*a*b*(e*x)^(2*n)*arctan(exp(c+d*x^n))/d/e/n/(x^n)
-b^2*(e*x)^(2*n)*ln(cosh(c+d*x^n))/d^2/e/n/(x^(2*n))-2*I*a*b*(e*x)^(2*n)*p
olylog(2,-I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))+2*I*a*b*(e*x)^(2*n)*polylog(2,
I*exp(c+d*x^n))/d^2/e/n/(x^(2*n))+b^2*(e*x)^(2*n)*tanh(c+d*x^n)/d/e/n/(x^n
)
```

### 3.77.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 501 vs.  $2(208) = 416$ .

Time = 5.47 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.41

$$\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

$$= \frac{x^{-2n} (ex)^{2n} \operatorname{csch}^5(c) \operatorname{sech}(c + dx^n) \left( -2b^2 dx^n \cosh(dx^n) \sqrt{-\operatorname{csch}^2(c)} + 2b^2 dx^n \cosh(2c + dx^n) \sqrt{-\operatorname{csch}^2(c)} \right)}{}$$

input `Integrate[(e*x)^(-1 + 2*n)*(a + b*Sech[c + d*x^n])^2,x]`

output

```
((e*x)^(2*n)*Csch[c]^5*Sech[c + d*x^n]*(-2*b^2*d*x^n*Cosh[d*x^n]*Sqrt[-Csch[c]^2] + 2*b^2*d*x^n*Cosh[2*c + d*x^n]*Sqrt[-Csch[c]^2] + 8*a*b*d*x^n*Cosh[c + d*x^n]*Log[1 - E^(-(d*x^n) - ArcTanh[Coth[c]])] + 8*a*b*ArcTanh[Coth[c]]*Cosh[c + d*x^n]*Log[1 - E^(-(d*x^n) - ArcTanh[Coth[c]])] - 8*a*b*d*x^n*Cosh[c + d*x^n]*Log[1 + E^(-(d*x^n) - ArcTanh[Coth[c]])] - 8*a*b*ArcTanh[Coth[c]]*Cosh[c + d*x^n]*Log[1 + E^(-(d*x^n) - ArcTanh[Coth[c]])] + 8*a*b*Cosh[c + d*x^n]*PolyLog[2, -E^(-(d*x^n) - ArcTanh[Coth[c]])] - 8*a*b*Cosh[c + d*x^n]*PolyLog[2, E^(-(d*x^n) - ArcTanh[Coth[c]])] - a^2*d^2*x^(2*n)*Sqrt[-Csch[c]^2]*Sinh[d*x^n] + 8*a*b*ArcTan[Sinh[c] + Cosh[c]*Tanh[(d*x^n)/2]]*ArcTanh[Coth[c]]*Sqrt[-Csch[c]^2]*Sinh[d*x^n] + 2*b^2*Sqrt[-Csch[c]^2]*Log[Cosh[c + d*x^n]]*Sinh[d*x^n] + a^2*d^2*x^(2*n)*Sqrt[-Csch[c]^2]*Sinh[2*c + d*x^n] - 8*a*b*ArcTan[Sinh[c] + Cosh[c]*Tanh[(d*x^n)/2]]*ArcTanh[Coth[c]]*Sqrt[-Csch[c]^2]*Sinh[2*c + d*x^n] - 2*b^2*Sqrt[-Csch[c]^2]*Log[Cosh[c + d*x^n]]*Sinh[2*c + d*x^n]))/(4*d^2*e*n*x^(2*n)*(-Csch[c]^2)^(5/2))
```

### 3.77.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.65, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {5963, 5959, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{2n-1} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

↓ 5963

---

3.77.  $\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n))^2 dx$

$$\begin{array}{c}
 \frac{x^{-2n}(ex)^{2n} \int x^{2n-1}(a + b\operatorname{sech}(dx^n + c))^2 dx}{e} \\
 \downarrow \text{5959} \\
 \frac{x^{-2n}(ex)^{2n} \int x^n(a + b\operatorname{sech}(dx^n + c))^2 dx^n}{en} \\
 \downarrow \text{3042} \\
 \frac{x^{-2n}(ex)^{2n} \int x^n(a + b\operatorname{csc}(idx^n + ic + \frac{\pi}{2}))^2 dx^n}{en} \\
 \downarrow \text{4678} \\
 \frac{x^{-2n}(ex)^{2n} \int (a^2x^n + b^2\operatorname{sech}^2(dx^n + c)x^n + 2ab\operatorname{sech}(dx^n + c)x^n) dx^n}{en} \\
 \downarrow \text{2009} \\
 \frac{x^{-2n}(ex)^{2n} \left( \frac{1}{2}a^2x^{2n} + \frac{4abx^n \arctan(e^{c+dx^n})}{d} - \frac{2iab \operatorname{PolyLog}(2, -ie^{dx^n+c})}{d^2} + \frac{2iab \operatorname{PolyLog}(2, ie^{dx^n+c})}{d^2} - \frac{b^2 \log(\cosh(c+dx^n))}{d^2} + \dots \right)}{en}
 \end{array}$$

input `Int[(e*x)^(-1 + 2*n)*(a + b*Sech[c + d*x^n])^2,x]`

output `((e*x)^(2*n)*((a^2*x^(2*n))/2 + (4*a*b*x^n*ArcTan[E^(c + d*x^n)])/d - (b^2*Log[Cosh[c + d*x^n]])/d^2 - ((2*I)*a*b*PolyLog[2, (-I)*E^(c + d*x^n)])/d^2 + ((2*I)*a*b*PolyLog[2, I*E^(c + d*x^n)])/d^2 + (b^2*x^n*Tanh[c + d*x^n])/d))/(e*n*x^(2*n))`

### 3.77.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

```
rule 5959 Int[(x_)^(m_)*((a_) + (b_)*Sech[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol]
  := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x]
  /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

```
rule 5963 Int[((e_)*(x_)^(m_)*((a_) + (b_)*Sech[(c_) + (d_)*(x_)^(n_)])^(p_), x_Symbol]
  := Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sech[c + d*x^n])^p, x], x]
  /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

### 3.77.4 Maple [F]

$$\int (ex)^{2n-1} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

```
input int((e*x)^(2*n-1)*(a+b*sech(c+d*x^n))^2,x)
```

```
output int((e*x)^(2*n-1)*(a+b*sech(c+d*x^n))^2,x)
```

### 3.77.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2972 vs.  $2(199) = 398$ .

Time = 0.32 (sec) , antiderivative size = 2972, normalized size of antiderivative = 14.29

$$\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n))^2 dx = \text{Too large to display}$$

```
input integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")
```

output  $1/2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + 4*b^2*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 + (a^2*d^2*cosh(n*log(x))^2 + 4*b^2*d*cosh(n*log(x)) + 4*b^2*c)*sinh((2*n - 1)*log(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 2*b^2*d*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh(n*log(x)) + 2*b^2*d)*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + 4*b^2*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 + (a^2*d^2*cosh(n*log(x))^2 + 4*b^2*d*cosh(n*log(x)) + 4*b^2*c)*sinh((2*n - 1)*log(e)) + 2*(a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 2*b^2*d*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh(n*log(x)) + 2*b^2*d)*sinh((2*n - 1)*log(e)))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a^2*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + 4*b^2*d*cosh((2*n - 1)*log(e))*cosh(n*log(x)) + 4*b^2*c*cosh((2*n - 1)*log(e)) + (a^2*d^2*cosh((2*n - 1)*log(e)) + a^2*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 + (a^2*d^2*cosh(n*log(x))^2 + 4*b^2*d*cosh(n*log(x)) + 4*b^2*c)*sinh((2*n - 1)*log(e)) + 2*(a^2*d^2*co...$

### 3.77.6 Sympy [F]

$$\int (ex)^{-1+2n} (a + b\operatorname{sech}(c + dx^n))^2 dx = \int (ex)^{2n-1} (a + b\operatorname{sech}(c + dx^n))^2 dx$$

input `integrate((e*x)**(-1+2*n)*(a+b*sech(c+d*x**n))**2,x)`

output `Integral((e*x)**(2*n - 1)*(a + b*sech(c + d*x**n))**2, x)`

**3.77.7 Maxima [F]**

$$\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n))^2 dx = \int (b \operatorname{sech}(dx^n + c) + a)^2 (ex)^{2n-1} dx$$

input `integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")`

output `4*a*b*e^(2*n)*integrate(e^(d*x^n + 2*n*log(x) + c)/(e*x*e^(2*d*x^n + 2*c) + e*x), x) + b^2*(2*e^(2*n)*e^(2*d*x^n + n*log(x) + 2*c)/(d*e*n*e^(2*d*x^n + 2*c) + d*e*n) - e^(2*n - 1)*log((e^(2*d*x^n + 2*c) + 1)*e^(-2*c))/(d^2*n)) + 1/2*(e*x)^(2*n)*a^2/(e*n)`

**3.77.8 Giac [F]**

$$\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n))^2 dx = \int (b \operatorname{sech}(dx^n + c) + a)^2 (ex)^{2n-1} dx$$

input `integrate((e*x)^(-1+2*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="giac")`

output `integrate((b*sech(d*x^n + c) + a)^2*(e*x)^(2*n - 1), x)`

**3.77.9 Mupad [F(-1)]**

Timed out.

$$\int (ex)^{-1+2n} (a + b \operatorname{sech}(c + dx^n))^2 dx = \int \left( a + \frac{b}{\cosh(c + dx^n)} \right)^2 (ex)^{2n-1} dx$$

input `int((a + b/cosh(c + d*x^n))^2*(e*x)^(2*n - 1),x)`

output `int((a + b/cosh(c + d*x^n))^2*(e*x)^(2*n - 1), x)`

### 3.78 $\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx$

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#### 3.78.1 Optimal result

Integrand size = 24, antiderivative size = 363

$$\begin{aligned} \int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx = & \frac{a^2(ex)^{3n}}{3en} + \frac{b^2x^{-n}(ex)^{3n}}{den} \\ & + \frac{4abx^{-n}(ex)^{3n} \arctan(e^{c+dx^n})}{den} \\ & - \frac{2b^2x^{-2n}(ex)^{3n} \log(1 + e^{2(c+dx^n)})}{d^2en} \\ & - \frac{4iabx^{-2n}(ex)^{3n} \operatorname{PolyLog}(2, -ie^{c+dx^n})}{d^2en} \\ & + \frac{4iabx^{-2n}(ex)^{3n} \operatorname{PolyLog}(2, ie^{c+dx^n})}{d^2en} \\ & - \frac{b^2x^{-3n}(ex)^{3n} \operatorname{PolyLog}(2, -e^{2(c+dx^n)})}{d^3en} \\ & + \frac{4iabx^{-3n}(ex)^{3n} \operatorname{PolyLog}(3, -ie^{c+dx^n})}{d^3en} \\ & - \frac{4iabx^{-3n}(ex)^{3n} \operatorname{PolyLog}(3, ie^{c+dx^n})}{d^3en} \\ & + \frac{b^2x^{-n}(ex)^{3n} \tanh(c + dx^n)}{den} \end{aligned}$$



output  $\frac{1}{3}a^2(e^x)^{3n}/e/n+b^2(e^x)^{3n}/d/e/n/(x^n)+4a*b*(e^x)^{3n}*\arctan(\exp(c+d*x^n))/d/e/n/(x^n)-2*b^2*(e^x)^{3n}*\ln(1+\exp(2*c+2*d*x^n))/d^2/e/n/(x^{2n})-4*I*a*b*(e^x)^{3n}*polylog(2,-I*\exp(c+d*x^n))/d^2/e/n/(x^{2n})+4*I*a*b*(e^x)^{3n}*polylog(2,I*\exp(c+d*x^n))/d^2/e/n/(x^{2n})-b^2*(e^x)^{3n}*polylog(2,-\exp(2*c+2*d*x^n))/d^3/e/n/(x^{3n})+4*I*a*b*(e^x)^{3n}*polylog(3,-I*\exp(c+d*x^n))/d^3/e/n/(x^{3n})-4*I*a*b*(e^x)^{3n}*polylog(3,I*\exp(c+d*x^n))/d^3/e/n/(x^{3n})+b^2*(e^x)^{3n}*tanh(c+d*x^n)/d/e/n/(x^n)$

### 3.78.2 Mathematica [F]

$$\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx = \int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx$$

input `Integrate[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n])^2,x]`

output `Integrate[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n])^2, x]`

### 3.78.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.65, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {5963, 5959, 3042, 4678, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int (ex)^{3n-1} (a + b\operatorname{sech}(c + dx^n))^2 dx \\ \downarrow \text{5963} \\ \frac{x^{-3n}(ex)^{3n} \int x^{3n-1} (a + b\operatorname{sech}(dx^n + c))^2 dx}{e} \\ \downarrow \text{5959} \\ \frac{x^{-3n}(ex)^{3n} \int x^{2n} (a + b\operatorname{sech}(dx^n + c))^2 dx^n}{en} \\ \downarrow \text{3042} \end{array}$$

---

3.78.  $\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx$

$$\frac{x^{-3n}(ex)^{3n} \int x^{2n} (a + b \csc (idx^n + ic + \frac{\pi}{2}))^2 dx^n}{en}$$

↓ 4678

$$\frac{x^{-3n}(ex)^{3n} \int (a^2x^{2n} + b^2\operatorname{sech}^2(dx^n + c)x^{2n} + 2ab\operatorname{sech}(dx^n + c)x^{2n}) dx^n}{en}$$

↓ 2009

$$\frac{x^{-3n}(ex)^{3n} \left( \frac{1}{3}a^2x^{3n} + \frac{4abx^{2n} \arctan(e^{c+dx^n})}{d} + \frac{4iab \operatorname{PolyLog}(3, -ie^{dx^n+c})}{d^3} - \frac{4iab \operatorname{PolyLog}(3, ie^{dx^n+c})}{d^3} - \frac{4iabx^n \operatorname{PolyLog}(2, -ie^{dx^n+c})}{d^2} \right)}{en}$$

input `Int[(e*x)^(-1 + 3*n)*(a + b*Sech[c + d*x^n])^2,x]`

output `((e*x)^(3*n)*((b^2*x^(2*n))/d + (a^2*x^(3*n))/3 + (4*a*b*x^(2*n)*ArcTan[E^(c + d*x^n)]/d - (2*b^2*x^n*Log[1 + E^(2*(c + d*x^n))])/d^2 - ((4*I)*a*b*x^n*PolyLog[2, (-I)*E^(c + d*x^n)]/d^2 + ((4*I)*a*b*x^n*PolyLog[2, I*E^(c + d*x^n)]/d^2 - (b^2*PolyLog[2, -E^(2*(c + d*x^n))])/d^3 + ((4*I)*a*b*PolyLog[3, (-I)*E^(c + d*x^n)]/d^3 - ((4*I)*a*b*PolyLog[3, I*E^(c + d*x^n)]/d^3 + (b^2*x^(2*n)*Tanh[c + d*x^n])/d))/(e*n*x^(3*n))`

### 3.78.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4678 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

---

3.78.  $\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx$

rule 5963 `Int[((e_)*(x_))^(m_)*((a_) + (b_)*Sech[(c_) + (d_)*(x_)^(n_)])^(p_),  
x_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*  
(a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]`

### 3.78.4 Maple [F]

$$\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

input `int((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n))^2,x)`

output `int((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n))^2,x)`

### 3.78.5 Fricas [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + dx^n))^2 dx = \text{Timed out}$$

input `integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")`

output `Timed out`

### 3.78.6 Sympy [F]

$$\int (ex)^{-1+3n} (a + b \operatorname{sech}(c + dx^n))^2 dx = \int (ex)^{3n-1} (a + b \operatorname{sech}(c + dx^n))^2 dx$$

input `integrate((e*x)**(-1+3*n)*(a+b*sech(c+d*x**n))**2,x)`

output `Integral((e*x)**(3*n - 1)*(a + b*sech(c + d*x**n))**2, x)`

**3.78.7 Maxima [F]**

$$\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx = \int (b\operatorname{sech}(dx^n + c) + a)^2 (ex)^{3n-1} dx$$

input `integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")`

output `-2*b^2*e^(3*n)*x^(2*n)/(d*e*n*e^(2*d*x^n + 2*c) + d*e*n) + 1/3*(e*x)^(3*n)*a^2/(e*n) + integrate(4*(a*b*d*e^(3*n)*e^(d*x^n + 3*n*log(x) + c) + b^2*e^(3*n)*x^(2*n))/(d*e*x*e^(2*d*x^n + 2*c) + d*e*x), x)`

**3.78.8 Giac [F]**

$$\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx = \int (b\operatorname{sech}(dx^n + c) + a)^2 (ex)^{3n-1} dx$$

input `integrate((e*x)^(-1+3*n)*(a+b*sech(c+d*x^n))^2,x, algorithm="giac")`

output `integrate((b*sech(d*x^n + c) + a)^2*(e*x)^(3*n - 1), x)`

**3.78.9 Mupad [F(-1)]**

Timed out.

$$\int (ex)^{-1+3n} (a + b\operatorname{sech}(c + dx^n))^2 dx = \int \left( a + \frac{b}{\cosh(c + dx^n)} \right)^2 (ex)^{3n-1} dx$$

input `int((a + b/cosh(c + d*x^n))^2*(e*x)^(3*n - 1),x)`

output `int((a + b/cosh(c + d*x^n))^2*(e*x)^(3*n - 1), x)`

### 3.79 $\int \frac{(ex)^{-1+n}}{a+b\operatorname{sech}(c+dx^n)} dx$

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#### 3.79.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{(ex)^{-1+n}}{a+b\operatorname{sech}(c+dx^n)} dx = \frac{(ex)^n}{aen} - \frac{2bx^{-n}(ex)^n \arctan\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}en}$$

output  $(e*x)^n/a/e/n-2*b*(e*x)^n*\arctan((a-b)^{(1/2)*\tanh(1/2*c+1/2*d*x^n)/(a+b)^{(1/2)})/a/d/e/n/(x^n)/(a-b)^{(1/2)/(a+b)^{(1/2)}$

#### 3.79.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^{-1+n}}{a+b\operatorname{sech}(c+dx^n)} dx = \frac{(ex)^n \left( d + cx^{-n} + \frac{2bx^{-n} \arctan\left(\frac{(-a+b) \tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} \right)}{aden}$$

input `Integrate[(e*x)^(-1 + n)/(a + b*Sech[c + d*x^n]),x]`

output  $((e*x)^n*(d + c/x^n + (2*b*ArcTan[((-a + b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*x^n))/(a*d*e*n)$

**3.79.3 Rubi [A] (verified)**

Time = 0.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$ , Rules used = {5963, 5959, 3042, 4270, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int \frac{(ex)^{n-1}}{a + b \operatorname{sech}(c + dx^n)} dx \\
 \downarrow \text{5963} \\
 \frac{x^{-n}(ex)^n \int \frac{x^{n-1}}{a + b \operatorname{sech}(dx^n + c)} dx}{e} \\
 \downarrow \text{5959} \\
 \frac{x^{-n}(ex)^n \int \frac{1}{a + b \operatorname{sech}(dx^n + c)} dx^n}{en} \\
 \downarrow \text{3042} \\
 \frac{x^{-n}(ex)^n \int \frac{1}{a + b \csc(idx^n + ic + \frac{\pi}{2})} dx^n}{en} \\
 \downarrow \text{4270} \\
 \frac{x^{-n}(ex)^n \left( \frac{x^n}{a} - \frac{\int \frac{1}{\frac{a \cosh(dx^n + c)}{b} + 1} dx^n}{a} \right)}{en} \\
 \downarrow \text{3042} \\
 \frac{x^{-n}(ex)^n \left( \frac{x^n}{a} - \frac{\int \frac{1}{\frac{a \sin(idx^n + ic + \frac{\pi}{2})}{b} + 1} dx^n}{a} \right)}{en} \\
 \downarrow \text{3138} \\
 \frac{x^{-n}(ex)^n \left( \frac{x^n}{a} + \frac{2i \int \frac{1}{(1 - \frac{a}{b})x^{2n} + \frac{a+b}{b}} d(i \tanh(\frac{1}{2}(dx^n + c)))}{ad} \right)}{en} \\
 \downarrow \text{221}
 \end{array}$$

$$\frac{x^{-n}(ex)^n \left( \frac{x^n}{a} - \frac{2b \arctan\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}} \right)}{en}$$

input `Int[(e*x)^(-1 + n)/(a + b*Sech[c + d*x^n]),x]`

output `((e*x)^n*(x^n/a - (2*b*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x^n)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(e*n*x^n)`

### 3.79.3.1 Defintions of rubi rules used

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3138 `Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

rule 4270 `Int[(csc[(c_.) + (d_.)*(x_)])*(b_.) + (a_)^(-1), x_Symbol] := Simp[x/a, x] - Simp[1/a Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

rule 5963 `Int[((e_)*(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]`

### 3.79.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.85 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.64

method	result
risch	$\frac{x e^{\frac{(-1+n)(-i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie x) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie x)^2 \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie x)^2 \pi - i \operatorname{csgn}(ie x)^3 \pi + 2 \ln(e) + 2 \ln(x))}{2}}}{an} - \frac{2b e^{-\frac{i \pi n \operatorname{csgn}(ie)}{2}}}{an}$

```
input int((e*x)^(-1+n)/(a+b*sech(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
output 1/a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*
csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2
*ln(x))-2*b/a/n*exp(-1/2*I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*
I*Pi*n*csgn(I*e)*csgn(I*e*x)^2)*exp(1/2*I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*ex
p(-1/2*I*Pi*n*csgn(I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))
*exp(-1/2*I*Pi*csgn(I*e)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)
)^2)*exp(1/2*I*Pi*csgn(I*e*x)^3)*e^n/e*exp(c)/d/(a^2*exp(2*c)-exp(2*c)*b^2
)^(1/2)*arctan(1/2*(2*a*exp(2*c+d*x^n)+2*exp(c)*b)/(a^2*exp(2*c)-exp(2*c)*
b^2)^(1/2))
```

### 3.79.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(78) = 156.

Time = 0.30 (sec) , antiderivative size = 511, normalized size of antiderivative = 5.87

$$\int \frac{(ex)^{-1+n}}{a + b \operatorname{sech}(c + dx^n)} dx$$

$$= \left[ \frac{(a^2 - b^2)d \cosh((n - 1) \log(e)) \cosh(n \log(x)) + (a^2 - b^2)d \cosh(n \log(x)) \sinh((n - 1) \log(e)) - (\sqrt{a^2 - b^2}) \cosh(n \log(x)) \cosh((n - 1) \log(e))}{(a^2 - b^2) \cosh(n \log(x)) \cosh((n - 1) \log(e)) + (a^2 - b^2) \cosh((n - 1) \log(e)) \cosh(n \log(x)) - (\sqrt{a^2 - b^2}) \cosh(n \log(x)) \sinh((n - 1) \log(e)) - (\sqrt{a^2 - b^2}) \sinh(n \log(x)) \cosh((n - 1) \log(e))} \right]$$

```
input integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n)),x, algorithm="fracas")
```



output `[((a^2 - b^2)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^2 - b^2)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) - (sqrt(-a^2 + b^2)*b*cosh((n - 1)*log(e)) + sqrt(-a^2 + b^2)*b*sinh((n - 1)*log(e)))*log((a*b + (b^2 + sqrt(-a^2 + b^2)*b)*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a^2 - b^2 - sqrt(-a^2 + b^2)*b)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sqrt(-a^2 + b^2)*a)/(a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + b)) + ((a^2 - b^2)*d*cosh((n - 1)*log(e)) + (a^2 - b^2)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))/((a^3 - a*b^2)*d*n), ((a^2 - b^2)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^2 - b^2)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + 2*(sqrt(a^2 - b^2)*b*cosh((n - 1)*log(e)) + sqrt(a^2 - b^2)*b*sinh((n - 1)*log(e)))*arctan(-(sqrt(a^2 - b^2)*a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sqrt(a^2 - b^2)*a*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + sqrt(a^2 - b^2)*b)/(a^2 - b^2)) + ((a^2 - b^2)*d*cosh((n - 1)*log(e)) + (a^2 - b^2)*d*sinh((n - 1)*log(e)))*sinh(n*log(x)))/((a^3 - a*b^2)*d*n)]`

### 3.79.6 Sympy [F]

$$\int \frac{(ex)^{-1+n}}{a + b \operatorname{sech}(c + dx^n)} dx = \int \frac{(ex)^{n-1}}{a + b \operatorname{sech}(c + dx^n)} dx$$

input `integrate((e*x)**(-1+n)/(a+b*sech(c+d*x**n)),x)`

output `Integral((e*x)**(n - 1)/(a + b*sech(c + d*x**n)), x)`

### 3.79.7 Maxima [F]

$$\int \frac{(ex)^{-1+n}}{a + b \operatorname{sech}(c + dx^n)} dx = \int \frac{(ex)^{n-1}}{b \operatorname{sech}(dx^n + c) + a} dx$$

input `integrate((e*x)**(-1+n)/(a+b*sech(c+d*x^n)),x, algorithm="maxima")`

output `-2*b*e^n*integrate(e^(d*x^n + n*log(x) + c)/(a^2*e*x*e^(2*d*x^n + 2*c) + 2*a*b*e*x*e^(d*x^n + c) + a^2*e*x), x) + e^(n - 1)*x^n/(a*n)`

## 3.79.8 Giac [F]

$$\int \frac{(ex)^{-1+n}}{a + b \operatorname{sech}(c + dx^n)} dx = \int \frac{(ex)^{n-1}}{b \operatorname{sech}(dx^n + c) + a} dx$$

input `integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n)),x, algorithm="giac")`

output `integrate((e*x)^(n - 1)/(b*sech(d*x^n + c) + a), x)`

## 3.79.9 Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 409, normalized size of antiderivative = 4.70

$$\int \frac{(ex)^{-1+n}}{a + b \operatorname{sech}(c + dx^n)} dx = \frac{x (ex)^{n-1}}{a n} \left( 2 \operatorname{atan} \left( \frac{a^2 e^{dx^n} e^c \left( \frac{2 b x (ex)^{n-1}}{a^4 d n x^n \sqrt{b^2 x^2 (ex)^{2n-2}} + \frac{2 b d n x^n (ex)^{1-n} \sqrt{b^2 x^2 (ex)^{2n-2}}}{a^2 x \sqrt{a^4 d^2 n^2 x^{2n} - a^2 b^2 d^2 n^2 x^{2n}}} \right)}{2} \right) \sqrt{a^4 d^2 n^2 x^{2n} - a^2 b^2 d^2 n^2 x^{2n}} \right)$$

input `int((e*x)^(n - 1)/(a + b/cosh(c + d*x^n)),x)`

output `(x*(e*x)^(n - 1))/(a*n) - ((2*atan((a^2*exp(d*x^n)*exp(c))*((2*b*x*(e*x)^(n - 1))/(a^4*d*n*x^n*(b^2*x^2*(e*x)^(2*n - 2))^(1/2)) + (2*b*d*n*x^n*(e*x)^(1 - n)*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(a^2*x*(a^4*d^2*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x^(2*n))^(1/2)*(a^2*d^2*n^2*x^(2*n)*(a^2 - b^2))^(1/2)))*(a^4*d^2*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x^(2*n))^(1/2))/2 + (a*d*n*x^n*(e*x)^(1 - n)*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(x*(a^2*d^2*n^2*x^(2*n)*(a^2 - b^2))^(1/2))) + 2*atan((x*(e*x)^(n - 1)*(a^2*d^2*n^2*x^(2*n)*(a^2 - b^2))^(1/2))/(a*d*n*x^n*(b^2*x^2*(e*x)^(2*n - 2))^(1/2)))*(b^2*x^2*(e*x)^(2*n - 2))^(1/2))/(a^4*d^2*n^2*x^(2*n) - a^2*b^2*d^2*n^2*x^(2*n))^(1/2)`

### 3.80 $\int \frac{(ex)^{-1+2n}}{a+b\operatorname{sech}(c+dx^n)} dx$

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#### 3.80.1 Optimal result

Integrand size = 24, antiderivative size = 307

$$\int \frac{(ex)^{-1+2n}}{a+b\operatorname{sech}(c+dx^n)} dx = \frac{(ex)^{2n}}{2aen} - \frac{bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{bx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} + \frac{bx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}$$

output  $1/2*(e*x)^{(2*n)}/a/e/n-b*(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)}))/a/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}+b*(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)}))/a/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}-b*(e*x)^{(2*n)}*\operatorname{polylog}(2,-a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)}))/a/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}+b*(e*x)^{(2*n)}*\operatorname{polylog}(2,-a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)}))/a/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}$

### 3.80.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.70 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.80

$$\int \frac{(ex)^{-1+2n}}{a + b \operatorname{sech}(c + dx^n)} dx$$

$$(ex)^{2n} (b + a \cosh(c + dx^n)) \left( 1 + \frac{2bx^{-2n} \left( 2(c+dx^n) \arctan\left(\frac{(a+b) \coth\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right) + 2(c-i \arccos(-\frac{b}{a})) \arctan\left(\frac{(a-b) \tanh\left(\frac{1}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\right)}{\right)}$$

input `Integrate[(e*x)^(-1 + 2*n)/(a + b*Sech[c + d*x^n]),x]`

output

```
((e*x)^(2*n)*(b + a*Cosh[c + d*x^n])*(1 + (2*b*(2*(c + d*x^n)*ArcTan[((a + b)*Coth[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + 2*(c - I*ArcCos[-(b/a)])*ArcTan[((a - b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + (ArcCos[-(b/a)] + 2*(ArcTan[((a + b)*Coth[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + ArcTan[((a - b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^2]*E^(-1/2*c - (d*x^n)/2))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Cosh[c + d*x^n]])] + (ArcCos[-(b/a)] - 2*(ArcTan[((a + b)*Coth[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + ArcTan[((a - b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^2]*E^((c + d*x^n)/2))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Cosh[c + d*x^n]])] - (ArcCos[-(b/a)] + 2*ArcTan[((a - b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[((a + b)*(-a + b + I*Sqrt[a^2 - b^2])*(-1 + Tanh[(c + d*x^n)/2]))/(a*(a + b + I*Sqrt[a^2 - b^2]*Tanh[(c + d*x^n)/2]))] - (ArcCos[-(b/a)] - 2*ArcTan[((a - b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[((a + b)*(a - b + I*Sqrt[a^2 - b^2])*(1 + Tanh[(c + d*x^n)/2]))/(a*(a + b + I*Sqrt[a^2 - b^2]*Tanh[(c + d*x^n)/2]))] + I*(PolyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b - I*Sqrt[a^2 - b^2])*Tanh[(c + d*x^n)/2]))/(a*(a + b + I*Sqrt[a^2 - b^2]*Tanh[(c + d*x^n)/2]))] - PolyLog[2, ((b + I*Sqrt[a^2 - b^2])*(a + b - I*Sqrt[a^2 - b^2])*Tanh[(c + d*x^n)/2]))/(a*(a + b + I*Sqrt[a^2 - b^2]*Tanh[(c + d*x^n)/2]))]))/(Sqrt[a^2 - b^2]*d^2*x^(2*n))*Sech[c + d*x^n]/(2*a*e^n*(a + b*Sech[c + d*x^n]))
```

### 3.80.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {5963, 5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(ex)^{2n-1}}{a + b \operatorname{sech}(c + dx^n)} dx \\
 & \quad \downarrow \text{5963} \\
 & \frac{x^{-2n}(ex)^{2n} \int \frac{x^{2n-1}}{a + b \operatorname{sech}(dx^n + c)} dx}{e} \\
 & \quad \downarrow \text{5959} \\
 & \frac{x^{-2n}(ex)^{2n} \int \frac{x^n}{a + b \operatorname{sech}(dx^n + c)} dx^n}{en} \\
 & \quad \downarrow \text{3042} \\
 & \frac{x^{-2n}(ex)^{2n} \int \frac{x^n}{a + b \csc(idx^n + ic + \frac{\pi}{2})} dx^n}{en} \\
 & \quad \downarrow \text{4679} \\
 & \frac{x^{-2n}(ex)^{2n} \int \left( \frac{x^n}{a} - \frac{bx^n}{a(b + a \cosh(dx^n + c))} \right) dx^n}{en} \\
 & \quad \downarrow \text{2009} \\
 & \frac{x^{-2n}(ex)^{2n} \left( -\frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{b \operatorname{PolyLog}\left(2, -\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{bx^n \log\left(\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}+1\right)}{ad\sqrt{b^2-a^2}} + \frac{bx^n \log\left(\frac{ae^{c+dx^n}}{\sqrt{b^2-a^2}+b}+1\right)}{ad\sqrt{b^2-a^2}} + \frac{x^{2n}}{2a} \right)}{en}
 \end{aligned}$$

input `Int[(e*x)^(-1 + 2*n)/(a + b*Sech[c + d*x^n]), x]`

```
output ((e*x)^(2*n)*(x^(2*n)/(2*a) - (b*x^n*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]]))/(a*Sqrt[-a^2 + b^2]*d) + (b*x^n*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]]))/(a*Sqrt[-a^2 + b^2]*d) - (b*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))))/(a*Sqrt[-a^2 + b^2]*d^2) + (b*PolyLog[2, -((a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]))))/(a*Sqrt[-a^2 + b^2]*d^2)))/(e*n*x^(2*n))
```

### 3.80.3.1 Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

```
rule 4679 Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sinn[e + f*x]^n)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]
```

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

```
rule 5963 Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

### 3.80.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.78 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.91

method	result
risch	$x e^{\frac{(2n-1)(-i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie) \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie) \pi - i \operatorname{csgn}(ie) \operatorname{csgn}(ie) \pi + 2 \ln(e) + 2 \ln(x))}{2}} - \frac{2b e^{-i \pi n \operatorname{csgn}(ie)}}{2an}$

```
input int((e*x)^(2*n-1)/(a+b*sech(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
output 1/2/a/n*x*exp(1/2*(2*n-1)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*
e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e
)+2*ln(x))-2*b/a*exp(-I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(I*Pi*n*
csgn(I*e)*csgn(I*e*x)^2)*exp(I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-I*Pi*n*c
sgn(I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(-1/2*I*Pi*
csgn(I*e)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*exp(1/2*I*
Pi*csgn(I*e*x)^3)*(e^n)^2/e*exp(c)/n/d^2*(1/2*x^n*d*(ln((-a*exp(2*c+d*x^n)
-exp(c)*b+(exp(2*c)*b^2-a^2*exp(2*c))^(1/2)))/(-exp(c)*b+(exp(2*c)*b^2-a^2*
exp(2*c))^(1/2)))-ln((a*exp(2*c+d*x^n)+exp(c)*b+(exp(2*c)*b^2-a^2*exp(2*c)
)^(1/2))/(exp(c)*b+(exp(2*c)*b^2-a^2*exp(2*c))^(1/2)))/(exp(2*c)*b^2-a^2*
exp(2*c))^(1/2)+1/2*(dilog((-a*exp(2*c+d*x^n)-exp(c)*b+(exp(2*c)*b^2-a^2*
exp(2*c))^(1/2)))/(-exp(c)*b+(exp(2*c)*b^2-a^2*exp(2*c))^(1/2))-dilog((a*
exp(2*c+d*x^n)+exp(c)*b+(exp(2*c)*b^2-a^2*exp(2*c))^(1/2))/(exp(c)*b+(exp(2*
c)*b^2-a^2*exp(2*c))^(1/2)))/(exp(2*c)*b^2-a^2*exp(2*c))^(1/2))
```

### 3.80.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1286 vs.  $2(287) = 574$ .

Time = 0.30 (sec) , antiderivative size = 1286, normalized size of antiderivative = 4.19

$$\int \frac{(ex)^{-1+2n}}{a+b\operatorname{sech}(c+dx^n)} dx = \text{Too large to display}$$

```
input integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n)),x, algorithm="fracas")
```

```

output 1/2*((a^2 - b^2)*d^2*cosh((2*n - 1)*log(e))*cosh(n*log(x))^2 + (a^2 - b^2)
*d^2*cosh(n*log(x))^2*sinh((2*n - 1)*log(e)) + ((a^2 - b^2)*d^2*cosh((2*n
- 1)*log(e)) + (a^2 - b^2)*d^2*sinh((2*n - 1)*log(e)))*sinh(n*log(x))^2 +
2*(a*b*sqrt(-(a^2 - b^2)/a^2)*cosh((2*n - 1)*log(e)) + a*b*sqrt(-(a^2 - b^
2)/a^2)*sinh((2*n - 1)*log(e)))*dilog(-(a*sqrt(-(a^2 - b^2)/a^2) + b)*cos
h(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqrt(-(a^2 - b^2)/a^2) + b
)*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + a)/a + 1) - 2*(a*b*sqrt(
-(a^2 - b^2)/a^2)*cosh((2*n - 1)*log(e)) + a*b*sqrt(-(a^2 - b^2)/a^2)*sinh
((2*n - 1)*log(e)))*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cosh(d*cosh(n*lo
g(x)) + d*sinh(n*log(x)) + c) + (a*sqrt(-(a^2 - b^2)/a^2) - b)*sinh(d*cos
h(n*log(x)) + d*sinh(n*log(x)) + c) - a)/a + 1) + 2*(a*b*c*sqrt(-(a^2 - b^2
)/a^2)*cosh((2*n - 1)*log(e)) + a*b*c*sqrt(-(a^2 - b^2)/a^2)*sinh((2*n - 1
)*log(e))*log(2*a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*a*sin
h(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*a*sqrt(-(a^2 - b^2)/a^2) +
2*b) - 2*(a*b*c*sqrt(-(a^2 - b^2)/a^2)*cosh((2*n - 1)*log(e)) + a*b*c*sqrt
(-(a^2 - b^2)/a^2)*sinh((2*n - 1)*log(e))*log(2*a*cosh(d*cosh(n*log(x)) +
d*sinh(n*log(x)) + c) + 2*a*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)
- 2*a*sqrt(-(a^2 - b^2)/a^2) + 2*b) + 2*(a*b*d*sqrt(-(a^2 - b^2)/a^2)*cos
h((2*n - 1)*log(e))*cosh(n*log(x)) + a*b*c*sqrt(-(a^2 - b^2)/a^2)*cosh((2*
n - 1)*log(e)) + (a*b*d*sqrt(-(a^2 - b^2)/a^2)*cosh(n*log(x)) + a*b*c*s...

```

### 3.80.6 Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{a + b \operatorname{sech}(c + dx^n)} dx = \int \frac{(ex)^{2n-1}}{a + b \operatorname{sech}(c + dx^n)} dx$$

```
input integrate((e*x)**(-1+2*n)/(a+b*sech(c+d*x**n)),x)
```

```
output Integral((e*x)**(2*n - 1)/(a + b*sech(c + d*x**n)), x)
```



**3.80.7 Maxima [F]**

$$\int \frac{(ex)^{-1+2n}}{a + b \operatorname{sech}(c + dx^n)} dx = \int \frac{(ex)^{2n-1}}{b \operatorname{sech}(dx^n + c) + a} dx$$

input `integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n)),x, algorithm="maxima")`

output `-2*b*e^(2*n)*integrate(e^(d*x^n + 2*n*log(x) + c)/(a^2*e*x*e^(2*d*x^n + 2*c) + 2*a*b*e*x*e^(d*x^n + c) + a^2*e*x), x) + 1/2*e^(2*n - 1)*x^(2*n)/(a*n)`

**3.80.8 Giac [F]**

$$\int \frac{(ex)^{-1+2n}}{a + b \operatorname{sech}(c + dx^n)} dx = \int \frac{(ex)^{2n-1}}{b \operatorname{sech}(dx^n + c) + a} dx$$

input `integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n)),x, algorithm="giac")`

output `integrate((e*x)^(2*n - 1)/(b*sech(d*x^n + c) + a), x)`

**3.80.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{(ex)^{-1+2n}}{a + b \operatorname{sech}(c + dx^n)} dx = \int \frac{(ex)^{2n-1}}{a + \frac{b}{\cosh(c+dx^n)}} dx$$

input `int((e*x)^(2*n - 1)/(a + b/cosh(c + d*x^n)),x)`

output `int((e*x)^(2*n - 1)/(a + b/cosh(c + d*x^n)), x)`

### 3.81 $\int \frac{(ex)^{-1+3n}}{a+b\operatorname{sech}(c+dx^n)} dx$

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#### 3.81.1 Optimal result

Integrand size = 24, antiderivative size = 452

$$\int \frac{(ex)^{-1+3n}}{a+b\operatorname{sech}(c+dx^n)} dx = \frac{(ex)^{3n}}{3aen} - \frac{bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{2bx^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} + \frac{2bx^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} + \frac{2bx^{-3n}(ex)^{3n} \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en} - \frac{2bx^{-3n}(ex)^{3n} \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}$$

output  $1/3*(e*x)^{(3*n)}/a/e/n-b*(e*x)^{(3*n)}*\ln(1+a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)})))/a/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}+b*(e*x)^{(3*n)}*\ln(1+a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)})))/a/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}-2*b*(e*x)^{(3*n)}*\text{polylog}(2,-a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)})))/a/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}+2*b*(e*x)^{(3*n)}*\text{polylog}(2,-a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)})))/a/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}+2*b*(e*x)^{(3*n)}*\text{polylog}(3,-a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)})))/a/d^3/e/n/(x^{(3*n)})/(-a^2+b^2)^{(1/2)}-2*b*(e*x)^{(3*n)}*\text{polylog}(3,-a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)})))/a/d^3/e/n/(x^{(3*n)})/(-a^2+b^2)^{(1/2)}$

### 3.81.2 Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{a + b\text{sech}(c + dx^n)} dx = \int \frac{(ex)^{-1+3n}}{a + b\text{sech}(c + dx^n)} dx$$

input `Integrate[(e*x)^(-1 + 3*n)/(a + b*Sech[c + d*x^n]),x]`

output `Integrate[(e*x)^(-1 + 3*n)/(a + b*Sech[c + d*x^n]), x]`

### 3.81.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {5963, 5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c} \int \frac{(ex)^{3n-1}}{a + b\text{sech}(c + dx^n)} dx \\ \downarrow \text{5963} \\ \frac{x^{-3n}(ex)^{3n} \int \frac{x^{3n-1}}{a+b\text{sech}(dx^n+c)} dx}{e} \\ \downarrow \text{5959} \\ \frac{x^{-3n}(ex)^{3n} \int \frac{x^{2n}}{a+b\text{sech}(dx^n+c)} dx^n}{en} \end{array}$$

---

3.81.  $\int \frac{(ex)^{-1+3n}}{a+b\text{sech}(c+dx^n)} dx$

$$\begin{array}{c}
 \downarrow \text{3042} \\
 \frac{x^{-3n}(ex)^{3n} \int \frac{x^{2n}}{a+b \csc(dx^n+ic+\frac{\pi}{2})} dx^n}{en} \\
 \downarrow \text{4679} \\
 \frac{x^{-3n}(ex)^{3n} \int \left( \frac{x^{2n}}{a} - \frac{bx^{2n}}{a(b+a \cosh(dx^n+c))} \right) dx^n}{en} \\
 \downarrow \text{2009} \\
 \frac{x^{-3n}(ex)^{3n} \left( \frac{2b \operatorname{PolyLog}\left(3, -\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{2b \operatorname{PolyLog}\left(3, -\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{2bx^n \operatorname{PolyLog}\left(2, -\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{2bx^n \operatorname{PolyLog}\left(2, -\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} \right)}{en}
 \end{array}$$

input `Int[(e*x)^(-1 + 3*n)/(a + b*Sech[c + d*x^n]), x]`

output `((e*x)^(3*n)*(x^(3*n)/(3*a) - (b*x^(2*n)*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]])/(a*Sqrt[-a^2 + b^2]*d) + (b*x^(2*n)*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]])/(a*Sqrt[-a^2 + b^2]*d) - (2*b*x^n*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (2*b*x^n*PolyLog[2, -((a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^2) + (2*b*PolyLog[3, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3) - (2*b*PolyLog[3, -((a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]))])/(a*Sqrt[-a^2 + b^2]*d^3))/(e*n*x^(3*n))`

### 3.81.3.1 Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 4679 `Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]`

---

3.81.  $\int \frac{(ex)^{-1+3n}}{a+b\operatorname{sech}(c+dx^n)} dx$

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol]
  := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x]
  /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

```
rule 5963 Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.),
  x_Symbol] := Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sech[c + d*x^n])^p, x], x]
  /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

### 3.81.4 Maple [F]

$$\int \frac{(ex)^{-1+3n}}{a + b \operatorname{sech}(c + dx^n)} dx$$

```
input int((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n)),x)
```

```
output int((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n)),x)
```

### 3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2005 vs.  $2(426) = 852$ .

Time = 0.32 (sec) , antiderivative size = 2005, normalized size of antiderivative = 4.44

$$\int \frac{(ex)^{-1+3n}}{a + b \operatorname{sech}(c + dx^n)} dx = \text{Too large to display}$$

```
input integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n)),x, algorithm="fricas")
```

```

output 1/3*((a^2 - b^2)*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x))^3 + (a^2 - b^2)
*d^3*cosh(n*log(x))^3*sinh((3*n - 1)*log(e)) + ((a^2 - b^2)*d^3*cosh((3*n
- 1)*log(e)) + (a^2 - b^2)*d^3*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^3 +
3*((a^2 - b^2)*d^3*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + (a^2 - b^2)*d^3
*cosh(n*log(x))*sinh((3*n - 1)*log(e)))*sinh(n*log(x))^2 + 6*(a*b*d*sqrt(-
(a^2 - b^2)/a^2)*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + a*b*d*sqrt(-(a^2
- b^2)/a^2)*cosh(n*log(x))*sinh((3*n - 1)*log(e)) + (a*b*d*sqrt(-(a^2 - b^
2)/a^2)*cosh((3*n - 1)*log(e)) + a*b*d*sqrt(-(a^2 - b^2)/a^2)*sinh((3*n -
1)*log(e)))*sinh(n*log(x))*dilog(-(a*sqrt(-(a^2 - b^2)/a^2) + b)*cosh(d*
cosh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqrt(-(a^2 - b^2)/a^2) + b)*si
nh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + a)/a + 1) - 6*(a*b*d*sqrt(-(
a^2 - b^2)/a^2)*cosh((3*n - 1)*log(e))*cosh(n*log(x)) + a*b*d*sqrt(-(a^2 -
b^2)/a^2)*cosh(n*log(x))*sinh((3*n - 1)*log(e)) + (a*b*d*sqrt(-(a^2 - b^2
)/a^2)*cosh((3*n - 1)*log(e)) + a*b*d*sqrt(-(a^2 - b^2)/a^2)*sinh((3*n - 1
)*log(e)))*sinh(n*log(x))*dilog(((a*sqrt(-(a^2 - b^2)/a^2) - b)*cosh(d*co
sh(n*log(x)) + d*sinh(n*log(x)) + c) + (a*sqrt(-(a^2 - b^2)/a^2) - b)*sinh
(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - a)/a + 1) - 3*(a*b*c^2*sqrt(-(
a^2 - b^2)/a^2)*cosh((3*n - 1)*log(e)) + a*b*c^2*sqrt(-(a^2 - b^2)/a^2)*si
nh((3*n - 1)*log(e))*log(2*a*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c
) + 2*a*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) + 2*a*sqrt(-(a^2 ...

```

### 3.81.6 Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{a + b \operatorname{sech}(c + dx^n)} dx = \int \frac{(ex)^{3n-1}}{a + b \operatorname{sech}(c + dx^n)} dx$$

```
input integrate((e*x)**(-1+3*n)/(a+b*sech(c+d*x**n)),x)
```

```
output Integral((e*x)**(3*n - 1)/(a + b*sech(c + d*x**n)), x)
```

**3.81.7 Maxima [F]**

$$\int \frac{(ex)^{-1+3n}}{a + b \operatorname{sech}(c + dx^n)} dx = \int \frac{(ex)^{3n-1}}{b \operatorname{sech}(dx^n + c) + a} dx$$

input `integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n)),x, algorithm="maxima")`

output `-2*b*e^(3*n)*integrate(e^(d*x^n + 3*n*log(x) + c)/(a^2*e*x*e^(2*d*x^n + 2*c) + 2*a*b*e*x*e^(d*x^n + c) + a^2*e*x), x) + 1/3*e^(3*n - 1)*x^(3*n)/(a*n)`

**3.81.8 Giac [F]**

$$\int \frac{(ex)^{-1+3n}}{a + b \operatorname{sech}(c + dx^n)} dx = \int \frac{(ex)^{3n-1}}{b \operatorname{sech}(dx^n + c) + a} dx$$

input `integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n)),x, algorithm="giac")`

output `integrate((e*x)^(3*n - 1)/(b*sech(d*x^n + c) + a), x)`

**3.81.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{(ex)^{-1+3n}}{a + b \operatorname{sech}(c + dx^n)} dx = \int \frac{(ex)^{3n-1}}{a + \frac{b}{\cosh(c+dx^n)}} dx$$

input `int((e*x)^(3*n - 1)/(a + b/cosh(c + d*x^n)),x)`

output `int((e*x)^(3*n - 1)/(a + b/cosh(c + d*x^n)), x)`

$$3.82 \quad \int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$$

3.82.1	Optimal result	471
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3.82.6	Sympy [F]	477
3.82.7	Maxima [F]	478
3.82.8	Giac [F]	478
3.82.9	Mupad [F(-1)]	478

### 3.82.1 Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx = \frac{(ex)^n}{a^2 e n} - \frac{2b(2a^2 - b^2) x^{-n} (ex)^n \arctan\left(\frac{\sqrt{a-b} \tanh(\frac{1}{2}(c+dx^n))}{\sqrt{a+b}}\right)}{a^2 (a-b)^{3/2} (a+b)^{3/2} den} + \frac{b^2 x^{-n} (ex)^n \tanh(c+dx^n)}{a (a^2 - b^2) den (a+b\operatorname{sech}(c+dx^n))}$$

output  $(e*x)^n/a^2/e/n-2*b*(2*a^2-b^2)*(e*x)^n*\arctan((a-b)^{(1/2)}*\tanh(1/2*c+1/2*d*x^n)/(a+b)^{(1/2)})/a^2/(a-b)^{(3/2)}/(a+b)^{(3/2)}/d/e/n/(x^n)+b^2*(e*x)^n*\tanh(c+d*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(a+b*\operatorname{sech}(c+d*x^n))$

### 3.82.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.48

$$\int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx = \frac{x^{-n} (ex)^n \left( a \left( (a^2 - b^2)^{3/2} (c + dx^n) + (4a^2 b - 2b^3) \arctan\left(\frac{(-a+b) \tanh(\frac{1}{2}(c+dx^n))}{\sqrt{a^2-b^2}}\right) \right) \cosh(c + dx^n) + b \left( (a^2 - b^2)^{3/2} (c + dx^n) + (4a^2 b - 2b^3) \arctan\left(\frac{(-a+b) \tanh(\frac{1}{2}(c+dx^n))}{\sqrt{a^2-b^2}}\right) \right) \right)}{a^2 (a-b) (a+b) \sqrt{a^2 - b^2} den (b)}$$

input `Integrate[(e*x)^(-1 + n)/(a + b*Sech[c + d*x^n])^2,x]`

---

3.82.  $\int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$



output  $((e*x)^n*(a*((a^2 - b^2)^{(3/2)}*(c + d*x^n) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*x^n] + b*((a^2 - b^2)^{(3/2)}*(c + d*x^n) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x^n)/2])/Sqrt[a^2 - b^2]] + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*x^n]))/(a^2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d*e*n*x^n*(b + a*Cosh[c + d*x^n]))$

### 3.82.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$ , Rules used = {5963, 5959, 3042, 4272, 25, 3042, 4407, 3042, 4318, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{n-1}}{(a + b\operatorname{sech}(c + dx^n))^2} dx$$

↓ 5963

$$\frac{x^{-n}(ex)^n \int \frac{x^{n-1}}{(a+b\operatorname{sech}(dx^n+c))^2} dx}{e}$$

↓ 5959

$$\frac{x^{-n}(ex)^n \int \frac{1}{(a+b\operatorname{sech}(dx^n+c))^2} dx^n}{en}$$

↓ 3042

$$\frac{x^{-n}(ex)^n \int \frac{1}{(a+b \csc(idx^n+ic+\frac{\pi}{2}))^2} dx^n}{en}$$

↓ 4272

$$\frac{x^{-n}(ex)^n \left( \frac{b^2 \tanh(c+dx^n)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^n))} - \frac{\int -\frac{a^2-b\operatorname{sech}(dx^n+c)a-b^2}{a+b\operatorname{sech}(dx^n+c)} dx^n}{a(a^2-b^2)} \right)}{en}$$

↓ 25

$$\frac{x^{-n}(ex)^n \left( \frac{\int \frac{a^2-b\operatorname{sech}(dx^n+c)a-b^2}{a+b\operatorname{sech}(dx^n+c)} dx^n}{a(a^2-b^2)} + \frac{b^2 \tanh(c+dx^n)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^n))} \right)}{en}$$

---

3.82.  $\int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$

$$\begin{array}{c}
 \downarrow \text{3042} \\
 \frac{x^{-n}(ex)^n \left( \frac{b^2 \tanh(c+dx^n)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^n))} + \frac{\int \frac{a^2-b \csc\left(ix^n+ic+\frac{\pi}{2}\right)a-b^2}{a+b \csc\left(ix^n+ic+\frac{\pi}{2}\right)} dx^n}{a(a^2-b^2)} \right)}{en} \\
 \downarrow \text{4407} \\
 \frac{x^{-n}(ex)^n \left( \frac{\left(\frac{a^2-b^2}{a}\right)x^n - \frac{b(2a^2-b^2) \int \frac{\operatorname{sech}(dx^n+c)}{a+b\operatorname{sech}(dx^n+c)} dx^n}{a}}{a(a^2-b^2)} + \frac{b^2 \tanh(c+dx^n)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^n))} \right)}{en} \\
 \downarrow \text{3042} \\
 \frac{x^{-n}(ex)^n \left( \frac{b^2 \tanh(c+dx^n)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^n))} + \frac{\left(\frac{a^2-b^2}{a}\right)x^n - \frac{b(2a^2-b^2) \int \frac{\csc\left(ix^n+ic+\frac{\pi}{2}\right)}{a+b \csc\left(ix^n+ic+\frac{\pi}{2}\right)} dx^n}{a}}{a(a^2-b^2)} \right)}{en} \\
 \downarrow \text{4318} \\
 \frac{x^{-n}(ex)^n \left( \frac{\left(\frac{a^2-b^2}{a}\right)x^n - \frac{(2a^2-b^2) \int \frac{1}{a \cosh\left(\frac{dx^n+c}{b}\right)+1} dx^n}{a}}{a(a^2-b^2)} + \frac{b^2 \tanh(c+dx^n)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^n))} \right)}{en} \\
 \downarrow \text{3042} \\
 \frac{x^{-n}(ex)^n \left( \frac{b^2 \tanh(c+dx^n)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^n))} + \frac{\left(\frac{a^2-b^2}{a}\right)x^n - \frac{(2a^2-b^2) \int \frac{1}{a \sin\left(\frac{ix^n+ic+\frac{\pi}{2}}{b}\right)+1} dx^n}{a}}{a(a^2-b^2)} \right)}{en} \\
 \downarrow \text{3138} \\
 \frac{x^{-n}(ex)^n \left( \frac{b^2 \tanh(c+dx^n)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^n))} + \frac{\left(\frac{a^2-b^2}{a}\right)x^n + \frac{2i(2a^2-b^2) \int \frac{1}{\left(1-\frac{a}{b}\right)x^{2n} + \frac{a+b}{b}} d\left(i \tanh\left(\frac{1}{2}(dx^n+c)\right)\right)}{ad}}{a(a^2-b^2)} \right)}{en} \\
 \downarrow \text{221}
 \end{array}$$

---

3.82.  $\int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$

$$x^{-n}(ex)^n \left( \frac{\frac{(a^2-b^2)x^n}{a} - \frac{2b(2a^2-b^2) \arctan\left(\frac{\sqrt{a-b} \tanh\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}}{a(a^2-b^2)} + \frac{b^2 \tanh(c+dx^n)}{ad(a^2-b^2)(a+b\operatorname{sech}(c+dx^n))} \right)$$

*en*

input `Int[(e*x)^(-1 + n)/(a + b*Sech[c + d*x^n])^2,x]`

output `((e*x)^n*(((a^2 - b^2)*x^n)/a - (2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x^n)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]*d))/(a*(a^2 - b^2)) + (b^2*Tanh[c + d*x^n])/(a*(a^2 - b^2)*d*(a + b*Sech[c + d*x^n])))/(e*n*x^n)`

### 3.82.3.1 Defintions of rubi rules used

rule 25 `Int[-(Fx_), x_Symbol] := Simp[Identity[-1] Int[Fx, x], x]`

rule 221 `Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

rule 3042 `Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]`

rule 3138 `Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d) Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

rule 4272 `Int[(csc[(c_.) + (d_.)*(x_)])*(b_.) + (a_)^(n_), x_Symbol] := Simp[b^2*Cot[c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Simp[1/(a*(n + 1)*(a^2 - b^2)) Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]`

rule 4318 `Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[1/b Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]`

rule 4407 `Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[c*(x/a), x] - Simp[(b*c - a*d)/a Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]`

rule 5959 `Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]`

rule 5963 `Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]`

### 3.82.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.45 (sec) , antiderivative size = 491, normalized size of antiderivative = 3.13

method	result
risch	$\frac{x e^{\frac{(-1+n)(-i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie x) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie x)^2 \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie x)^2 \pi - i \operatorname{csgn}(ie x)^3 \pi + 2 \ln(e) + 2 \ln(x))}{2}}}{a^2 n} - \frac{2 b^2 e^{\frac{(-1+n)(-i \operatorname{csgn}(ie) \operatorname{csgn}(ix) \operatorname{csgn}(ie x) \pi + i \operatorname{csgn}(ie) \operatorname{csgn}(ie x)^2 \pi + i \operatorname{csgn}(ix) \operatorname{csgn}(ie x)^2 \pi - i \operatorname{csgn}(ie x)^3 \pi + 2 \ln(e) + 2 \ln(x))}{2}}}{a^2 n}$

input `int((e*x)^(-1+n)/(a+b*sech(c+d*x^n))^2,x,method=_RETURNVERBOSE)`

$$3.82. \int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$$

output `1/a^2/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*ln(x)))-2*b^2*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*ln(x)))*x*(b*exp(c+d*x^n)+a)/a^2/(a^2-b^2)/d/n/(x^n)/(2*b*exp(c+d*x^n)+a*exp(2*c+2*d*x^n)+a)-2*b/a^2*(2*a^2-b^2)/(a^2-b^2)/n*exp(-1/2*I*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*n*csgn(I*e)*csgn(I*e*x)^2)*exp(1/2*I*Pi*n*csgn(I*x)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*n*csgn(I*e*x)^3)*exp(1/2*I*Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(-1/2*I*Pi*csgn(I*e)*csgn(I*e*x)^2)*exp(-1/2*I*Pi*csgn(I*x)*csgn(I*e*x)^2)*exp(1/2*I*Pi*csgn(I*e*x)^3)*e^n/e*exp(c)/d/(a^2*exp(2*c)-exp(2*c)*b^2)^(1/2)*arctan(1/2*(2*a*exp(2*c+d*x^n)+2*exp(c)*b)/(a^2*exp(2*c)-exp(2*c)*b^2)^(1/2))`

### 3.82.5 Fracas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1758 vs.  $2(148) = 296$ .

Time = 0.32 (sec) , antiderivative size = 3547, normalized size of antiderivative = 22.59

$$\int \frac{(ex)^{-1+n}}{(a + b\operatorname{sech}(c + dx^n))^2} dx = \text{Too large to display}$$

input `integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n))^2,x, algorithm="fracas")`

output

```

[((a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + ((a^5
- 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2*a^3*
b^2 + a*b^4)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^2 + a
*b^4)*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n - 1)*lo
g(e))*sinh(n*log(x)))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (
(a^5 - 2*a^3*b^2 + a*b^4)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) + (a^5 - 2
*a^3*b^2 + a*b^4)*d*cosh(n*log(x))*sinh((n - 1)*log(e)) + ((a^5 - 2*a^3*b^
2 + a*b^4)*d*cosh((n - 1)*log(e)) + (a^5 - 2*a^3*b^2 + a*b^4)*d*sinh((n -
1)*log(e))*sinh(n*log(x)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^
2 + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh((n - 1)*log(e))*cosh(n*log(x)) - (
a^2*b^3 - b^5)*cosh((n - 1)*log(e)) - (a^2*b^3 - b^5 - (a^4*b - 2*a^2*b^3
+ b^5)*d*cosh(n*log(x)))*sinh((n - 1)*log(e)) + ((a^4*b - 2*a^2*b^3 + b^5)
*d*cosh((n - 1)*log(e)) + (a^4*b - 2*a^2*b^3 + b^5)*d*sinh((n - 1)*log(e))
)*sinh(n*log(x))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c) - 2*(a^3*b
^2 - a*b^4)*cosh((n - 1)*log(e)) - (((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*co
sh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*sinh((n - 1)*log(e)
))*cosh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + ((2*a^3*b - a*b^3)*s
qrt(-a^2 + b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*
sinh((n - 1)*log(e)))*sinh(d*cosh(n*log(x)) + d*sinh(n*log(x)) + c)^2 + (2
*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*cosh((n - 1)*log(e)) + (2*a^3*b - a*b^...

```

### 3.82.6 Sympy [F]

$$\int \frac{(ex)^{-1+n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{n-1}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

input `integrate((e*x)**(-1+n)/(a+b*sech(c+d*x**n))**2,x)`

output `Integral((e*x)**(n - 1)/(a + b*sech(c + d*x**n))**2, x)`

## 3.82.7 Maxima [F]

$$\int \frac{(ex)^{-1+n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{n-1}}{(b \operatorname{sech}(dx^n + c) + a)^2} dx$$

input `integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")`

output `-2*(2*a^2*b*e^n*e^c - b^3*e^n*e^c)*integrate(e^(d*x^n + n*log(x))/((a^5*e*e^(2*c) - a^3*b^2*e*e^(2*c))*x*e^(2*d*x^n) + 2*(a^4*b*e*e^c - a^2*b^3*e*e^c)*x*e^(d*x^n) + (a^5*e - a^3*b^2*e)*x), x) - (2*a*b^2*e^n - (a^3*d*e^n - a*b^2*d*e^n)*x^n - (a^3*d*e^n*e^(2*c) - a*b^2*d*e^n*e^(2*c))*e^(2*d*x^n + n*log(x)) + 2*(b^3*e^n*e^c - (a^2*b*d*e^n*e^c - b^3*d*e^n*e^c)*x^n)*e^(d*x^n))/(a^5*d*e*n - a^3*b^2*d*e*n + (a^5*d*e*n*e^(2*c) - a^3*b^2*d*e*n*e^(2*c))*e^(2*d*x^n) + 2*(a^4*b*d*e*n*e^c - a^2*b^3*d*e*n*e^c)*e^(d*x^n))`

## 3.82.8 Giac [F]

$$\int \frac{(ex)^{-1+n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{n-1}}{(b \operatorname{sech}(dx^n + c) + a)^2} dx$$

input `integrate((e*x)^(-1+n)/(a+b*sech(c+d*x^n))^2,x, algorithm="giac")`

output `integrate((e*x)^(n - 1)/(b*sech(d*x^n + c) + a)^2, x)`

## 3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{n-1}}{\left(a + \frac{b}{\cosh(c + dx^n)}\right)^2} dx$$

input `int((e*x)^(n - 1)/(a + b/cosh(c + d*x^n))^2,x)`

output `int((e*x)^(n - 1)/(a + b/cosh(c + d*x^n))^2, x)`

---

3.82.  $\int \frac{(ex)^{-1+n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$

$$3.83 \quad \int \frac{(ex)^{-1+2n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$$

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### 3.83.1 Optimal result

Integrand size = 24, antiderivative size = 717

$$\begin{aligned}
 \int \frac{(ex)^{-1+2n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx &= \frac{(ex)^{2n}}{2a^2en} + \frac{b^3x^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} \\
 &\quad - \frac{2bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} \\
 &\quad - \frac{b^3x^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} \\
 &\quad + \frac{2bx^{-n}(ex)^{2n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} \\
 &\quad - \frac{b^2x^{-2n}(ex)^{2n} \log(b+a\cosh(c+dx^n))}{a^2(a^2-b^2)d^2en} \\
 &\quad + \frac{b^3x^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2en} \\
 &\quad - \frac{2bx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} \\
 &\quad - \frac{b^3x^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2en} \\
 &\quad + \frac{2bx^{-2n}(ex)^{2n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} \\
 &\quad + \frac{b^2x^{-n}(ex)^{2n} \sinh(c+dx^n)}{a(a^2-b^2)den(b+a\cosh(c+dx^n))}
 \end{aligned}$$

output  $\frac{1}{2}*(e*x)^{(2*n)}/a^2/e/n-b^2*(e*x)^{(2*n)}*\ln(b+a*\cosh(c+d*x^n))/a^2/(a^2-b^2)/d^2/e/n/(x^{(2*n)})+b^3*(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d/e/n/(x^n)-b^3*(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d/e/n/(x^n)+b^3*(e*x)^{(2*n)}*\text{polylog}(2,-a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^2/e/n/(x^{(2*n)})-b^3*(e*x)^{(2*n)}*\text{polylog}(2,-a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)}))/a^2/(-a^2+b^2)^{(3/2)}/d^2/e/n/(x^{(2*n)})+b^2*(e*x)^{(2*n)}*\sinh(c+d*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(b+a*\cosh(c+d*x^n))-2*b*(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)}))/a^2/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}+2*b*(e*x)^{(2*n)}*\ln(1+a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)}))/a^2/d/e/n/(x^n)/(-a^2+b^2)^{(1/2)}-2*b*(e*x)^{(2*n)}*\text{polylog}(2,-a*\exp(c+d*x^n)/(b-(-a^2+b^2)^{(1/2)}))/a^2/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}+2*b*(e*x)^{(2*n)}*\text{polylog}(2,-a*\exp(c+d*x^n)/(b+(-a^2+b^2)^{(1/2)}))/a^2/d^2/e/n/(x^{(2*n)})/(-a^2+b^2)^{(1/2)}$

### 3.83.2 Mathematica [A] (verified)

Time = 6.46 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.65

$$\int \frac{(ex)^{-1+2n}}{(a + b\operatorname{sech}(c + dx^n))^2} dx$$

$$= \frac{x^{-2n}(ex)^{2n}(b + a \cosh(c + dx^n)) \operatorname{sech}^2(c + dx^n) \left( \frac{4b^2 de^{2c} x^n (b + a \cosh(c + dx^n))}{(a^2 - b^2)(1 + e^{2c})} + \frac{2b(b + a \cosh(c + dx^n)) (b\sqrt{-a^2 + b^2} \log(a + b \operatorname{sech}(c + dx^n)))}{(a^2 - b^2)(1 + e^{2c})} \right)}{(a + b \operatorname{sech}(c + dx^n))^2}$$

input `Integrate[(e*x)^(-1 + 2*n)/(a + b*Sech[c + d*x^n])^2,x]`

output  $((e*x)^{(2*n)}*(b + a*\Cosh[c + d*x^n])*Sech[c + d*x^n]^2*((4*b^2*d*E^{(2*c)}*x^n*(b + a*\Cosh[c + d*x^n]))/((a^2 - b^2)*(1 + E^{(2*c)})) + (2*b*(b + a*\Cosh[c + d*x^n])*(b*\sqrt{-a^2 + b^2}*\Log[a + 2*b*E^{(c + d*x^n)} + a*E^{(2*(c + d*x^n))}] + (2*a^2 - b^2)*(d*x^n*\Log[1 + (a*E^{(c + d*x^n)})/(b - \sqrt{-a^2 + b^2}]]) + \text{PolyLog}[2, (a*E^{(c + d*x^n)})/(-b + \sqrt{-a^2 + b^2}]]) - (2*a^2 - b^2)*(d*x^n*\Log[1 + (a*E^{(c + d*x^n)})/(b + \sqrt{-a^2 + b^2}]]) + \text{PolyLog}[2, -((a*E^{(c + d*x^n)})/(b + \sqrt{-a^2 + b^2}]])]))/(-a^2 + b^2)^{(3/2)} + (2*b^2*d*x^n*Sech[c]*(b*\sinh[c] - a*\sinh[d*x^n]))/((-a + b)*(a + b)) + (2*b^2*d*x^n*(b + a*\Cosh[c + d*x^n])*Tanh[c])/(-a^2 + b^2) + (d*x^n*(b + a*\Cosh[c + d*x^n]))*((a^2 - b^2)*d*x^n + 2*b^2*Tanh[c])/((a - b)*(a + b)))/((2*a^2*d^2*e*n*x^{(2*n)}*(a + b*Sech[c + d*x^n])^2)$

3.83.  $\int \frac{(ex)^{-1+2n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$

### 3.83.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 563, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {5963, 5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(ex)^{2n-1}}{(a + b\operatorname{sech}(c + dx^n))^2} dx \\
 & \quad \downarrow \text{5963} \\
 & \frac{x^{-2n}(ex)^{2n} \int \frac{x^{2n-1}}{(a+b\operatorname{sech}(dx^n+c))^2} dx}{e} \\
 & \quad \downarrow \text{5959} \\
 & \frac{x^{-2n}(ex)^{2n} \int \frac{x^n}{(a+b\operatorname{sech}(dx^n+c))^2} dx^n}{en} \\
 & \quad \downarrow \text{3042} \\
 & \frac{x^{-2n}(ex)^{2n} \int \frac{x^n}{(a+b\csc(idx^n+ic+\frac{\pi}{2}))^2} dx^n}{en} \\
 & \quad \downarrow \text{4679} \\
 & \frac{x^{-2n}(ex)^{2n} \int \left( -\frac{2bx^n}{a^2(b+a\cosh(dx^n+c))} + \frac{x^n}{a^2} + \frac{b^2x^n}{a^2(b+a\cosh(dx^n+c))^2} \right) dx^n}{en} \\
 & \quad \downarrow \text{2009} \\
 & \frac{x^{-2n}(ex)^{2n} \left( -\frac{2b\operatorname{PolyLog}\left(2, -\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}} + \frac{2b\operatorname{PolyLog}\left(2, -\frac{ae^{dx^n+c}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}} - \frac{b^2\log(a\cosh(c+dx^n)+b)}{a^2d^2(a^2-b^2)} - \frac{2bx^n\log\left(\frac{ae^{c+dx^n}}{b-\sqrt{b^2-a^2}}+1\right)}{a^2d\sqrt{b^2-a^2}} + \right)}{en}
 \end{aligned}$$

input `Int[(e*x)^(-1 + 2*n)/(a + b*Sech[c + d*x^n])^2, x]`

---

3.83.  $\int \frac{(ex)^{-1+2n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$

```
output ((e*x)^(2*n)*(x^(2*n)/(2*a^2) + (b^3*x^n*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]])]/(a^2*(-a^2 + b^2)^(3/2)*d) - (2*b*x^n*Log[1 + (a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]])]/(a^2*Sqrt[-a^2 + b^2]*d) - (b^3*x^n*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]])]/(a^2*(-a^2 + b^2)^(3/2)*d) + (2*b*x^n*Log[1 + (a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]])]/(a^2*Sqrt[-a^2 + b^2]*d) - (b^2*Log[b + a*Cosh[c + d*x^n]])/(a^2*(a^2 - b^2)*d^2) + (b^3*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (2*b*PolyLog[2, -((a*E^(c + d*x^n))/(b - Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^2) - (b^3*PolyLog[2, -((a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]))]/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (2*b*PolyLog[2, -((a*E^(c + d*x^n))/(b + Sqrt[-a^2 + b^2]))]/(a^2*Sqrt[-a^2 + b^2]*d^2) + (b^2*x^n*Sinh[c + d*x^n])/(a*(a^2 - b^2)*d*(b + a*Cosh[c + d*x^n])))/(e*n*x^(2*n))
```

### 3.83.3.1 Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 3042 Int[u_, x_Symbol] := Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinearQ[u, x]
```

```
rule 4679 Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGtQ[m, 0]
```

```
rule 5959 Int[(x_)^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[1/n Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sech[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m + 1)/n], 0] && IntegerQ[p]
```

```
rule 5963 Int[((e_)*(x_))^(m_.)*((a_.) + (b_.)*Sech[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] := Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*(a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

**3.83.4 Maple [F]**

$$\int \frac{(ex)^{2n-1}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

input `int((e*x)^(2*n-1)/(a+b*sech(c+d*x^n))^2,x)`

output `int((e*x)^(2*n-1)/(a+b*sech(c+d*x^n))^2,x)`

**3.83.5 Fricas [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 9020 vs.  $2(681) = 1362$ .

Time = 0.42 (sec) , antiderivative size = 9020, normalized size of antiderivative = 12.58

$$\int \frac{(ex)^{-1+2n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \text{Too large to display}$$

input `integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")`

output `Too large to include`

**3.83.6 Sympy [F]**

$$\int \frac{(ex)^{-1+2n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{2n-1}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

input `integrate((e*x)**(-1+2*n)/(a+b*sech(c+d*x**n))**2,x)`

output `Integral((e*x)**(2*n - 1)/(a + b*sech(c + d*x**n))**2, x)`

**3.83.7 Maxima [F]**

$$\int \frac{(ex)^{-1+2n}}{(a + b\operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{2n-1}}{(b\operatorname{sech}(dx^n + c) + a)^2} dx$$

input `integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")`

output `-1/2*(4*a*b^2*e^(2*n)*x^n - (a^3*d*e^(2*n) - a*b^2*d*e^(2*n))*x^(2*n) - (a^3*d*e^(2*n)*e^(2*c) - a*b^2*d*e^(2*n)*e^(2*c))*e^(2*d*x^n + 2*n*log(x)) + 2*(2*b^3*e^(2*n)*e^(n*log(x) + c) - (a^2*b*d*e^(2*n)*e^c - b^3*d*e^(2*n)*e^c)*x^(2*n))*e^(d*x^n))/(a^5*d*e*n - a^3*b^2*d*e*n + (a^5*d*e*n*e^(2*c) - a^3*b^2*d*e*n*e^(2*c))*e^(2*d*x^n) + 2*(a^4*b*d*e*n*e^c - a^2*b^3*d*e*n*e^c)*e^(d*x^n)) - integrate(-2*(a*b^2*e^(2*n)*x^n + (b^3*e^(2*n)*e^(n*log(x) + c) - (2*a^2*b*d*e^(2*n)*e^c - b^3*d*e^(2*n)*e^c)*x^(2*n))*e^(d*x^n))/(a^5*d*e*n*e^(2*c) - a^3*b^2*d*e*n*e^(2*c))*x*e^(2*d*x^n) + 2*(a^4*b*d*e*n*e^c - a^2*b^3*d*e*n*e^c)*x*e^(d*x^n) + (a^5*d*e - a^3*b^2*d*e)*x), x)`

**3.83.8 Giac [F]**

$$\int \frac{(ex)^{-1+2n}}{(a + b\operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{2n-1}}{(b\operatorname{sech}(dx^n + c) + a)^2} dx$$

input `integrate((e*x)^(-1+2*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="giac")`

output `integrate((e*x)^(2*n - 1)/(b*sech(d*x^n + c) + a)^2, x)`

**3.83.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{(ex)^{-1+2n}}{(a + b\operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{2n-1}}{\left(a + \frac{b}{\cosh(c+dx^n)}\right)^2} dx$$

input `int((e*x)^(2*n - 1)/(a + b/cosh(c + d*x^n))^2,x)`

output `int((e*x)^(2*n - 1)/(a + b/cosh(c + d*x^n))^2, x)`

---

3.83.  $\int \frac{(ex)^{-1+2n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$

$$3.84 \quad \int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$$

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---


$$3.84. \quad \int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$$

## 3.84.1 Optimal result

Integrand size = 24, antiderivative size = 1284

$$\begin{aligned}
\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx &= \frac{(ex)^{3n}}{3a^2en} + \frac{b^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)den} \\
&\quad - \frac{2b^2x^{-2n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2en} \\
&\quad + \frac{b^3x^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} \\
&\quad - \frac{2bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} \\
&\quad - \frac{2b^2x^{-2n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^2en} \\
&\quad - \frac{b^3x^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} \\
&\quad + \frac{2bx^{-n}(ex)^{3n} \log\left(1 + \frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} \\
&\quad - \frac{2b^2x^{-3n}(ex)^{3n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3en} \\
&\quad + \frac{2b^3x^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2en} \\
&\quad - \frac{4bx^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} \\
&\quad - \frac{2b^2x^{-3n}(ex)^{3n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(a^2-b^2)d^3en} \\
&\quad - \frac{2b^3x^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2en} \\
&\quad + \frac{4bx^{-2n}(ex)^{3n} \operatorname{PolyLog}\left(2, -\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} \\
&\quad - \frac{2b^3x^{-3n}(ex)^{3n} \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3en} \\
&\quad + \frac{4bx^{-3n}(ex)^{3n} \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^n}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en} \\
&\quad - \frac{2b^3x^{-3n}(ex)^{3n} \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3en} \\
&\quad + \frac{4bx^{-3n}(ex)^{3n} \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en} \\
3.84. \int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx &+ \frac{2b^3x^{-3n}(ex)^{3n} \operatorname{PolyLog}\left(3, -\frac{ae^{c+dx^n}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3en}
\end{aligned}$$



output  $\frac{1}{3} \frac{(ex)^{3n}}{a^2 e/n + b^2 (ex)^{3n} / a^2 / (a^2 - b^2) / d / e/n / (x^n)^{-2} b^2 (ex)^{3n} \ln(1 + a \exp(c + dx^n) / (b - (a^2 + b^2)^{1/2}))} / a^2 / (a^2 - b^2) / d^2 / e/n / (x^{2n}) + b^3 (ex)^{3n} \ln(1 + a \exp(c + dx^n) / (b - (a^2 + b^2)^{1/2}))} / a^2 / (-a^2 + b^2)^{3/2} / d / e/n / (x^n)^{-2} b^2 (ex)^{3n} \ln(1 + a \exp(c + dx^n) / (b + (a^2 + b^2)^{1/2}))} / a^2 / (a^2 - b^2) / d^2 / e/n / (x^{2n}) - b^3 (ex)^{3n} \ln(1 + a \exp(c + dx^n) / (b + (a^2 + b^2)^{1/2}))} / a^2 / (-a^2 + b^2)^{3/2} / d / e/n / (x^n)^{-2} b^2 (ex)^{3n} \operatorname{polylog}(2, -a \exp(c + dx^n) / (b - (a^2 + b^2)^{1/2}))} / a^2 / (a^2 - b^2) / d^3 / e/n / (x^{3n}) + 2 b^3 (ex)^{3n} \operatorname{polylog}(2, -a \exp(c + dx^n) / (b - (a^2 + b^2)^{1/2}))} / a^2 / (-a^2 + b^2)^{3/2} / d^2 / e/n / (x^{2n}) - 2 b^2 (ex)^{3n} \operatorname{polylog}(2, -a \exp(c + dx^n) / (b + (a^2 + b^2)^{1/2}))} / a^2 / (a^2 - b^2) / d^3 / e/n / (x^{3n}) - 2 b^3 (ex)^{3n} \operatorname{polylog}(2, -a \exp(c + dx^n) / (b + (a^2 + b^2)^{1/2}))} / a^2 / (-a^2 + b^2)^{3/2} / d^2 / e/n / (x^{2n}) - 2 b^3 (ex)^{3n} \operatorname{polylog}(3, -a \exp(c + dx^n) / (b - (a^2 + b^2)^{1/2}))} / a^2 / (-a^2 + b^2)^{3/2} / d^3 / e/n / (x^{3n}) + 2 b^3 (ex)^{3n} \operatorname{polylog}(3, -a \exp(c + dx^n) / (b + (a^2 + b^2)^{1/2}))} / a^2 / (-a^2 + b^2)^{3/2} / d^3 / e/n / (x^{3n}) + b^2 (ex)^{3n} \sinh(c + dx^n) / a / (a^2 - b^2) / d / e/n / (x^n) / (b + a \cosh(c + dx^n)) - 2 b (ex)^{3n} \ln(1 + a \exp(c + dx^n) / (b - (a^2 + b^2)^{1/2}))} / a^2 / d / e/n / (x^n) / (-a^2 + b^2)^{1/2} + 2 b (ex)^{3n} \ln(1 + a \exp(c + dx^n) / (b + (a^2 + b^2)^{1/2}))} / a^2 / d / e/n / (x^n) / (-a^2 + b^2)^{1/2} - 4 b (ex)^{3n} \operatorname{polylog}(2, -a \exp(c + dx^n) / (b - (a^2 + b^2)^{1/2}))} / a^2 / d^2 / e/n / (x^{2n}) / (-a^2 + b^2)^{1/2} + 4 b (ex)^{3n} \operatorname{polylog}(2, -a \exp(c + dx^n) / (b + (a^2 + b^2)^{1/2}))} / a^2 / d^2 \dots$

### 3.84.2 Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{-1+3n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

input `Integrate[(ex)^(-1 + 3n)/(a + b*Sech[c + d*x^n])^2,x]`

output `Integrate[(ex)^(-1 + 3n)/(a + b*Sech[c + d*x^n])^2, x]`

### 3.84.3 Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 1019, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {5963, 5959, 3042, 4679, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{(ex)^{3n-1}}{(a + b\operatorname{sech}(c + dx^n))^2} dx \\
 & \quad \downarrow \text{5963} \\
 & \frac{x^{-3n}(ex)^{3n} \int \frac{x^{3n-1}}{(a+b\operatorname{sech}(dx^n+c))^2} dx}{e} \\
 & \quad \downarrow \text{5959} \\
 & \frac{x^{-3n}(ex)^{3n} \int \frac{x^{2n}}{(a+b\operatorname{sech}(dx^n+c))^2} dx^n}{en} \\
 & \quad \downarrow \text{3042} \\
 & \frac{x^{-3n}(ex)^{3n} \int \frac{x^{2n}}{(a+b\operatorname{csc}(idx^n+ic+\frac{\pi}{2}))^2} dx^n}{en} \\
 & \quad \downarrow \text{4679} \\
 & \frac{x^{-3n}(ex)^{3n} \int \left( -\frac{2bx^{2n}}{a^2(b+a\cosh(dx^n+c))} + \frac{x^{2n}}{a^2} + \frac{b^2x^{2n}}{a^2(b+a\cosh(dx^n+c))^2} \right) dx^n}{en} \\
 & \quad \downarrow \text{2009} \\
 & x^{-3n}(ex)^{3n} \left( -\frac{2b^2 \log\left(\frac{e^{dx^n+c}+a}{b-\sqrt{b^2-a^2}}+1\right)x^n}{a^2(a^2-b^2)d^2} - \frac{2b^2 \log\left(\frac{e^{dx^n+c}-a}{b+\sqrt{b^2-a^2}}+1\right)x^n}{a^2(a^2-b^2)d^2} - \frac{4b \operatorname{PolyLog}\left(2, -\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^n}{a^2\sqrt{b^2-a^2}d^2} + \frac{2b^3 \operatorname{PolyLog}\left(2, -\frac{ae^{dx^n+c}}{b-\sqrt{b^2-a^2}}\right)x^n}{a^2(b^2-a^2)^{3/2}d^2} \right)
 \end{aligned}$$

input `Int[(e*x)^(-1 + 3*n)/(a + b*Sech[c + d*x^n])^2, x]`

---

3.84.  $\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$



rule 5963 `Int[((e_)*(x_))^(m_)*((a_) + (b_)*Sech[(c_) + (d_)*(x_)^(n_)])^(p_),  
x_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]) Int[x^m*  
(a + b*Sech[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]`

### 3.84.4 Maple [F]

$$\int \frac{(ex)^{-1+3n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

input `int((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n))^2,x)`

output `int((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n))^2,x)`

### 3.84.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \text{Timed out}$$

input `integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="fricas")`

output `Timed out`

### 3.84.6 Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{3n-1}}{(a + b \operatorname{sech}(c + dx^n))^2} dx$$

input `integrate((e*x)**(-1+3*n)/(a+b*sech(c+d*x**n))**2,x)`

output `Integral((e*x)**(3*n - 1)/(a + b*sech(c + d*x**n))**2, x)`

**3.84.7 Maxima [F]**

$$\int \frac{(ex)^{-1+3n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{3n-1}}{(b \operatorname{sech}(dx^n + c) + a)^2} dx$$

input `integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="maxima")`

output `-1/3*(6*a*b^2*e^(3*n)*x^(2*n) - (a^3*d*e^(3*n) - a*b^2*d*e^(3*n))*x^(3*n) - (a^3*d*e^(3*n)*e^(2*c) - a*b^2*d*e^(3*n)*e^(2*c))*e^(2*d*x^n + 3*n*log(x)) + 2*(3*b^3*e^(3*n)*e^(2*n*log(x) + c) - (a^2*b*d*e^(3*n)*e^c - b^3*d*e^(3*n)*e^c)*x^(3*n))*e^(d*x^n))/(a^5*d*e*n - a^3*b^2*d*e*n + (a^5*d*e*n*e^(2*c) - a^3*b^2*d*e*n*e^(2*c))*e^(2*d*x^n) + 2*(a^4*b*d*e*n*e^c - a^2*b^3*d*e*n*e^c)*e^(d*x^n)) - integrate(-2*(2*a*b^2*e^(3*n)*x^(2*n) + (2*b^3*e^(3*n)*e^(2*n*log(x) + c) - (2*a^2*b*d*e^(3*n)*e^c - b^3*d*e^(3*n)*e^c)*x^(3*n))*e^(d*x^n))/((a^5*d*e*e^(2*c) - a^3*b^2*d*e*e^(2*c))*x*e^(2*d*x^n) + 2*(a^4*b*d*e*e^c - a^2*b^3*d*e*e^c)*x*e^(d*x^n) + (a^5*d*e - a^3*b^2*d*e)*x), x)`

**3.84.8 Giac [F]**

$$\int \frac{(ex)^{-1+3n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{3n-1}}{(b \operatorname{sech}(dx^n + c) + a)^2} dx$$

input `integrate((e*x)^(-1+3*n)/(a+b*sech(c+d*x^n))^2,x, algorithm="giac")`

output `integrate((e*x)^(3*n - 1)/(b*sech(d*x^n + c) + a)^2, x)`

**3.84.9 Mupad [F(-1)]**

Timed out.

$$\int \frac{(ex)^{-1+3n}}{(a + b \operatorname{sech}(c + dx^n))^2} dx = \int \frac{(ex)^{3n-1}}{\left(a + \frac{b}{\cosh(c+dx^n)}\right)^2} dx$$

input `int((e*x)^(3*n - 1)/(a + b/cosh(c + d*x^n))^2,x)`

output `int((e*x)^(3*n - 1)/(a + b/cosh(c + d*x^n))^2, x)`

---

3.84.  $\int \frac{(ex)^{-1+3n}}{(a+b\operatorname{sech}(c+dx^n))^2} dx$

## APPENDIX

4.1 Listing of Grading functions . . . . .	493
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## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*           Small rewrite of logic in main function to make it*)
(*           match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
```

```

(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A"," "}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
        ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
      ,(*ELSE*)(*result does not contains complex*)
      If[leafCountResult<=2*leafCountOptimal,
        finalresult={"A"," "}
        ,(*ELSE*)
        finalresult={"B","Leaf count is larger than twice the leaf count of optimal.$
      ]
    ]
    ,(*ELSE*)(*expnResult>expnOptimal*)
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      finalresult={"C","Result contains higher order function than in optimal. Order "<
      ,
      finalresult={"F","Contains unresolved integral."}
    ]
  ];

  finalresult
]

```

```

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType,expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]],2]],
            Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
            If[ElementaryFunctionQ[Head[expn]],
              Max[3,ExpnType[expn[[1]]],
            If[SpecialFunctionQ[Head[expn]],
              Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
            If[HypergeometricFunctionQ[Head[expn]],
              Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
            If[AppellFunctionQ[Head[expn]],
              Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
            If[Head[expn]===RootSum,
              Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
            If[Head[expn]===Integrate || Head[expn]===Int,
              Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
            9]]]]]]]]]]

```



```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1}, func]

```

### 4.1.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B","result has leaf size over 500,000. Avoiding possible recursion issues";
      fi;

      leaf_count_optimal := leafcount(optimal);
      ExpnType_result := ExpnType(result);
      ExpnType_optimal := ExpnType(optimal);

      if debug then
          print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

```

```

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;
  if is_contains_complex(result) then
    if is_contains_complex(optimal) then
      if debug then
        print("both result and optimal complex");
      fi;
      if leaf_count_result<=2*leaf_count_optimal then
        return "A"," ";
      else
        return "B",cat("Both result and optimal contain complex but leaf count of
                        convert(leaf_count_result,string)," vs. $2 (" ,
                        convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
        end if
      else #result contains complex but optimal is not
        if debug then
          print("result contains complex but optimal is not");
        fi;
        return "C","Result contains complex when optimal does not.";
      fi;
    else # result do not contain complex
      # this assumes optimal do not as well. No check is needed here.
      if debug then
        print("result do not contain complex, this assumes optimal do not as well"
      fi;
      if leaf_count_result<=2*leaf_count_optimal then
        if debug then
          print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A"," ";
      else
        if debug then
          print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of o
                        convert(leaf_count_result,string),"$ vs. $2(",
                        convert(leaf_count_optimal,string),"=" ,convert(2*leaf_cou
        fi;
      fi;
    fi;
  fi;

```

```

else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C",cat("Result contains higher order function than in optimal. Order ",
    convert(ExpnType_result,string)," vs. order ",
    convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  end if
end proc:

```

```

elif type(expn, ``~`) then
  if type(op(2,expn), 'integer') then
    ExpnType(op(1,expn))
  elif type(op(2,expn), 'rational') then
    if type(op(1,expn), 'rational') then
      1
    else
      max(2, ExpnType(op(1,expn)))
    end if
  else
    max(3, ExpnType(op(1,expn)), ExpnType(op(2,expn)))
  end if
elif type(expn, ``+`) or type(expn, ``*`) then
  max(ExpnType(op(1,expn)), max(ExpnType(rest(expn))))
elif ElementaryFunctionQ(op(0,expn)) then
  max(3, ExpnType(op(1,expn)))
elif SpecialFunctionQ(op(0,expn)) then
  max(4, apply(max, map(ExpnType, [op(expn)])))
elif HypergeometricFunctionQ(op(0,expn)) then
  max(5, apply(max, map(ExpnType, [op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6, apply(max, map(ExpnType, [op(expn)])))
elif op(0,expn)='int' then
  max(8, apply(max, map(ExpnType, [op(expn)]))) else
  9
end if
end proc:

```

```

ElementaryFunctionQ := proc(func)
  member(func, [
    exp, log, ln,
    sin, cos, tan, cot, sec, csc,
    arcsin, arccos, arctan, arccot, arcsec, arccsc,
    sinh, cosh, tanh, coth, sech, csch,
    arcsinh, arccosh, arctanh, arccoth, arcsech, arccsch])
end proc:

```

```

SpecialFunctionQ := proc(func)
  member(func, [
    erf, erfc, erfi,
    FresnelS, FresnelC,
    Ei, Ei, Li, Si, Ci, Shi, Chi,

```

```

        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
    member(func, [Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
    member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
    if nops(u)=2 then
        op(2,u)
    else
        apply(op(0,u),op(2..nops(u),u))
    end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma] [LeafCount] (u);
end proc:

```

### 4.1.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#          Port of original Maple grading function by
#          Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#          added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):

```

```
if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
        return True
    else:
        return False
else:
    return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```

return 1
elif isinstance(expn,list):
    return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
elif is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
    else:
        return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
elif isinstance(expn,Pow): #type(expn,``^`)
    if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
        return expnType(expn.args[0]) #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or type(expn,``*`)
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]]
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

*#main function*



```

def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is large"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
        else:
            grade = "C"
            grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

```

```
return grade, grade_annotation
```

#### 4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#      Albert Rich to use with Sagemath. This is used to
#      grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#      'arctan2', 'floor', 'abs', 'log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#      issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```

        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

```

```

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric', 'hypergeometric_M', 'hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']  #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=", expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-try:
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception, AttributeError in is_atom")
        print ("caught exception" , type(error).__name__ )
        return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:  #isinstance(expn,list):

```

```

    return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
elif is_sqrt(expn):
    if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],Rational):
        return 1
    else:
        return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
elif expn.operator() == operator.pow: #instance(expn,Pow)
    if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
        return expnType(expn.operands()[0]) #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational: #instance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #instance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #instance(expn,Add) or inst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

```

```

if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)",type(result))
    print("type(optimal)",type(optimal))

leaf_count_result = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

expnType_result = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = " "
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation = "Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = " "
            else:
                grade = "B"
                grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal." + str(leaf
    else:
        grade = "C"
        grade_annotation = "Result contains higher order function than in optimal. Order " + str(expnType_resu

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

```

```
return grade, grade_annotation
```